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# THE

# PRACTICAL ARITHMETIC

## ORAL AND WRITTEN

BY

**FRANK H. HALL**

AUTHOR OF THE "WERNER ARITHMETICS"

NEW YORK ·:· CINCINNATI ·:· CHICAGO
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PRACTICAL ARITHMETIC.
E-P 1

# PREFACE

In accordance with the opinions of leading students of pedagogy and with the courses of study in the best schools of this country, the first two books of this series are arranged on the spiral plan, and the third book, for the seventh and eighth grades, on the topical plan.

The Werner Arithmetic, Book III, of which this is a revision, was a combination of the spiral and the topical methods, with emphasis on the latter. Further experience and observation lead the author to a strictly topical arrangement of the subjects in this book.

No effort whatever has been made to follow the lead of some recently published arithmetics and make the book an encyclopedia of geographical, historical, and statistical number facts. It is a pedagogical axiom that the concrete illustrations of number relations that are most helpful in giving the pupil a clear conception of the facts and processes of arithmetic are those with which the student is already somewhat familiar or those with which he feels the need of becoming familiar.

The correlation of the work in arithmetic with the work in geography and history is often desirable and helpful; but this should be incidental — should be brought to the attention of the pupil in connection with the quantitative incidents of geography and history, and not forced upon him in the arithmetic class.

The very best problems that can be given to the pupil to illustrate and enforce mathematical principles are those which may be selected by the teacher from the immediate surroundings of the student. The best problems with which to teach arithmetic from a book on that subject are those which are common to the environment of the largest numbers of pupils who will use the book.

Gradation is the keynote of the entire series. In the primary and the intermediate arithmetic everything is subordinated to the idea of a step-by-step advancement. The practical arithmetic makes classification the keynote, and puts great emphasis upon accuracy in the mechanical processes employed in the solution of the simple, everyday problems of the home, the shop, the store, and the farm.

FRANK H. HALL.

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# PRACTICAL ARITHMETIC

## NOTATION AND NUMERATION

### NOTATION

NOTE. — The definition of notation and other terms with which the pupil has become familiar while in the grades below, will be found on pages 337–350.

Write in figures:

1. One hundred seventy-five million.
2. Three hundred twenty-four thousand.
3. Five hundred seventy-eight.
4. Eight hundred twenty-four million six hundred seventy-five thousand four hundred twenty-two.

*a*. Find the sum of the above numbers.

Write in figures, using the decimal point:

5. Two hundred seventy-five millionths.
6. Two hundred seventy-five hundred-thousandths.
7. Two hundred seventy-five ten-thousandths.
8. Two hundred seventy-five thousandths.

*b*. Find the sum of the above numbers.

Write in figures, using the decimal point:

9. One hundred forty-six and three tenths.
10. Two hundred fifty-four and two hundredths.
11. Six hundred ninety-six and five thousandths.

*c*. Find the sum of the above numbers.

Write in figures:

**1.** Four million forty thousand four.

**2.** Forty million four hundred thousand forty.

**3.** Four hundred million four thousand four hundred.

*a.* Find the sum of the above numbers.

Write in figures, using the decimal point:

**4.** Two hundred and eight thousandths.*

**5.** Two hundred eight thousandths.

**6.** Six hundred and fifteen thousandths.

**7.** Six hundred fifteen thousandths.

*b.* Find the sum of the above numbers.

Write in figures, using the decimal point:

**8.** Four hundred seventy-eight tenths.†

**9.** Seven hundred forty-five hundredths.

**10.** Two thousand one hundred thirty-five thousandths.

*c.* Find the sum of the above numbers.

Write in figures, using the decimal point when necessary:

**11.** Two hundred fifty-four thousand one hundred sixty.

**12.** One hundred seventy-five and two hundred six thousandths.

*d.* Find the sum of the above numbers.

---

* In expressions like this, the word *and* may be thought of as standing for the decimal point in the figure notation.

† After writing the number, read it in the usual way — forty-seven and eight tenths.

Read each of the following in the usual way, using the word *and* only in place of the decimal point. 546.85 is five hundred forty-six *and* eighty-five hundredths.

**1.** 546.85    **2.** 453.15    **3.** 729.43
**4.** 2746.2    **5.** 7253.8    **6.** 2040.8
**7.** 24.006    **8.** 75.944    **9.** 37.055

*a.* Copy the above numbers, arranging them with reference to convenience in adding, and find their sum.

Observe that any mixed decimal may be read as though it were integral by giving the name of the units denoted by the right-hand figure to the entire number; thus, 21.8 is 218 tenths; 5.36 is 536 hundredths; 24.305 is 24305 thousandths.

Read each of the following as though it were integral, giving the name of the units denoted by the right-hand figure to the entire number:

**10.** 27.5    **11.** 2.75    **12.** 6.275
**13.** 72.5    **14.** 7.25    **15.** 1.725
**16.** 52.3    **17.** 5.23    **18.** 5.523

*b.* Copy the above numbers, and find their sum.

Observe that any part of the consecutive figures of a number may be read by giving the name of the units denoted by the last figure of the part, to the entire part, thus: 24.65 may be thought of and read as 246 tenths and 5 hundredths; 14.275 as 142 tenths and 75 thousandths, or as 1427 hundredths and 5 thousandths.

In a similar manner read each of the following:

**19.** 3.76    **20.** 32.46    **21.** 6.425
**22.** 4.25    **23.** 63.37    **24.** 1.346
**25.** 6.25    **26.** 67.54    **27.** 3.575

*c.* Copy the above numbers, and find their sum.

Observe that in reading such numbers as the following it is only by the use or the omission of the word *and* that one can distinguish between the number (or numbers) on the left and the number in the same line on the right.

Read the following:

**1.** 200.006 .206
**2.** 400.0005 .0405
**3.** 500.025 .525
**4.** 800 and 24 824.

*a.* Copy the above numbers, arranging them with reference to convenience in adding, and find their sum.

Read the following by telling the number of units in each order; thus, 346.25 is 3 hundreds, 4 tens, 6 primary units, 2 tenths, and 5 hundredths.

**5.** 467.34 **6.** 24.176 **7.** 3.2461
**8.** 147.21 **9.** 30.108 **10.** 2.3046

*b.* Copy the above numbers and find their sum.

Observe that the units of any one order may be thought of as an equal number of tenths of one of the next higher order; thus, in the number 324.576, the 6 thousandths are 6 tenths of a hundredth; the 7 hundredths are 7 tenths of a tenth; the 5 tenths are 5 tenths of one primary unit; the 4 units are 4 tenths of a ten; the 2 hundreds are 2 tenths of a thousand.

Again, the units of any one order may be thought of as ten times as many units of the next lower order; thus, in the number 253.475, the 2 hundreds are 20 tens; the 5 tens are 50 primary units; the 3 primary units are 30 tenths; the 7 hundredths are 70 thousandths.

Tell the value of some of the units of the different orders in the following numbers in units of the next higher and the next lower orders:

**11.** 345.678 **12.** 453.876 **13.** 268.754

# ADDITION

**Note.** For definitions of terms see page 339.

## Examples of Addition

| *a* | *b* | *c* |
|---|---|---|
| 3754 acres | 37.426 | $24.305 |
| 2662 acres | 1.48 | 6.752 |
| 1457 acres | 375.062 | 375.08 |
| 3214 acres | 46.3 | 152.6 |
| 11087 acres | 460.268 | $558.737 |

**1.** In each of the above examples, tell which are the addends, and which is the sum.

In written problems in addition the figures that stand for units of the same order are usually written in the same column.

**2.** In example *b* what figures represent units of the second decimal order? Of the third decimal order?

**3.** In example *c* what figures represent units of the first integral order? Of the second integral order?

*Only like numbers can be added.*

*The denomination of the sum is the same as that of the addends.*

There are forty-five **primary facts of addition.** These are given in full on page 340. The nine primary facts often requiring special drill for their complete mastery are given below.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 7 | 8 | 9 | 8 | 9 | 8 | 9 | 9 | 9 |
| 6 | 5 | 4 | 6 | 5 | 7 | 6 | 7 | 8 |
| 13 | 13 | 13 | 14 | 14 | 15 | 15 | 16 | 17 |

Copy the following and *check*. Then add at least twice (upward and downward) before showing the answers to the teacher.

*a.* Add 3465, 4268, 3279, 6534, 5731.
*b.* Add 5732, 6721, 3466, 4269, 6535.
*c.* Add 2768, 5329, 4685, 3752, 8467.
*d.* Add 4671, 5315, 6248, 1533, 7232.

| *e* | *f* | *g* | *h* |
|---|---|---|---|
| 375 | 436 | 625 | 564 |
| 506 | 307 | 494 | 693 |
| 258 | 316 | 742 | 684 |
| 327 | 498 | 673 | 502 |
| 580 | 736 | 420 | 264 |

*i.* Find the sum of the eight sums, *a* to *h*.

Add the following. To facilitate the reëxamination of any part of the work, write the footing of each column by itself; then find the sum of the footings as indicated in the example.

| *j* | *k* | *l* | *m* |
|---|---|---|---|
| 9756 | 3268 | 16732 | 483268 |
| 3842 | 4709 | 25291 | 374709 |
| 1596 | 1076 | 38924 | 261076 |
| 3874 | 3462 | 46538 | 153462 |
| 18 | 1586 | 18414 | 481586 |
| 25 | 2735 | 27265 | 372735 |
| 28 | 3406 | 36594 | 263406 |
| 16 | 1590 | 48410 | 151590 |
| 19068 | | | |

*n.* Find the sum of the three sums, *k* to *m*.

The addition of decimals is in no way different from the addition of simple numbers; but care must be exercised in writing the numbers preparatory to adding them. The decimal points in the other addends and in the sum should be directly under the one first written. This, however, is simply a matter of convenience. It is quite possible to add either simple numbers or decimals wherever they may be written; but to guard against error and to make the work less burdensome they should usually be so written that the units of the same order will fall in the same column.

Copy the following and *check*. Then add at least twice before showing the answers to your teacher:

*a*. Add 274.36, 21.37, 38.007, and .275.

*b*. Add 78.63, 61.993, .725, and 724.64.

*c*. Add .7, .84, .375, .0275, and .25326.

*d*. Add .16, .625, .9725, .74674, and .3.

*e*. Add 46.07, 14.003, 52.0006, and 28.

*f*. Add 85.997, 47.9994, 72, and 53.93.

*g*. Find the sum of two hundred and six thousandths, and two hundred six thousandths.

*h*. Find the sum of seven hundred ninety-eight and six hundredths, and nine hundred ninety-four thousandths.

*i*. Find the sum of eight hundred and forty-six thousandths, and eight hundred forty-six thousandths.

*j*. Find the sum of eighty-four and five tenths, and seventy-five and six hundredths.

*k*. Find the sum of the ten sums, *a* to *j*.*

---

* *To the Pupil.* Can you solve these ten problems and find the sum of the ten sums on first trial without an error?

Add the following:

To facilitate the reëxamination of any part of the work, as well as to enable the accountant to carry on such computations in the midst of interruptions, the footing of each column may be written by itself as suggested on page 14, or it may be written as given in the example below.

| *a* |  | *b* | *c* | *d* |
|---:|---|---:|---:|---:|
| $324.45 |  | $256.35 | $275 | $733.65 |
| 28.47 |  | 145.24 | 146 | 854.76 |
| 375.28 |  | 321.75 | 281 | 678.25 |
| 6.94 |  | 286.44 | 675 | 713.56 |
| 175.89 |  | 156.52 | 284 | 843.18 |
| 22.56 | 64* | 244.31 | 725 | 755.69 |
| 475.39 | 50 | 986.24 | 854 | 13.76 |
| 302.21 | 47 | 275.46 | 719 | 724.54 |
| 354.48 | 37 | 383.27 | 325 | 616.73 |
| 411.37 | 24 | 152.10 | 716 | 847.90 |
| $2477.04 |  |  |  |  |

*e.* Find the sum of the three sums, *b* to *d*.

Write the following and *check*. Then add:

*f.* Three hundred thirty-six dollars and twenty-five cents. Two hundred five dollars and seven cents. Forty-three dollars and thirty-six cents.

---

* 64 is the sum of the first or right hand column. 50 is the sum of the second column including the 6 carried from the footing of the first column. 47 is the sum of the third column including the 5 carried from the footing of the second column, etc. Hence the figures of the answer to the problem are made up of the figures of the footing of the last column (24) to which must be annexed in reverse order the right-hand figures of each of the other footings; thus, 2 4 7 7 0 4. The decimal point and the dollar sign must be written in their proper places, $2477.04.

There is no process in arithmetic more important than addition of simple numbers, decimals, and United States money. The one thing especially desirable at this stage of the instruction is that the pupil should perfect himself in this work. No pains should be spared in the acquisition of the power to add accurately. In the mechanical processes of arithmetic nothing short of accuracy is commendable. One figure wrong in one problem in ten is failure. The young man or young woman who cannot make figures with such care and apply such tests to his results as to insure accuracy, is worthless as an accountant.

Add the following by column and by line:

| | *a* | *b* | *c* | *d* | *e* | *f* |
|---|---|---|---|---|---|---|
| *g.* | $34.25 | $146.10 | $321.94 | $7.36 | $31.37 | $4.20 |
| *h.* | 55.32 | 241.34 | 465.74 | 3.27 | 52.30 | 5.20 |
| *i.* | 65.25 | 853.90 | 678.06 | 2.64 | 68.63 | 5.80 |
| *j.* | 44.68 | 758.66 | 524.26 | 6.73 | 47.70 | 4.80 |
| *k.* | 37.50 | 254.90 | 175.32 | 8.21 | 35.27 | 8.05 |
| *l.* | 14.64 | 541.39 | 371.46 | 5.55 | 51.15 | 6.84 |
| *m.* | 73.25 | 627.31 | 808.82 | 7.46 | 38.94 | 5.47 |
| *n.* | 62.50 | 745.10 | 824.68 | 1.79 | 64.73 | 4.32 |
| *o.* | 85.36 | 458.61 | 628.54 | 4.45 | 54.32 | 1.95 |
| *p.* | 26.75 | 372.69 | 191.18 | 6.54 | 48.85 | 3.16 |
| *q.* | 58.42 | 164.94 | 765.43 | 2.54 | 61.06 | 4.53 |
| *r.* | 41.58 | 876.54 | 325.16 | 5.05 | 38.83 | 6.66 |
| *s.* | 98.76 | 833.06 | 674.84 | 4.95 | 61.17 | 3.34 |
| *t.* | 46.34 | 153.21 | 346.25 | 8.32 | 43.20 | 6.21 |

*u.* Find the sum of the sums of the columns, *a* to *f*.

*v.* Find the sum of the sums of the lines *g* to *t*.

The usual method of testing the work in addition is to add a second time, reversing the order — adding upward, then downward. While this is not an absolute proof of the correctness of the result, if the operations are performed with a good degree of care and the results are alike, there can be small chance for error.

## CASTING OUT THE 9'S

Another method of testing, sometimes employed, is known as proof by *casting out the 9's.* The following will make clear the principle upon which this is based: *The excess of 9's in the sum must exactly equal the excess of 9's in the addends; that is, if the addends are together a certain number of 9's and 4 over, then the sum must be a certain number of 9's and 4 over. If the excess, 4, is the same in each case, there can be little doubt as to the accuracy of the sum.*

But it is not necessary to divide each number by 9 to find the excess of 9's. In the number 8765, the 8000 is a certain number of 9's and 8 over; the 7 hundred is a certain number of 9's and 7 over; the 60 is a certain number of 9's and 6 over; the 5 is no 9's at all and 5 over. Then 8765 is a certain number of 9's and 8 + 7 + 6 + 5 over. But 8 + 7 + 6 + 5 = 26. 26 is a certain number of 9's and 8 over. So the excess of 9's in the number 8765 is 8; and the *excess of 9's in any number is equal to the excess of 9's in the sum of its digits.*

### EXAMPLE

| | | |
|---|---|---|
| 9824 | . . . . 9 out; 8 + 2, 9 out = 1; 1 + 4 = excess, 5 | excess, 4. |
| 6483 | . . . . 6 + 4, 9 out = 1; 1 + 8, out; excess, 3 | |
| 9762 | . . . . 9 out; 7 + 6, 9 out = 4; 4 + 2 = excess, 6 | |
| 8468 | . . . . 8 + 4, 9 out = 3; 3 + 6, out; excess, 8 | |
| 34,537 | . . . . 3 + 4 + 5, 9 out = 3; 3 + 3 + 7, 9 out = excess . . . 4. | |

Or,

| | |
|---|---|
| 9 + 8 + 2 + 4 = 23 | 2 + 3 = 5, excess. |
| 6 + 4 + 8 + 3 = 21 | 2 + 1 = 3, excess. |
| 9 + 7 + 6 + 2 = 24 | 2 + 4 = 6, excess. |
| etc. | etc. |

# SUBTRACTION

**Note.** For definitions of terms see page 340.

## Examples of Subtraction

| *a* | *b* | *c* |
|---|---|---|
| 2687 acres | 57.38 | $675.46 |
| 1298 acres | 28.146 | 282.75 |
| 1389 acres | 29.234 | $392.71 |

**1.** Tell which is the minuend, which the subtrahend, and which the difference, in each of the above:

Observe that in written problems in subtraction the subtrahend is usually written under the minuend, and the difference under the subtrahend; and that, as in addition, the units of the same order are written in the same column.

**2.** In example *b*, what figures represent units of the third decimal order? Of the second integral order? Of the first decimal order? Of the first integral order?

*Only like numbers can be subtracted.*

*The denomination of the difference is the same as that of the minuend and the subtrahend.*

There are eighty-one **primary facts of subtraction** — twice as many less 9, as there are primary facts of addition. See page 340.

The primary facts of subtraction should be learned while learning the forty-five primary addition facts. $8 + 5 = 13$; $13 - 8 = 5$, and $13 - 5 = 8$. The eighteen subtraction facts that require special drill are suggested by the nine addition facts given on page 13.

Copy the following and *check*. Then solve, and prove by one of the methods of proof given on page 24:

*a*. From 35642 subtract 12456.

*b*. From 87544 subtract 64358.

*c*. From 90070 subtract 13256.

*d*. From the sum of 8539, 2647, 3984, 1461, 7353, 6016, and 2364, subtract 22364.

*e*. From the sum of 1352, 3425, 2640, 3724, 6575, 7360, and 6276, subtract 21352.

*f*. From 8 thousand 1 hundred 64, subtract 3 thousand 2 hundred 75.

*g*. From 6 thousand 7 hundred 25, subtract 1 thousand 8 hundred 36.

*h*. From seven thousand four hundred sixty-five, subtract two thousand three hundred fifty-four.

*i*. Find the sum of the eight differences, *a* to *h*.*

It is sometimes convenient to find the difference of two numbers by observing how many must be *added* to the smaller number to give a sum equal to the larger number. Thus, to find the difference of 280 and 500, it may be observed that 20 added to 280 makes 300, and 200 added to 300 makes 500; hence the difference is 20 + 200, or 220.

1. The difference of 760 and 1000 is ——.
2. The difference of 1290 and 1500 is ——.
3. The difference of 430 and 710 is ——.
4. The difference of 325 and 500 is ——.
5. The difference of 548 and 900 is ——.

*j*. Find the sum of the five differences, 1 to 5.

---

* *To the Pupil.* Do not allow yourself to make one error. Find the eight differences and their sum accurately, *on first trial.*

**The subtraction of decimals is in no way different from the subtraction of simple numbers; but care must be exercised in writing the numbers preparatory to subtracting them. The decimal point in the subtrahend should be directly under the decimal point in the minuend. The decimal point in the difference should be written under the other decimal points.**

Copy the following and *check*. Then subtract, and prove by one of the methods given on page 24:

*a.* From 124.905 subtract 15.245.

*b.* From 306.002 subtract 27.36.

*c.* From 875.095 subtract 84.755.

*d.* From 693.998 subtract 72.64.

*e.* Find the sum of the four differences, *a* to *d*.

### MISCELLANEOUS PROBLEMS

1. The sum of two numbers is 12; one of the numbers is 5; the other number is ——.

*f.* The sum of two numbers is 3.7464; one of the numbers is 1.3521. What is the other number?

2. The difference of two numbers is 8; the less number is 11; the other number is ——.

*g.* The difference of two numbers is 2.3254; the less number is 7.6746. What is the other number?

3. The difference of two numbers is 7; the greater number is 19; the other number is ——.

*h.* The difference of two numbers is 2.3943; the greater number is 10. What is the less number?

4. The difference is 6; the subtrahend is 2; the minuend is ——.

$$\begin{array}{r} ? \\ 2 \\ \hline 6 \end{array}$$

Copy the following and *check*. Then subtract, and prove by one of the methods given on page 24:

*a.* From $1346.75 subtract $850.42.

*b.* From $3407.08 subtract $506.37.

*c.* From $8653.25 subtract $149.58.

*d.* From $6592.92 subtract $493.63.

*e.* From $5463.27 subtract $635.21.

*f.* From $4536.73 subtract $364.79.

*g.* Find the sum of the six differences, *a* to *f*.

In selling goods it is quite common to determine the amount of change due the customer by *adding* to the cost of the goods enough to make the sum equal the amount paid. Thus, if the goods cost $2.65 and the customer gives the salesman a five-dollar bill, the salesman puts down a dime and says—"75"; a quarter and says—"three dollars"; $2 and says—"five dollars." He has given the customer 10¢, 25¢, and $2.00, $2.35, the difference of $5 and $2.65.

1. A customer bought $7.25 worth of goods. He gave the salesman a ten-dollar bill. He should receive in change ——.

2. Bill, $11.40. Salesman received $15. Change?

3. Bill, $7.32. Salesman received $10. Change?

4. Bill, $16.70. Salesman received $20. Change?

5. Bill, $12.62. Salesman received $15. Change?

6. Bill, $2.55. Salesman received $20. Change?

7. Bill, $8.25. Salesman received $10. Change?

8. Bill, $11.30. Salesman received $15. Change?

*h.* Find the sum of the eight answers, 1 to 8.

## Bank Accounts

*a.* On the morning of Jan. 5, Mr. Albert Carter had to his credit in the bank $824.70. During the day he deposited $61.40, and "checked out" as follows: $69.50, $78.25, $81.66, and $93.77. How much had Mr. Carter to his credit in the bank at the close of the day?

| Debit | | Credit | |
|---|---|---|---|
| 69 | 50 | 824 | 70 |
| 78 | 25 | 61 | 40 |
| 81 | 66 | | |
| 93 | 77 | | |
| | | | |

Note. The pupil should copy the above lines and figures, solve the problem, and write his answer (in red ink, if convenient) as the balance in the debit column. He should then "foot" the two columns and write the sums (in black ink) in the proper places. The footings should be alike.

*b.* Mr. Smith's account for Jan. 5 was as follows:
Amount to his credit in the morning, $136.25.
Deposited during the day, $94.21 and $56.45.
Checked out, $11.15, $24.30, $53.25, and $15.05.
Find the balance of his account at the close of the day.

*c.* Mr. Harvey's account for Jan. 5 was as follows:
Amount to his credit in the morning, $55.20.
Deposited during the day, $250.
Checked out, $34.65. Balance?

*d.* Find the total amount to the credit of Carter, Smith, and Harvey on the morning of Jan. 5.

*e.* Find the sum of the deposits of Carter, Smith, and Harvey, Jan. 5.

The usual method of testing the work in subtraction is to add the subtrahend and the difference. If the sum is like the minuend, the work is supposed to be right.

## CASTING OUT THE 9's *

Another method of proof is sometimes employed known as proof by casting out the 9's. (See page 18.) The excess of 9's in the minuend should exactly equal the sum of the excess of 9's in the subtrahend and the difference together.

**1.**

9472 . 9 out; 4 + 7, 9 out = 2; 2 + 2 = excess, . . . . . . . . . . . 4
3791 . 3 + 7, 9 out = 1; 1 + 9, 9 out = 1; 1 + 1 = excess, 2 }
5681 . 5 + 6, 9 out = 2; 2 + 8, 9 out = 1; 1 + 1 = excess, 2 } excess, 4

**2.**

58.312 . . . 5 + 8, 9 out = 4; 4 + 3 + 1 + 2, 9 out = excess . . . 1
34.295 . . . 3 + 4 + 2, 9 out = 0; 9 out; excess, 5 }
24.017 . . . 2 + 4 + 1 + 7, 9 out = excess . . . . 5 } excess . . . 1

Solve, and prove by casting out the 9's:

*a.* From 958436 take 154018. *b.* 83.27 − 15.436

*c.* From 879514 take 252192. *d.* 62.573 − 33.3333

*e.* Find the sum of the four differences.

* This may be omitted if the teacher does not care to have the pupils consider this topic. The proof by casting out the 9's is an instructive figure drill and a practical principle. Some accountants constantly cast out the 9's (or the 11's) as a means of preventing errors. A system for checking up the work by bookkeepers has been devised and sold in some parts of the country, which is nothing more than an ingenious application of this principle.

# MULTIPLICATION

**Note.** For definitions of terms see page 340.

## Examples of Multiplication

| *a* | *b* | *c* |
|---|---|---|
| 4865 acres | 37.258 | $375.42 |
| 3 | 3 | 3 |
| 14595 acres | 111.774 | $1126.26 |

**1.** Point out the multiplicand, the multiplier, and the product in each of the above examples.

Observe that in written problems in multiplication we usually write the multiplier under the multiplicand, and begin with the right-hand figure of each. In doing such work without the aid of a pencil, it is better to begin with the left-hand figure of the multiplicand. Thus, if required to multiply 4865 acres by 3 without the aid of a pencil, one would say,— Three times 4 thousand acres are 12 thousand acres; 3 times 8 hundred acres are 2 thousand 4 hundred acres, which added to 12 thousand acres make 14 thousand 4 hundred acres, etc.

*The multiplier is always an abstract number.*

*The denomination of the product is always the same as that of the multiplicand.*

There are 64 **primary facts of multiplication** (see pages 341, 342). These make the usual multiplication table, beginning with 2 times 2, and ending with 9 times 9. There are other facts of multiplication that should be memorized; but these 64 facts must be mastered as a preparation for solving multiplication problems. The pupils must also know that once 1 is 1, once 2 is 2, etc.; and that 2 times 0 is 0, etc.; but these can hardly be said to require a memory effort, and hence are not included in the primary facts.

Remembering that the writing of one zero to the right of an integral number multiplies the number by 10, and that 20 times a number is 10 times 2 times the number, and that 30 times a number is 10 times 3 times the number, etc., solve the following. Prove by one of the methods given on page 31.

*a.* Multiply 3256 by 10 *b.* 3256 × 20
*c.* Multiply 4208 by 10 *d.* 4208 × 30
*e.* Multiply 6744 by 10 *f.* 6744 × 20
*g.* Find the sum of the six products, *a* to *f*.

Remembering that the writing of two zeros to the right of an integral number multiplies the number by 100, three zeros by 1000, etc., solve the following.

*h.* Multiply 324 by 100 *i.* 324 × 400
*j.* Multiply 324 by 1000 *k.* 324 × 5000
*l.* Multiply 676 by 100 *m.* 676 × 400
*n.* Find the sum of the six products, *h* to *m*.

1. Find the product of 3426 and 57.

**OPERATION**

$$\begin{array}{r} 3426\\ \underline{57}\\ 23982\\ \underline{17130\phantom{0}}\\ 195282 \end{array}$$

**EXPLANATION**

Seven times 3426 equals 23982. Fifty times 3426 equals 171300. Fifty times the number plus 7 times the number equals 57 times the number. Therefore, adding 23982 and 171300 gives 57 times 3426.

The product of 3426 and 57 is 195282.

*o.* Multiply 3241 by 27 *p.* 6759 × 27
*q.* Multiply 4328 by 36 *r.* 5672 × 36
*s.* Multiply 3479 by 54 *t.* 6521 × 54
*u.* Find the sum of the six products, *o* to *t*.

**1.** Find the product of 728.37 and .6.

OPERATION

$$\begin{array}{r} 72^{\vee}8.37 \\ .6 \\ \hline 437.022 \end{array}$$

EXPLANATION

To multiply by .6, means to take 6 times 1 tenth of the number. One tenth of 728.37 is 72.837. Six times 72.837 equals 437.022.

NOTE 1. The separatrix is used to indicate the place of the decimal point in the number denoting 1 tenth of the multiplicand.

NOTE 2. The decimal point should be written in the product when, in the process of multiplication, the place is reached where it belongs. Do not multiply all the figures and then attempt to determine the place of the point.

**2.** Find the product of 746.2 and .25.

OPERATION

$$\begin{array}{r} 7^{\vee}46.2\phantom{0} \\ .25\phantom{0} \\ \hline 37.310 \\ 149.24\phantom{00} \\ \hline 186.550 \end{array}$$

EXPLANATION

To multiply by .25, means to take 25 times 1 hundredth of the number.

One hundredth of 746.2 is 7.462. Five times 7.462 equals 37.310. Twenty times 7.462 equals 149.24. 37.310 + 149.24 = 186.550.

NOTE 3. For further direction in regard to the use of the separatrix, see note on page 73 of the Intermediate Arithmetic.

*a.* Multiply 324.6 by .7 *b.* 324.6 × .27

*c.* Multiply 324.6 by 2.7 *d.* 324.6 × 27

*e.* Multiply 324.6 by 5.4 *f.* 324.6 × 54

*g.* Multiply 324.6 by 7.3 *h.* 324.6 × 73

*i.* Multiply 324.6 by 4.6 *j.* 324.6 × 46

*k.* Find the sum of the ten products, *a* to *j*.

*l.* Multiply five hundred thirty-five and six tenths by three and four tenths.

Multiply:

*a.* 3.246 by 10
*b.* 51.24 by 10
*c.* .8847 by 10
*d.* 3.012 by 10
*e.* 423.1 by 10
*f.* 5.213 by 20
*g.* 64.16 by 20
*h.* .0562 by 40
*i.* Find the sum of the eight products, *a* to *h*.

Multiply:

*j.* .5214 by 100
*k.* 324.6 by 100
*l.* 15.16 by 100
*m.* 3.184 by 100
*n.* .0037 by 100
*o.* .4051 by 200
*p.* 421.3 by 200
*q.* 32.51 by 400
*r.* Find the sum of the eight products, *j* to *q*.

Multiply:

*s.* 6245 by 72
*t.* 3.157 by 7.2
*u.* 317.6 by .72
*v.* 824.5 by .072
*w.* 38.16 by 720
*x.* .7124 by 6
*y.* .4715 by .6
*z.* .2616 by .06
*aa.* Find the sum of the eight products, *s* to *z*.

Multiply:

*bb.* Six thousand by six thousandths.

*cc.* Sixty-six thousand by sixty-six thousandths.

*dd.* The sum of seventy-eight thousandths and forty-five hundredths by their difference.

*To the Teacher.* It may be unnecessary for all the pupils to solve all the problems on this page. The judgment of the teacher must determine how much of such figure drill is necessary with each class.

**1.** Find the product of $34.50 and 53.4.

**OPERATION**

$$\begin{array}{r} \$34.50 \\ 53.4 \\ \hline 13.800 \\ 103.50\phantom{0} \\ 1725.0\phantom{00} \\ \hline \$1842.300 \end{array}$$

**EXPLANATION**

To multiply by 53.4, means to take 53 times the multiplicand plus 4 tenths of the multiplicand.

One tenth of $34.50 is $3.45.

Four tenths of $34.50 is $13.80.

Three times $34.50 is $103.50.

Fifty times $34.50 is $1725.

The sum of these three partial products is $1842.30.

Practical application of the foregoing.

If one acre of land costs $34.50, how much will 53.4 acres cost?
One tenth of an acre costs $3.45.
Four tenths of an acre cost $13.80.
Three acres cost $103.50.
Fifty acres cost $1725.
53.4 acres cost $1842.30.

*a.* Complete the following bill, and find the amount of it.

SCHOOL FOR THE BLIND,

1907. *To* GEO. E. SYBRANT, *Dr.*

| | | | | | |
|---|---|---|---|---|---|
| Dec. | 4 | 27 bbl. Apples | @ $2.25 | | |
| " | 10 | 56 bu. Potatoes | @ .52 | | |
| " | 12 | 13 bu. Beans | @ 1.75 | | |
| " | 16 | 34 bu. Turnips | @ .35 | | |
| " | 18 | 50 bbl. Flour | @ 4.90 | | |
| " | 21 | 74 lb. Butter | @ .19 | | |
| " | 22 | 53 lb. Tea | @ .42 | | |
| " | 24 | 37 bu. Onions | @ .55 | | |
| " | 31 | 58 lb. Ham | @ .14 | | |
| " | " | 36 lb. Lard | @ .12 | | |
| " | " | 42 lb. Coffee | @ .23 | | |

*To the Teacher.* This page is for drill. If pupils have not had such problems in the grades below, the page may be omitted until the book is reviewed. Whenever the work is done, *require pupils to give an approximate answer to each problem on the page before attempting its solution with the aid of a pencil.*

Find the cost of:

*a.* 26420 lb. hard coal at $7.75 per ton.

*b.* 54650 lb. soft coal at $4.60 per ton.

*c.* 146½ cd. wood at $5.40 per cord.

Find the cost of:

*d.* 8250 ft. lumber at $24.50 per thousand.

*e.* 6540 ft. lumber at $27.25 per thousand.

*f.* 5240 ft. lumber at $32.50 per thousand.

*g.* 8620 brick at $7.50 per thousand.

*h.* 1440 brick at $12 per thousand.

Find the cost of:

*i.* 7250 lb. beef at $8.75 per cwt.

*j.* 5150 lb. pork at $7.25 per cwt.

*k.* 1220 lb. mutton at $9.10 per cwt.

*l.* 624 lb. veal at $8.30 per cwt.

Find the cost of:

*m.* 175 bu. corn at 34.5 cents a bushel.

*n.* 230 bu. oats at 22½ cents a bushel.

*o.* 30480 lb. hay at $8.50 per ton.

*p.* 12800 lb. straw at $4.25 per ton.

*q.* 56 bu. potatoes at 46 cents a bushel.

*r.* 24 hogs (4360 lb.) at $5.75 per cwt.

*s.* 15 cattle (13640 lb.) at $4.35 per cwt.

The usual method of testing the work in multiplication is to divide the product by the multiplier; the quotient should be like the multiplicand.

## CASTING OUT THE 9'S

This proof is based on the fact that if the excess of 9's in the multiplicand is multiplied by the excess of 9's in the multiplier, the excess of 9's in this new product equals the excess of 9's in the original product.

1.

$$\begin{array}{r l l r l l} 3523 & \ldots & \text{Excess of 9's,} & 4 & & \\ \underline{6} & & & \underline{6} & & \\ 21138 & \ldots & \text{Excess, 6} & 24 & \ldots & \text{Excess 6} \end{array}$$

2.

$$\begin{array}{r l l r l l} 5729.3\,* & \ldots & \text{Excess of 9's,} & 8 & & \\ \underline{.25} & \ldots & \text{Excess} & \underline{7} & & \\ 286.465 & & & 56 & \ldots & \text{Excess 2} \\ \underline{1145.86\phantom{0}} & & & & & \\ 1432.325 & \ldots & \text{Excess, 2} & & & \end{array}$$

Solve the following multiplication problems, and prove by casting out the 9's.

*a.* Multiply 2275.23 by 12

*b.* Multiply 8427 by 125

*c.* Multiply 13604.1 by 37

*d.* Multiply 25162.03 by 26

*e.* Multiply .2036 by 51

*f.* Find the sum of the five products, *a* to *e*, and prove by casting out the 9's.

---

*Observe that this method of proof is no check on the use of the decimal point; neither would an error made by a transposition of figures be detected.

**The careful accountant often tests his work by solving a second time in another way. Some of these tests are given below.**

*a.* Multiply 5286 by 9, and prove by subtracting 5286 from 10 times 5286.

*b.* Multiply 8437 by 15, and prove by taking 10 times 8437 and adding to the product half of 10 times 8437.

*c.* Multiply 5836 by 16, and prove by finding the sum of 10 times 5836 and 5 times 5836 (half of 10 times 5836) and 5836.

*d.* Multiply 4378 by 25, and prove by taking 1 fourth of a hundred times 4378.

*e.* Multiply 2750 by 75, and prove by taking 3 fourths of a hundred times 2750.

*f.* Multiply 5462 by 19, and prove by subtracting 5462 from 20 times 5462.

*g.* Multiply 7140 by 48, and prove by subtracting twice 7140 from 50 times 7140.

*h.* Multiply 6175 by 35, and prove by finding the sum of 10 times 6175 and 10 times 6175 and 10 times 6175 and 1 half of 10 times 6175.

*i.* Multiply 4352 by 12, and prove by adding 10 times 4352 and once 4352 and once 4352.

*j.* Multiply 3246 by 99, and prove by subtracting 3246 from 100 times 3246.

*k.* Multiply 6372 by 21, and prove by adding 6372 to 1 fifth of 100 times 6372.

*l.* Multiply 1357 by 24, and prove by subtracting 1357 from 1 fourth of 100 times 1357.

# DIVISION

NOTE. For definitions of terms see page 342.

## EXAMPLES OF DIVISION

*a*

$5)$1565
313 times

*b*

5)$1565
$313

*c*

2 bu.)246 bu.
123 times

*d*

2)246 bu.
143 bu.

**1.** Tell which is the dividend, which the divisor, and which the quotient in each of the above examples.

Observe that in written problems in division the divisor is usually written to the left of the dividend. The quotient in short division is written under the dividend; in long division (see next page) it may be written either over, or to the right of, the dividend.

*The dividend may always be thought of as concrete; that is, as standing for a number of things.*

*If the divisor is concrete, it must be like the dividend, and the quotient will show how many times the divisor is contained in the dividend.*

*If the divisor is abstract, the quotient will be like the dividend — will be a part of the dividend.*

There are 64 **primary facts of division.** These are the converse of the 64 facts of multiplication given on pages 341, 342.

The primary facts of division should be learned while learning the 64 multiplication facts. These facts should be observed in their double aspect. The pupil must know that since 3 times 5 are 15, 5 is contained in 15 three times, and 1 third of 15 is 5.

All division problems may be regarded as requirements to find how many times one number is contained in another number. Referring to example *b*, page 33: If one were required to find 1 fifth of 1565 silver dollars, he might first take 5 dollars from the 1565 dollars and put one of the dollars in each of five places. He might then take another 5 dollars from the number of dollars to be divided and put 1 dollar with each of the dollars first taken. In this manner he might continue to distribute fives of dollars until all of the dollars had been placed in the five piles. He might then count the dollars in one of the piles. Observe, then, that 1 fifth of 1565 dollars is as many dollars as 5 dollars is contained times in 1565 dollars. It is contained 313 times; hence 1 fifth of $1565 is $313.

1. Find the quotient of 43720 divided by 8.

SHORT DIVISION

$$\begin{array}{r} \underline{8)43720} \\ 5465 \end{array}$$

LONG DIVISION

$$\begin{array}{r} 5465 \\ 8\overline{)43720} \\ \underline{40}\phantom{000} \\ 37\phantom{00} \\ \underline{32}\phantom{00} \\ 52\phantom{0} \\ \underline{48}\phantom{0} \\ 40 \\ \underline{40} \end{array}$$

Observe that in long division the product obtained by multiplying the divisor by a figure of the quotient is written under the partial dividend, subtracted from it, and to the remainder thus obtained the next figure of the dividend is annexed, — "brought down." In short division this work of multiplying, subtracting, and annexing is done "mentally."

*a.* 93492 ÷ 49
*b.* 92169 ÷ 77
*c.* 72855 ÷ 45
*d.* 34694 ÷ 38
*e.* 54875 ÷ 25
*f.* 5904 ÷ 328
*g.* 7693 ÷ 157
*h.* 8190 ÷ 546
*i.* 8448 ÷ 128
*j.* 3072 ÷ 128

*k.* Find the sum of the ten quotients, *a* to *j*.

Observe that when the divisor is integral and the dividend a mixed decimal, the decimal point should be written in the quotient when, in the process of division, the decimal point in the dividend is reached.

**1.**

$$\begin{array}{r} 5\underline{)7143.5}\\ 1428.7 \end{array}$$

**2.**

$$\begin{array}{r} 7\underline{)423.43}\\ 60.49 \end{array}$$

**3.**

$$\begin{array}{r} 6\underline{)53.274}\\ 8.879 \end{array}$$

**4.**

$$\begin{array}{r} 5\underline{)8.230}\\ 1.646 \end{array}$$

**5.**

$$\begin{array}{r} 4\underline{)36.732}\\ 9.183 \end{array}$$

**6.**

$$\begin{array}{r} 2\underline{)3.4267}\\ 1.71335 \end{array}$$

**7.**

$$\begin{array}{r} 175.46\\ 25\overline{)4386.50}\\ \underline{25}\phantom{86.50}\\ 188\phantom{6.50}\\ \underline{175}\phantom{6.50}\\ 136\phantom{.50}\\ \underline{125}\phantom{.50}\\ 115\phantom{0}\\ \underline{100}\phantom{0}\\ 150\\ \underline{150} \end{array}$$

*a.* Divide 78.36 by 4* *b.* 962.45 ÷ 5

*c.* Divide 32.76 by 6 *d.* 35.861 ÷ 7

*e.* Divide 47.28 by 8 *f.* 6.2793 ÷ 3

*g.* Find the sum of the six quotients, *a* to *f*.

*h.* Divide 37.275 by 25 *i.* 1464.4 ÷ 28

*j.* Divide 148.24 by 34 *k.* 67.08 ÷ 43

*l.* Divide 450.0 by 12 *m.* 486.75 ÷ 15

*n.* Find the sum of the six quotients, *h* to *m*.

---

* This problem may be thought of as a requirement to find 1 fourth of 78.36; or, it may be thought of as a requirement to find how many times 4 is contained in 78.36. In either case if the attention for a moment rests upon the integral part of the dividend, the place of the decimal point in the quotient is easily apparent.

Observe that when the divisor is wholly or in part a decimal fraction, a separatrix ($^{\vee}$) should be placed after that figure of the dividend which is of the same order as the right-hand figure of the divisor. If there is no such figure in the dividend, then one or more zeros should be annexed and the separatrix placed after that zero in the dividend that is of the same order as the right-hand figure of the divisor. When, in the process of division, the separatrix is reached, a decimal point should be written in the quotient.

**1.** Divide 37.44 by 2.4.

EXPLANATION

This problem may be thought of as a requirement to find how many times 24 tenths are contained in 374 tenths. *Illustrative Problem:* If 37.44 acres of land are divided into lots, each containing 2.4 acres, how many lots?

*Ans.* Fifteen lots, with land enough remaining for 6 tenths of another lot. *Question:* How much land in the fractional lot? *Ans.* 6 tenths of 2.4 acres = ——.

OPERATION

$$\begin{array}{r}
15.6 \\
2.4\overline{)37.4^{\vee}4} \\
\underline{24\phantom{.4^{\vee}4}} \\
13.4\phantom{^{\vee}4} \\
\underline{12.0\phantom{^{\vee}4}} \\
1.44 \\
\underline{1.44}
\end{array}$$

**2.** Divide 54 by 2.4.

EXPLANATION

This problem may be thought of as a requirement to find how many times 24 tenths are contained in 540 tenths. *Illustrative Problem:* If 54 acres of land are divided into lots, each containing 2.4 acres, how many lots? *Ans.* Twenty-two lots, with land enough remaining for 5 tenths of another lot. *Question:* How much land in the fractional lot? *Ans.* 5 tenths of 2.4 acres = ——.

OPERATION

$$\begin{array}{r}
22.5 \\
2.4\overline{)54.0^{\vee}}\phantom{0} \\
\underline{48\phantom{.0^{\vee}}}\phantom{0} \\
6.0\phantom{0} \\
\underline{4.8\phantom{0}} \\
1.20 \\
\underline{1.20}
\end{array}$$

**3.**

$$\begin{array}{r} .5\overline{)40.2^{\vee}65} \\ 80.53 \end{array}$$

**4.**

$$\begin{array}{r} .05\overline{)706.20^{\vee}} \\ 14124. \end{array}$$

**5.**

$$\begin{array}{r} .005\overline{)38.000^{\vee}} \\ 7600. \end{array}$$

*a.* Divide 19278 by 2.7 *b.* 32.130 ÷ .45

*c.* Divide 24.255 by 6.3 *d.* 257.04 ÷ .36

*e.* Find the sum of the four quotients, *a* to *d*.

To divide a number by 10, move the decimal point one place to the left. To divide a number by 20, take 1 half of 1 tenth of the number. To divide a number by 30, take 1 third of 1 tenth of the number, etc.

1. Divide 347.48 by 20.

EXPLANATION

In practice, when the zero in the divisor is canceled, the separatrix should be written in the dividend as indicated in the example. 1 tenth of 347.48 is 34.748. 1 half of 34.748 is 17.374.

OPERATION

$$\begin{array}{r} 2\cancel{0})\underline{34^{\vee}7.48} \\ 17.374 \end{array}$$

*f.* Divide 34.26 by 10 *g.* 34.26 ÷ 20

*h.* Divide 572.4 by 10 *i.* 572.4 ÷ 30

*j.* Find the sum of the four quotients, *f* to *i*.

To divide a number by 100, move the decimal point two places to the left. The pupil should now be able to make a rule for dividing by 200, 300, etc.

*k.* Divide 4526 by 100 *l.* 4526 ÷ 200

*m.* Divide 347.8 by 100 *n.* 347.8 ÷ 400

*o.* Find the sum of the four quotients, *k* to *n*.

Make a rule for dividing by 1000, 2000, 3000, etc.

*p.* Divide 4279 by 1000 *q.* 5228 ÷ 2000

*r.* Divide 538.6 by 1000 *s.* 478.6 ÷ 2000

*t.* Find the sum of the four quotients, *p* to *s*.

**OPERATIONS**

**1.** Divide $754.65 by $.27.*

EXPLANATION

This means, — *Find how many times* 27 ¢ *are contained in* 75465 ¢. 27 ¢ are contained in 75465 ¢, 2795 times. *Illustrative Problem:* At 27 ¢ a bushel, how many bushels of oats can be bought for $754.65? *Ans.* As many bushels as 27 ¢ is contained times in $754.65. It is contained 2795 times; therefore 2795 bushels can be bought.

$$\begin{array}{r} 2795.\\ \$.27\overline{)\$754.65^{\vee}}\\ \underline{54}\phantom{.65}\\ 214\phantom{.5}\\ \underline{189}\phantom{.5}\\ 256\\ \underline{243}\\ 135\\ \underline{135} \end{array}$$

**2.** Divide $754.65 by 27.

EXPLANATION

This means, — *Find one 27th of* $754.65. One 27th of $754.65 is $27.95.

NOTE. One might find one 27th of $754.65 by finding how many times $27 are contained in $754.65. See page 34.

*Illustrative Problem:* If 27 acres of land are worth $754.65, how much is 1 acre worth?

$$\begin{array}{r} \$27.95\\ 27\overline{)\$754.65}\\ \underline{54}\phantom{.65}\\ 214\phantom{.5}\\ \underline{189}\phantom{.5}\\ 256\\ \underline{243}\\ 135\\ \underline{135} \end{array}$$

**3.** Divide $754.65 by .27.

EXPLANATION

This means, — *Find* 100 *27ths of* $754.65. One 27th of $754.65 is $27.95. 100 27ths of $754.65 is 100 times $27.95, or $2795.

NOTE. In practice we find one 27th of 100 times the number to be divided.

*Illustrative Problem:* If .27 of an acre of land is worth $754.65, how much is 1 acre worth at the same rate? It is worth 100 times one 27th of $754.65; or, it is worth one 27th of 100 times $754.65.

$$\begin{array}{r} \$2795\\ .27\overline{)\$754.65^{\vee}}\\ \underline{54}\phantom{.65}\\ 214\phantom{.5}\\ \underline{189}\phantom{.5}\\ 256\\ \underline{243}\\ 135\\ \underline{135} \end{array}$$

---

* For directions for "pointing off" in such problems as this, see page 36.

For each of the following *make a business problem;* then estimate the answer; then solve, observing the directions given on pages 35 and 36 for "pointing off." Compare the answer obtained with the estimate.

*a.* Divide $436.10 by $.35*
*b.* $436.10 ÷ 35
*c.* Divide $436.10 by $35
*d.* $436.10 ÷ $3.50
*e.* Divide $436.10 by 3.5
*f.* Divide $78 by $.24
*g.* $78 ÷ 24
*h.* Divide $78 by $24
*i.* $78 ÷ $2.40
*j.* Divide $78 by 2.4

*Estimate the answer* to each of the following before attempting the exact solution:

*k.* If 24.5 acres cost $882, what is the cost of 1 acre?

*l.* A lot of land at $52.50 per acre cost $645.75. How many acres?

*m.* Paid $791.70 for oats at $.325 a bushel. How many bushels?

*n.* Received $469.80 for 1080 bushels of corn. How much per bushel?

*o.* Paid $2340 for 1.8 acres of land. How much per acre?

*p.* Divide $2340 by $1.8. Tell the meaning.

* *To the Teacher.* Problem *a* means, — *Find how many times* 35 ¢ *are contained in* 43610 ¢. Problem *b* means, — *Find one* 35*th of* $436.10. Problem *c* means, — *Find how many times* $35 *are contained in* $436.10. Problem *d* means, — *Find how many times* $3.50 *are contained in* $436.10. Problem *e* means, — *Find ten* 35*ths of* $436.10. What is really done in the solution of problem *e* is to find one 35th of 10 times $436.10.

The usual method of testing the work in division is by multiplying the quotient by the divisor. If the product is like the dividend, the result is supposed to be right.

## CASTING OUT THE 9'S

This proof is based on the fact that if the excess of 9's in the divisor is multiplied by the excess of 9's in the quotient, the excess of 9's in the product thus obtained equals the excess of 9's in the dividend.

**1.**

```
      245
138)33810
    276
    ---
     621
     552
     ---
      690
      690
      ---
```

**PROOF**

| | |
|---|---|
| Excess in 138, the divisor . . . | 3 |
| Excess in 245, the quotient . . . | 2 |
| Excess in product, (3 × 2) . . . | 6 |
| Excess in 33810, the dividend . . | 6 |

**2.**

```
      2484.3
.15)372.64‸5
    30
    --
     72
     60
     --
     126
     120
     ---
       64
       60
       --
        45
        45
        --
```

**PROOF**

| | |
|---|---|
| Excess in .15, the divisor . . | 6 |
| Excess in 2484.3 the quotient . . | 3 |
| Excess in product, (6 × 3) . . | 0 |
| Excess in 372.645, the dividend, 0 | |

NOTE. This method of proof does not reveal errors in "pointing off."

1. How many barrels of apples at $2.50 a barrel must be given for 5 tons of coal at $6 a ton?

*a.* How many barrels of apples at $3.75 a barrel must be given for 12 tons of coal at $6.25 a ton?

2. A man earns $60 a month and spends $500 a year. How much does he save each year?

*b.* A man earns $72.50 a month and spends $650 a year. How much does he save each year?

3. The difference of two numbers is 8. The smaller number is 12. What is the larger number?

*c.* The difference of two numbers is 11.5. The smaller number is 49.34. What is the larger number?

4. Estimate, without making any figures, the average daily sales of a merchant whose receipts for one week were as follows: Monday, $124.50; Tuesday, $96.25; Wednesday, $51.40; Thursday, $75.50; Friday, $103.75; Saturday $148.60.

*d.* Find the exact average daily sales of the merchant whose receipts for one week are given in No. 4.

5. The sum of two numbers is 17. One of the numbers is 8. What is the other number?

*e.* The sum of two numbers is 94.3. One of the numbers is 46.15. What is the other number?

6. When corn costs 40¢ a bushel, how many bushels can be bought for $8?

*f.* When corn costs 35¢ a bushel, how many bushels can be bought for $1000?

7. Without making figures, estimate the answer to the following: At 52¢ a bushel, how many bushels of potatoes can be bought for $1000?

1. A bushel of oats weighs 32 lb. How many bushels in 640 lb.? Without making figures, estimate the number of bushels of oats in a load weighing 3150 lb.

*a.* Find the number of whole bushels in 3150 lb. of oats.

*b.* Find the number of whole bushels in 4260 lb. of oats.

2. Estimate the value at 6¢ a pound of 4 hogs weighing about 225 lb. each.

*c.* Find the exact value of the 4 hogs mentioned in No. 2 if they weigh, respectively, 215 lb., 236 lb., 220 lb., and 235 lb.

3. Estimate the value of 3.46 acres of land at $80 per acre.

*d.* Find the exact value of the land mentioned in problem 3.

4. Would a load of wheat at 61¢ a bushel, weighing 2400 lb., be worth more or less than $24?

*e.* Find the exact value of 2400 lb. of wheat at 61¢ a bushel.

*f.* Multiply 42375 by 342, and prove by casting out the 9's.

*g.* Add the following, and prove by casting out the 9's: 38275, 64291, 53286, 143725, 166891.

*h.* From the sum of 8379 and 5246 subtract their difference.

5. The product of two numbers is 36. One of the numbers is 9. What is the other number?

*i.* The product of two numbers is 38346. One of the numbers is 83. What is the other number?

*j.* The product of two numbers is 6850. One of the numbers is 12½. What is the other number?

# DIVISIBILITY OF NUMBERS

## NUMBERS DIVISIBLE BY 2

An integral number is exactly divisible by 2 if the right-hand figure is 0, or if the number expressed by its right-hand figure is exactly divisible by 2.

EXPLANATORY NOTE. Every integral number that may be expressed by two or more figures may be regarded as made up of a certain number of tens and a certain number (0 to 9) of primary units, thus: 485 is made up of 48 tens and 5 units; 4260 is made up of 426 tens and 0 units; 27562 is made up of 2756 tens and 2 units. But ten is exactly divisible by 2; so any number of tens or any number of tens plus any number of twos is exactly divisible by 2.

**1.** Tell which of the following are exactly divisible by 2, and why: 387, 5846, 2750, 2834.

## NUMBERS DIVISIBLE BY $2\frac{1}{2}$

Any number, integral or mixed, is exactly divisible by $2\frac{1}{2}$ if the part of the number expressed by figures to the right of the tens' figure is exactly divisible by $2\frac{1}{2}$. Show why this statement is correct, employing the thought process given in the "Explanatory Note" above.

**2.** Tell which of the following are exactly divisible by $2\frac{1}{2}$, and why: 485, 470, 365, $472\frac{1}{2}$, $3847\frac{1}{2}$

## NUMBERS DIVISIBLE BY $3\frac{1}{3}$

Any number is exactly divisible by $3\frac{1}{3}$ if the part of the number expressed by figures to the right of the tens' figure is exactly divisible by $3\frac{1}{3}$.

**3.** Tell which of the following are exactly divisible by $3\frac{1}{3}$, and why: 780, $283\frac{1}{3}$, $576\frac{2}{3}$, 742, 80.

### Numbers divisible by 5

Any integral number is exactly divisible by 5 if its right-hand figure is 0 or 5.

### Numbers divisible by 10

Any integral number is exactly divisible by 10 if its right-hand figure is 0.

*a.* How many times is 2½ contained in 582½?*

*b.* How many times is 2½ contained in 375?

*c.* How many times is 2½ contained in 467½?

*d.* How many times is 2½ contained in 4680?

*e.* How many times is 3⅓ contained in 786⅔?†

*f.* How many times is 3⅓ contained in 543⅓?

*g.* How many times is 3⅓ contained in 8640?

### Numbers divisible by 25

Any integral number is exactly divisible by 25 if its two right-hand figures are zeros or if the part of the number expressed by its two right-hand figures is exactly divisible by 25.

Explanatory Note. Every integral number expressed by three or more figures may be regarded as made up of a certain number of hundreds and a certain number (0 to 99) of primary units; thus 4624 is made up of 46 hundreds and 24 units; 38425 is made up of 384 hundreds and 25 units; 8400 is made up of 84 hundreds and 0 units. But a hundred is exactly divisible by 25; so any number of hundreds or any number of hundreds plus any number of 25's is exactly divisible by 25.

1. Tell which of the following are exactly divisible by 25, and why: 37625, 34836, 27950, 38575.

---

* 2½ is contained in 582½, (4 × 58) + 1 times. Why?

† 3⅓ is contained in 786⅔, (3 × 78) + 2 times. Why?

### Numbers divisible by 33⅓

Any number is exactly divisible by 33⅓ if that part of the number expressed by the figures to the right of the hundreds' figure is exactly divisible by 33⅓.

**1.** Show why the statement made in the foregoing paragraph is correct, employing the thought process given in the "Explanatory Note" on the preceding page.

**2.** Tell which of the following are exactly divisible by 33⅓, and why : 36466⅔, 2375, 46833⅓, 38900, 46820.

### Numbers divisible by 12½

Any number, integral or mixed, is exactly divisible by 12½ if the part of the number expressed by the figures to the right of the hundreds' figure is exactly divisible by 12½.

**3.** Tell which of the following are exactly divisible by 12½, and why : 375, 837½, 6450, 4329, 7467½, 3487½, 3425.

### Numbers divisible by 16⅔

Any number, integral or mixed, is exactly divisible by 16⅔ if

. . . . . . . . . . . . . . . .

**4.** Tell which of the following are exactly divisible by 16⅔, and why: 4633⅓, 5460, 2350, 37400, 27583⅓, 25416⅔.

### Numbers divisible by 20

Any integral number is exactly divisible by 20 if the number expressed by its two right-hand figures is exactly divisible by 20.

**5.** Tell which of the following are exactly divisible by 20, and why : 3740, 2650, 3860, 29480, 3470.

**6.** Tell which of the following are exactly divisible by 50, and why : 2560, 3450, 6800, 27380, 25450.

*a.* How many times is 25 contained in 2450?[*]
*b.* How many times is 25 contained in 3775?
*c.* How many times is $33\frac{1}{3}$ contained in $4666\frac{2}{3}$?[†]
*d.* How many times is $33\frac{1}{3}$ contained in $3433\frac{1}{3}$?
*e.* How many times is $12\frac{1}{2}$ contained in $4737\frac{1}{2}$?[‡]
*f.* How many times is $12\frac{1}{2}$ contained in $3662\frac{1}{2}$?
*g.* How many times is $16\frac{2}{3}$ contained in $2533\frac{1}{3}$?
*h.* How many times is $16\frac{2}{3}$ contained in 4550?

### NUMBERS EXACTLY DIVISIBLE BY 9

**Any number is exactly divisible by 9 if the sum of its digits is exactly divisible by 9.**

**EXPLANATORY NOTE. Any number more than nine is a certain number of nines and as many over as the number indicated by the sum of its digits. Thus 20 is two nines and 2 over; 41 is four nines and 4 + 1 over; 42 is four nines and 4 + 2 over; 200 is twenty-two nines and 2 over; 300 is thirty-three nines and 3 over; 320 is a certain number of nines and 3 + 2 over; 321 is a certain number of nines and 3 + 2 + 1 over.**

**326 is a certain number of nines and 3 + 2 + 6 over; but 3 + 2 + 6 = 11, or another nine and 2 over.**

1. Read the "Explanatory Note" carefully, and tell which of the following are exactly divisible by 9: 3256, 4266, 2314, 2574.

2. Remembering that there are eleven 9's and 1 over in each hundred, and one 9 and 1 over in each ten, tell how many times 9 is contained in each of the following: 645, 428, 575, 396, 254.

---

[*] 25 is contained in 2450, (4 × 24) + 2 times. Why?
[†] $33\frac{1}{3}$ is contained in $4666\frac{2}{3}$, (3 × 46) + 2 times. Why?
[‡] $12\frac{1}{2}$ is contained in $4737\frac{1}{2}$, (8 × 47) + 3 times. Why?

## PRIME FACTORS AND EXACT DIVISORS

Any integral number is exactly divisible by each of its prime factors and by the product of any two or more of its prime factors. Thus 30 $(2 \times 3 \times 5)$ is exactly divisible by 2, by 3, by 5, and by $(2 \times 3)$ 6, and by $(2 \times 5)$ 10, and by $(3 \times 5)$ 15.

1. The exact integral divisors of 36 $(2 \times 2 \times 3 \times 3)$ are 2, 3, —, —, —, —, and —.

## PRIME FACTORS, COMMON DIVISORS, AND GREATEST COMMON DIVISORS

Any prime factor or any product of two or more prime factors common to two or more numbers is a common divisor of the numbers. Thus the numbers 30 $(2 \times 3 \times 5)$ and 40 $(2 \times 2 \times 2 \times 5)$ have the factors 2 and 5 in common. So the common divisors of 30 and 40 are 2, 5, and 10, and the greatest common divisor is 10.

2. Find the g. c. d. of 50, 75, and 125.*

OPERATION NO. 1

$$\begin{aligned} 50 &= 2 \times 5 \times 5 \\ 75 &= 3 \times 5 \times 5 \\ 125 &= 5 \times 5 \times 5 \end{aligned}$$

$5 \times 5 = 25$, g. c. d.

OPERATION NO. 2

$$\begin{array}{r|rrr} 5 & 50 & 75 & 125 \\ \hline 5 & 10 & 15 & 25 \\ \hline & 2 & 3 & 5 \end{array}$$

$5 \times 5 = 25$, g. c. d.

**RULE.** *To find the greatest common divisor of two or more numbers, find the product of the prime factors common to the numbers.*

3. Find the g. c. d. of 80, 100, 140.
4. Find the g. c. d. of 48, 60, 72.
5. Find the g. c. d. of 64, 96, 256.

---

* The letters g. c. d. are sometimes used instead of the words *greatest common divisor.*

1. Find the g. c. d. of 640 and 760.

OPERATION

$$\begin{array}{l} 640)760(1 \\ \quad\;\underline{640} \\ \quad\;120)640(5 \\ \qquad\quad\;\underline{600} \\ \qquad\quad\;\;40)120(3 \\ \qquad\qquad\quad\underline{120} \end{array}$$

EXPLANATION

The number 760 is an integral number of times the g. c. d., *whatever that may be;* so is the number 640. We make an incomplete division of 760 by 640 and have as a remainder the number 120. Since 640 and 760 are each an integral number of times the g. c. d., their difference, 120, must be an integral number of times the g. c. d.; for, taking an integral number of times a thing from an integral number of times a thing must leave an integral number of times the thing. Therefore no number greater than 120 can be the g. c. d. But if 120 is an exact divisor of 640, it is also an exact divisor of 760, for it will be contained one more time in 760 than in 640. We make the trial and find that 120 is not an exact divisor of 640; there is a remainder of 40. Since 600 (120 × 5) and 640 are each an integral number of times the g. c. d., 40 must be an integral number of times the g. c. d. But if 40 is an exact divisor of 120 it is an exact divisor of 600 (120 × 5) and of 640 (40 more than 600) and of 760 (120 more than 640). We make the trial and find that it is an exact divisor of 120, and is therefore the g. c. d of 640 and 760.

From the foregoing make a rule for finding the g. c. d. of two numbers and apply it to the following:

Find the g. c. d.

*a.* Of 380 and 240
*b.* Of 540 and 450
*c.* Of 275 and 155
*d.* Of 320 and 860
*e.* Of 144 and 96
*f.* Of 475 and 350
*g.* Of 1728 and 288
*h.* Of 390 and 520
*i.* Of 650 and 175
*j.* Of 450 and 600
*k.* Find the sum of the ten results, *a* to *j*.

Find the g. c. d. of the following by the method given on page 47. Prove the work by the method given on page 48.

*a.* 121, 132 *b.* 380, 520 *c.* 288, 432
*d.* 216, 324 *e.* 326, 340 *f.* 585, 648
*g.* Find the sum of the six answers, *a* to *f*.

Find the g. c. d. of 120, 180, and 250 by the method given on page 47. Prove the work by first finding the g. c. d. of two of the numbers, and then of the g.c.d. found and the third number, employing the method given on page 48.

OPERATION

See page 47.

$$\begin{array}{r|rrr} 5 & 120 & 180 & 250 \\ \hline 2 & 24 & 36 & 50 \\ \hline & 12 & 18 & 25 \end{array}$$

G. c. d. = 5 × 2 = 10

EXPLANATION

See page 48.

G. c. d. of 120 and 180 = 60.
G. c. d. of 60 and 250 = 10.

Applying the methods suggested above, find the g. c. d. of the following:

*h.* 288, 432, 1152 *i.* 210, 294, 462
*j.* 175, 225, 1125 *k.* 144, 818, 827
*l.* Find the sum of the four answers, *h* to *k*.

Remembering that the g. c. d. of any two numbers cannot exceed their difference, find by inspection and trial the g. c. d. of the following:

*m.* 24, 82, 26, 96 *n.* 45, 55, 9265
*o.* 11, 15, 48, 99 *p.* 275, 625, 300
*q.* Find the sum of the four answers, *m* to *p*.

**To find the least common multiple (l. c. m.) of two or more numbers:**

1. Find the l. c. m of 24, 35, 36, and 50.

OPERATION

$$24 = 2 \times 2 \times 2 \times 3$$
$$35 = 5 \times 7$$
$$36 = 2 \times 2 \times 3 \times 3$$
$$50 = 2 \times 5 \times 5$$
$$(2 \times 2 \times 2)(3 \times 3)(5 \times 5) \times 7 = 12600$$

EXPLANATION

24 has as prime factors, 3 2's; 36 has 2 3's; 50 has 2 5's; 35 has 1 7. All these must appear as factors of the l. c. m.

RULE. *Resolve each number into its prime factors. Take as factors of the l. c. m. the greatest number of 2's, 3's, 5's, 7's, etc., found in any one of the numbers.*

Find the l. c. m. of the following:

*a.* 48, 60

*b.* 60, 75

*c.* 20, 30, 40

*d.* 50, 60

*e.* 30, 40

*f.* 40, 50, 60

*g.* Find the sum of the six answers, *a* to *f*.

Observe that the l. c. m. must contain every prime factor common to two or more of the numbers and every uncommon prime factor found in any of them. Hence the following:

2. Find the l. c. m. of 21, 30, 35.

OPERATION

$$\begin{array}{r|rrr} 3 & 21 & 30 & 35 \\ \hline 7 & 7 & 10 & 35 \\ \hline 5 & 1 & 10 & 5 \\ \hline & 1 & 2 & 1 \end{array}$$

EXPLANATION

The prime factors common to two or more of the numbers are 3, 7, and 5. The uncommon prime factor is 2, found in the number 30. The l. c. m. is the product of 3, 7, 5, and 2, which is 210.

$$(3 \times 7 \times 5) \times 2 = 210, \text{ l. c. m.}$$

Find the l. c. m. of the following:

*h.* 18, 20

*i.* 24, 32

*j.* 36, 72, 24

*k.* 16, 38

*l.* 46, 86

*m.* 45, 81, 27

1. Find several common multiples of 6 and 8.

*a.* Find several common multiples of 18 and 30.

2. Find the l. c. m. of 6 and 8.

*b.* Find the l. c. m. of 18 and 30.

3. Find several common divisors of 12 and 18.

*c.* Find several common divisors of 48 and 60.

4. Find the g. c. d. of 12 and 18.

*d.* Find the g. c. d. of 48 and 60.

5. Name two numbers whose product is their l. c. m.[*]

6. Find the l. c. m. of the following:

*e.* 10, 20, 24
*f.* 20, 30, 40, 50
*g.* 50, 60, 70
*h.* 27, 54, 63, 72
*i.* 18, 12, 39
*j.* 72, 96, 144, 120
*k.* 21, 45, 63
*l.* 75, 125, 150, 175

Remembering that the g. c. d. of two numbers is never greater than their difference, find by inspection and trial the g. c. d. of the following:

*m.* 45 and 50
*n.* 60 and 84
*o.* 55 and 75
*p.* 75 and 125
*q.* 40 and 60
*r.* 90 and 120
*s.* 48 and 84
*t.* 52 and 54
*u.* 97 and 101

7. Remembering that 25 is contained exactly 4 times in 100, tell without the aid of a pencil how many times 25 is contained in 825.

8. Remembering that $16\frac{2}{3}$ is contained exactly 6 times in 100, tell without the aid of a pencil how many times $16\frac{2}{3}$ is contained in $933\frac{1}{3}$.

---

[*] The numbers must contain no common factor; that is, they must be prime to each other.

1. 643,265,245,350. Without dividing, tell whether this number is exactly divisible by 9; by 5; by 10; by 25; by 50; by 12½; by 16⅔.*

A number is made up of the following prime factors: 2, 2, 3, 3, 5, 7.

*a.* What is the number?

*b.* Is the number whose prime factors are given above exactly divisible by 35?

*c.* How many times is 70 (2 × 5 × 7) contained in the number?

*d.* How many times is 45 (3 × 3 × 5) contained in the number?

*e.* How many times is 36 (2 × 2 × 3 × 3) contained in the number?

Resolve the number 836 into its prime factors, and write these factors on paper or on the blackboard. Then answer the following:

*f.* How many times is 19 contained in 836?

*g.* How many times is 209 (11 × 19) contained in 836?

*h.* How many times is 418 (19 × 11 × 2) contained in 836?

*i.* Multiply 8246 by 39. Prove the work by multiplying 8246 by 13, and the product thus obtained by 3.

*j.* Multiply 9463 by 48. Prove the work by multiplying 9463 by 12, and the product thus obtained by 4.

*k.* Multiply 3462 by 96. Prove the work by multiplying 3462 by 8 and the product thus obtained by 12.

* A careful study of pages 43–47 will enable the pupil to make the statements called for with little hesitation.

# FRACTIONS

There are three aspects in which fractions should now be considered:

## I. The Fractional Unit Aspect

The numerator tells the number of things, and the denominator indicates their name. In the fraction $\frac{5}{7}$ there are 5 things (magnitudes) called sevenths. In this fraction there are 5 fractional units, each of which is 1 seventh of another unit called the *unit of the fraction*. The denominator shows the number of parts into which the unit of the fraction is divided; the numerator shows how many of these parts (fractional units) are taken.

## II. The Division Aspect

The numerator is thought of as a *dividend*, the denominator, a *divisor*, and the fraction itself, a *quotient:* thus, in the fraction $\frac{5}{7}$, the dividend is 5, the divisor, 7, and the quotient, $\frac{5}{7}$. In the case of an improper fraction, as $\frac{8}{4}$, it may be more readily seen by the pupil that the numerator is the dividend, the denominator, the divisor, and the fraction ($\frac{8}{4}=2$) the quotient; but the division relation is in every fraction, whether proper or improper, common or decimal, simple or complex.

## III. The Ratio Aspect

The numerator of a fraction may be thought of as an *antecedent*, the denominator, a *consequent*, and the fraction itself, a *ratio:* thus, in the fraction $\frac{5}{7}$, 5 is the antecedent, 7 the consequent, and $\frac{5}{7}$ the ratio. This relation may be more readily seen by the pupil in the case of an improper fraction. In the fraction $\frac{12}{4}$, 12 is the antecedent; 4 the consequent; $\frac{12}{4}$, or 3, the ratio.

**To reduce a fraction to its lowest terms:**

1. Reduce $\frac{160}{200}$ to its lowest terms.

OPERATION NO. 1

$$\frac{160}{200} = \frac{16}{20} = \frac{8}{10} = \frac{4}{5}$$

OPERATION NO. 2

$$\frac{160 \div 40}{200 \div 40} = \frac{4}{5}$$

Observe that in the above operations the form of the fraction is changed without changing its value; for in $\frac{4}{5}$ there are 1 fortieth as many parts, and each part is 40 times as large as in $\frac{160}{200}$. Moreover, the ratio of the numerator to the denominator is unchanged.

RULE. *Divide the terms of the fraction by any common factor, and divide the terms of the fraction thus obtained by any common factor, and so continue until the terms are prime to each other;*

Or, — *Divide the terms of the fraction by their g. c. d.*

2. Reduce $\frac{20}{170}$ to its lowest terms.

3. Reduce $\frac{36}{42}$ to its lowest terms.

Reduce each of the following to its lowest terms:

*a.* $\frac{275}{375}$ *b.* $\frac{520}{650}$ *c.* $\frac{156}{270}$ *d.* $\frac{235}{340}$

*e.* $\frac{56}{210}$ *f.* $\frac{36}{180}$ *g.* $\frac{171}{405}$ *h.* $\frac{63}{204}$

*i.* $\frac{12\frac{1}{2}}{100}$\* *j.* $\frac{37\frac{1}{2}}{100}$ *k.* $\frac{2\frac{1}{2}}{4}$† *l.* $\frac{87\frac{1}{2}}{100}$

*m.* Find the sum of the twelve results, *a* to *l*.‡

---

* Divide each term by 12½. This involves the reduction of a complex fraction to a simple fraction; but it will lead to thoughtful work for the pupil to solve such problems in this manner.

† Divide each term by ½.

‡ If the pupil has not had sufficient practice in addition of fractions to do this, the finding of the sum may be omitted until the book is reviewed.

**To reduce an improper fraction to a whole or a mixed number:**

1. Reduce $\frac{265}{8}$ to a mixed number.

OPERATION

$$8\overline{)265}$$
$$33\tfrac{1}{8}$$

EXPLANATION

Since there are 8 eighths in a whole, there are as many wholes in 265 eighths as 8 is contained times in 265. It is contained $33\frac{1}{8}$ times; so in 265 eighths there are $33\frac{1}{8}$ wholes.

NOTE. Let it be remembered that every fraction may be thought of as a division problem. See page 53.

RULE. *Divide the numerator by the denominator. If the denominator is not an exact divisor of the numerator, complete the division by writing the remainder over the denominator.*

Reduce each of the following to a whole or a mixed number:

*a.* $\frac{340}{8}$ *b.* $\frac{145}{5}$ *c.* $\frac{252}{9}$ *d.* $\frac{327}{6}$

*e.* $\frac{525}{7}$ *f.* $\frac{250}{4}$ *g.* $\frac{85}{10}$ *h.* $\frac{156}{12}$

*i.* $\frac{345}{6}$ *j.* $\frac{326}{4}$ *k.* $\frac{132}{6}$ *l.* $\frac{371}{7}$

*m.* Find the sum of the twelve answers, *a* to *l*.

Reduce each of the following to a whole or a mixed number:

*n.* $\frac{65}{59}$ *o.* $\frac{94}{25}$ *p.* $\frac{37}{12}$ *q.* $\frac{86}{15}$

*r.* $\frac{49}{38}$ *s.* $\frac{75}{37}$ *t.* $\frac{89}{34}$ *u.* $\frac{155}{26}$

*v.* $\frac{57}{19}$ *w.* $\frac{92}{23}$ *x.* $\frac{85}{17}$ *y.* $\frac{96}{16}$

*z.* $\frac{75}{21}$ *aa.* $\frac{63}{21}$ *bb.* $\frac{94}{21}$ *cc.* $\frac{104}{21}$

*dd.* Find the sum of the last four answers, *z* to *cc*.

**To reduce a mixed number to an improper fraction:**

**1.** Reduce $25\frac{3}{7}$ to an improper fraction.

OPERATION

$$\begin{array}{l} 25\frac{3}{7} = \frac{178}{7} \\ \underline{\phantom{2}7} \\ 178 \end{array}$$

EXPLANATION

There are 7 times as many sevenths as wholes in a number. 7 times 25 = 175. 175 + 3 = 178. Hence $25\frac{3}{7} = \frac{178}{7}$.

RULE. *Multiply the integer by the denominator of the fraction, and to the product add the numerator of the fraction; then write the sum over the denominator.*

Reduce each of the following to an improper fraction:

| | | | |
|---|---|---|---|
| *a.* $24\frac{5}{6}$ | *b.* $35\frac{3}{7}$ | *c.* $75\frac{1}{6}$ | *d.* $64\frac{4}{7}$ |
| *e.* $38\frac{1}{5}$ | *f.* $64\frac{2}{9}$ | *g.* $61\frac{4}{5}$ | *h.* $35\frac{7}{9}$ |
| *i.* $53\frac{2}{3}$ | *j.* $46\frac{1}{4}$ | *k.* $46\frac{1}{3}$ | *l.* $53\frac{3}{4}$ |

*m.* On counting the money in a bank, it was found that there were $425\frac{1}{4}$ dollars in "quarters." How many "quarters"?

*n.* Nine hundred thirty-seven half dollars are how many dollars?

*o.* How many fifth-acre lots can be made from a piece of land containing $93\frac{2}{5}$ acres?

*p.* In 235 fifth-acre lots there are how many acres?

*q.* A furnace in a certain manufactory consumes 1 fourth of a ton of coal each day. How many tons of coal are required in a year of 365 days?

*r.* A "nickel" is 1 twentieth of a dollar. How many "nickels" are equivalent to $234\frac{1}{2}$ dollars?

**To reduce a fraction to higher terms, one of which is specified:**

1. Reduce $\frac{5}{8}$ to 120ths.

OPERATION

$120 \div 8 = 15$ $\quad$ $15 \times 5 = 75$, the new numerator.
Hence $\frac{5}{8} = \frac{75}{120}$.

NOTE 1. Observe that in the above operation the form of the fraction is changed without changing its value; for in $\frac{75}{120}$ there are 15 times as many parts, each of which is 1 fifteenth as large as in $\frac{5}{8}$. Moreover, the ratio of the numerator to the denominator is unchanged.

2. Reduce $\frac{5}{8}$ to a fraction whose numerator is 30.*

OPERATION

$30 \div 5 = 6$ $\quad$ $6 \times 8 = 48$, the new denominator.
Hence $\frac{5}{8} = \frac{30}{48}$.

NOTE 2. By a reasoning process similar to that employed in Note 1, show that in the above reduction the form of the fraction has been changed without changing its value.

RULE. *Divide the specified term by the corresponding term of the given fraction, and multiply the other term of the fraction by the quotient thus obtained. The result will be the required term of the fraction in its new form.*

Reduce each of the following to 80ths:

*a.* $\frac{5}{8}$ *b.* $\frac{11}{16}$ *c.* $\frac{9}{20}$ *d.* $\frac{37}{40}$
*e.* $\frac{3}{40}$ *f.* $\frac{3}{8}$ *g.* $\frac{5}{16}$ *h.* $\frac{11}{20}$

*i.* Find the sum of the eight results, *a* to *h*.

---

* This problem has no bearing on any work that immediately follows. It may be omitted until this part of the book is reviewed, if the teacher so directs.

**To reduce two or more fractions whose denominators are unlike to equivalent fractions having a common denominator:**

1. Reduce $\frac{5}{12}$, $\frac{2}{3}$, and $\frac{3}{8}$ to equivalent fractions having a common denominator.

96 is a common multiple of 12, 3, and 8.
48 is a common multiple of 12, 3, and 8.
24 is a common multiple of 12, 3, and 8.

Any of these multiples may be used in the solution of this problem; that is, each of these fractions may be changed to 96ths, or to 48ths, or to 24ths, or to any one of an infinite number of multiples.

OPERATION

$$48 \div 12 = 4 \qquad \frac{5 \times 4}{12 \times 4} = \frac{20}{48}$$

$$48 \div 3 = 16 \qquad \frac{2 \times 16}{3 \times 16} = \frac{32}{48}$$

$$48 \div 8 = 6 \qquad \frac{3 \times 6}{8 \times 6} = \frac{18}{48}$$

NOTE. The answers would have been correct had these fractions been reduced to 24ths or to 96ths or to 120ths.

RULE. *Find by inspection and trial a common multiple of the denominators; then change each fraction to an equivalent fraction having the common multiple for its denominator.*

Reduce each of the following groups to equivalent fractions having a common denominator:

*a.* $\frac{3}{4}$, $\frac{2}{5}$, $\frac{1}{2}$ *b.* $\frac{5}{8}$, $\frac{2}{3}$, $\frac{1}{4}$ *c.* $\frac{4}{5}$, $\frac{3}{10}$, $\frac{1}{2}$

Two fractions whose denominators are unlike may be reduced to equivalent fractions having a common denominator by multiplying the denominators together for the common denominator and multiplying each numerator by the denominator of the other fraction for the new numerators.

**To reduce two or more fractions whose denominators are unlike to equivalent fractions having their *least common denominator*:**

1. Reduce $\frac{11}{30}$, $\frac{9}{40}$, and $\frac{37}{60}$ to equivalent fractions having their least common denominator.

OPERATION

The l. c. m. of 30, 40, and 60 is 120

$$\frac{11}{30}=\frac{44}{120} \qquad \frac{9}{40}=\frac{27}{120} \qquad \frac{37}{60}=\frac{74}{120}$$

RULE. *Find the l. c. m. of the denominators; then change each fraction to an equivalent fraction having the l. c. m. for its denominator.*

Reduce each of the following groups to equivalent fractions having their least common denominator:

| | |
|---|---|
| *a.* $\frac{11}{16}$ and $\frac{9}{20}$ | *b.* $\frac{4}{15}$, $\frac{2}{5}$, and $\frac{11}{20}$ |
| *c.* $\frac{19}{25}$ and $\frac{13}{20}$ | *d.* $\frac{6}{25}$, $\frac{1}{5}$, and $\frac{7}{20}$ |
| *e.* $\frac{23}{30}$ and $\frac{17}{40}$ | *f.* $\frac{7}{30}$, $\frac{2}{5}$, and $\frac{23}{40}$ |

Remembering that the g. c. d. of two numbers is never greater than their difference, reduce the following to their lowest terms:

*g.* $\frac{91}{98}$ *h.* $\frac{104}{112}$ *i.* $\frac{156}{168}$ *j.* $\frac{3420}{3438}$

*k.* Without dividing, tell how many times 25 is contained in 4275.

*l.* Reduce $\dfrac{37\frac{1}{2}}{62\frac{1}{2}}$ to its simplest form.*

---

* Divide the numerator and the denominator by $\frac{1}{2}$, and reduce the fraction thus obtained to its lowest terms. Or, divide the numerator and the denominator by $12\frac{1}{2}$.

**To find the sum of two or more fractions:**

1. Add $\frac{11}{45}$, $\frac{17}{30}$, and $\frac{53}{60}$.

OPERATION

(1) The l. c. m. of 45, 30, and 60 is 180.

(2) $\frac{11}{45}=\frac{44}{180}$  $\frac{17}{30}=\frac{102}{180}$  $\frac{53}{60}=\frac{159}{180}$

(3) $\frac{44}{180}+\frac{102}{180}+\frac{159}{180}=\frac{305}{180}$

NOTE. If the work that precedes this article has been well done, no explanation of the foregoing will be necessary. Pupils have already learned (presumably before using this book) (1) that fractions may be reduced to higher terms, (2) that two or more fractions whose denominators are not alike may be reduced to higher terms with like denominators, (3) that a common denominator of two or more fractions with unlike denominators is a common multiple of the given denominators, and (4) that in reducing a fraction to higher terms the numerator and denominator must be multiplied by the same number. The simple problem of adding 44 180ths, 102 180ths, and 159 180ths is not unlike the problem of adding 44 apples, 102 apples, and 159 apples.

RULE. *Reduce the fractions, if necessary, to equivalent fractions having a common denominator, add their numerators, and write their sum over the common denominator.*

Find the sum of:

*a*, $\frac{5}{16}$ and $\frac{7}{20}$
*b*. $\frac{11}{16}$, $\frac{1}{2}$, and $\frac{13}{20}$
*c*. $\frac{4}{15}$ and $\frac{9}{18}$
*d*. $\frac{11}{15}$, $\frac{1}{3}$, and $\frac{9}{18}$
*e*. $\frac{11}{21}$ and $\frac{5}{18}$
*f*. $\frac{10}{21}$, $\frac{1}{4}$, and $\frac{13}{18}$
*g*. $\frac{7}{15}$ and $\frac{13}{45}$
*h*. $\frac{8}{15}$, $\frac{2}{3}$, and $\frac{32}{45}$
*i*. $\frac{5}{14}$ and $\frac{7}{10}$
*j*. $\frac{9}{14}$, $\frac{1}{4}$, and $\frac{3}{10}$

*k*. Find the sum of the ten sums, *a* to *j*.

**To find the difference of two fractions:**

1. From $\frac{11}{25}$ subtract $\frac{9}{35}$.

OPERATION

The l. c. m. of 25 and 35 is 175.

$$\frac{11}{25} = \frac{77}{175} = 77\ 175\text{ths}$$
$$\frac{9}{35} = \frac{45}{175} = \underline{45\ 175\text{ths}}$$
$$32\ 175\text{ths}$$
$$\frac{11}{25} - \frac{9}{35} = \frac{32}{175}$$

Compare the following:

77 175ths – 45 175ths = 32 175ths
77 apples – 45 apples = 32 apples

RULE. *Reduce the fractions, if necessary, to equivalent fractions having a common denominator, find the difference of their numerators, and write it over the common denominator.*

Find the difference of:

*a.* $\frac{3}{5}$ and $\frac{5}{18}$   *b.* $\frac{9}{20}$ and $\frac{1}{3}$   *c.* $\frac{5}{9}$ and $\frac{1}{2}$
*d.* $\frac{2}{5}$ and $\frac{7}{18}$   *e.* $\frac{11}{20}$ and $\frac{1}{6}$   *f.* $\frac{4}{9}$ and $\frac{1}{3}$
*g.* Find the sum of the six differences, *a* to *f*.
*h.* From $38\frac{5}{9}$ subtract $14\frac{1}{2}$.   *i.* $54\frac{3}{5} - 12\frac{7}{20}$
*j.* From $46\frac{3}{7}$ subtract $25\frac{1}{4}$.   *k.* $32\frac{3}{8} - 26\frac{1}{5}$

*l.* From a piece of land containing $36\frac{3}{5}$ acres there were sold two lots, each containing $3\frac{1}{10}$ acres. How many acres remained unsold?

*m.* Mr. Jones owned $47\frac{1}{2}$ acres of land. He sold $15\frac{3}{8}$ acres. How many acres did he then have?

*n.* Mr. George owns $95\frac{5}{8}$ acres. Mr. Henry owns $48\frac{2}{5}$ acres. Mr. George owns how many acres more than Mr. Henry?

**To subtract one mixed number from another when the fraction in the subtrahend is greater than the fraction in the minuend:**

1. From $58\frac{3}{8}$ take $32\frac{2}{3}$.

OPERATION

$$
\begin{aligned}
58\tfrac{3}{8} = 57\tfrac{11}{8} &= 57 \text{ and } 33 \text{ 24ths}\\
32\tfrac{2}{3} &= \underline{32 \text{ and } 16 \text{ 24ths}}\\
&\phantom{=}\ 25 \text{ and } 17 \text{ 24ths}\\
58\tfrac{3}{8} - 32\tfrac{2}{3} &= 25\tfrac{17}{24}
\end{aligned}
$$

EXPLANATION

I can think of $58\frac{3}{8}$ as 57 and $1\frac{3}{8}$, or $57\frac{11}{8}$. After changing $\frac{11}{8}$ and $\frac{2}{3}$ to 24ths, the subtraction may be performed in the usual manner.

Find the difference of:

*a.* $24\frac{2}{7}$ and $16\frac{2}{5}$
*b.* $35\frac{1}{4}$ and $26\frac{1}{2}$
*c.* $29\frac{3}{8}$ and $15\frac{1}{2}$
*d.* $28\frac{3}{7}$ and $14\frac{3}{5}$
*e.* $46\frac{19}{56}$ and $18\frac{1}{2}$
*f.* $36\frac{9}{28}$ and $8\frac{1}{2}$
*g.* $52\frac{5}{8}$ and $31\frac{3}{5}$
*h.* $65\frac{1}{2}$ and $22\frac{3}{4}$
*i.* $47\frac{3}{8}$ and $18\frac{2}{5}$
*j.* $34\frac{1}{2}$ and $27\frac{1}{4}$

*k.* Find the sum of the ten differences, *a* to *j*.

Reduce to simplest form:

*l.* $5\frac{1}{2} + 3\frac{1}{2} - 5\frac{1}{4}$
*m.* $6\frac{2}{5} - 3\frac{1}{8} + 4\frac{1}{5}$
*n.* $2\frac{1}{5} - 1\frac{1}{4} + 3\frac{2}{5}$
*o.* $7\frac{3}{4} + 3\frac{3}{8} - 1\frac{3}{5}$
*p.* $6\frac{1}{2} - 3\frac{1}{4} + 5\frac{1}{5}$
*q.* $8\frac{2}{5} + 3\frac{1}{5} - 5\frac{1}{4}$
*r.* $5\frac{2}{5} - 2\frac{1}{4} + 6\frac{3}{5}$
*s.* $4\frac{1}{4} + 3\frac{3}{4} - 6\frac{1}{2}$
*t.* $7\frac{1}{2} - 8\frac{1}{4} + 3\frac{3}{4}$
*u.* $2\frac{1}{4} + 3\frac{1}{4} + 4\frac{1}{2}$
*v.* $3\frac{1}{4} + 4\frac{1}{2} - 3\frac{1}{8} - 2\frac{3}{8} + 1\frac{3}{4}$
*w.* $6\frac{3}{5} - 2\frac{1}{4} - 1\frac{3}{4} - 1\frac{21}{40} + 2\frac{1}{4}$
*x.* $5\frac{1}{2} + 4\frac{3}{8} + 2\frac{1}{8} + 3\frac{7}{8} + 3\frac{1}{4}$
*y.* Find the sum of the thirteen results, *l* to *x*.

**To multiply a fraction by an integer:**

1. Multiply $\frac{7}{24}$ by 6.

OPERATION NO. 1

$\frac{7}{24} \times 6 = \frac{42}{24} = 1\frac{3}{4}$

OPERATION NO. 2

$\frac{7}{24} \times 6 = \frac{7}{4} = 1\frac{3}{4}$

Observe that by the first operation we obtain $\frac{42}{24}$; that in $\frac{42}{24}$ there are 6 times as many parts as there are in $\frac{7}{24}$, and that the parts are of the same size as those in $\frac{7}{24}$.

Observe that by the second operation we obtain $\frac{7}{4}$; that in $\frac{7}{4}$ there are the same number of parts as there are in $\frac{7}{24}$, and that the parts are 6 times as great as those in $\frac{7}{24}$.

RULE. *Multiply the numerator or divide the denominator by the integer.*

Find the product:

*a.* $\frac{5}{12} \times 4$ *b.* $\frac{5}{4} \times 8$ *c.* $\frac{7}{12} \times 4$

*d.* $\frac{8}{15} \times 6$ *e.* $\frac{3}{5} \times 9$ *f.* $\frac{7}{15} \times 6$

*g.* $\frac{11}{20} \times 5$ *h.* $\frac{3}{4} \times 8$ *i.* $\frac{9}{20} \times 5$

*j.* $\frac{17}{30} \times 7$ *k.* $\frac{2}{5} \times 9$ *l.* $\frac{13}{30} \times 7$

*m.* Find the sum of the twelve products, *a* to *l*.

Find the product:

*n.* $3\frac{1}{2} \times 7$ *o.* $4\frac{2}{5} \times 5$ *p.* $6\frac{6}{10} \times 5$

*q.* $5\frac{3}{8} \times 6$ *r.* $1\frac{7}{8} \times 4$ *s.* $2\frac{13}{20} \times 4$

*t.* $7\frac{7}{20} \times 4$ *u.* $5\frac{3}{5} \times 5$ *v.* $4\frac{5}{8} \times 6$

*w.* $3\frac{4}{10} \times 5$ *x.* $8\frac{1}{8} \times 4$ *y.* $6\frac{1}{2} \times 7$

*z.* Find the sum of the twelve products, *n* to *y*.

Find the cost of:

*aa.* 240 eggs at $\frac{3}{4}$ of a cent each.

*bb.* 144 pens at $\frac{3}{8}$ of a cent each.

*cc.* 175 apples at $\frac{2}{5}$ of a cent each.

**To multiply an integer or a mixed number by a fraction:** [*]

1. Multiply 24 by $\frac{3}{4}$.

OPERATION NO. 1

$24 \div 4 = 6$

$6 \times 3 = 18$, *Ans.*

OPERATION NO. 2

$24 \times 3 = 72$

$72 \div 4 = 18$, *Ans.*

RULE. *Divide the multiplicand by the denominator of the fraction, and multiply the quotient thus obtained by the numerator of the fraction.*

Observe that in practice we may, if more convenient, multiply the multiplicand by the numerator of the fraction and divide the product thus obtained by the denominator of the fraction. To multiply 12 by $\frac{3}{4}$, we may take 3 times 1 fourth of 12, or 1 fourth of 3 times 12, as we choose. The result is correct in each instance.

Find the product:

*a.* $345 \times \frac{1}{2}$ *b.* $263 \times \frac{1}{3}$ *c.* $263 \times \frac{1}{4}$

*d.* $345 \times \frac{3}{10}$ *e.* $263 \times \frac{3}{4}$ *f.* $576 \times \frac{3}{8}$

*g.* $345 \times \frac{1}{5}$ *h.* $263 \times \frac{2}{3}$ *i.* $576 \times \frac{5}{8}$

*j.* Find the sum of the nine products, *a* to *i*.

2. Multiply $246\frac{1}{2}$ by $\frac{2}{3}$.

OPERATION NO. 1

$246\frac{1}{2} \div 3 = 82\frac{1}{6}$

$82\frac{1}{6} \times 2 = 164\frac{1}{3}$, *Ans.*

OPERATION NO. 2

$246\frac{1}{2} \times 2 = 493$

$493 \div 3 = 164\frac{1}{3}$, *Ans.*

*k.* [illegible] *l.* $523\frac{2}{3} \times \frac{1}{3}$ *m.* $523\frac{2}{3} \times \frac{1}{4}$

*n.* [illegible] $\times \frac{3}{10}$ *o.* $523\frac{2}{3} \times \frac{3}{4}$ *p.* $153\frac{1}{2} \times \frac{3}{8}$

---

[*] *To the Teacher.* Be sure that the pupil understands that to [illegible] by a fraction is to take such part of the multiplicand as [illegible] by the fraction.

## To multiply a fraction by a fraction:

**1.** Multiply $\frac{2}{5}$ by $\frac{3}{4}$.

| OPERATION | EXPLANATION |
|---|---|
| $\frac{2}{5} \times \frac{3}{4} = \frac{6}{20} = \frac{3}{10}$ | To multiply $\frac{2}{5}$ by $\frac{3}{4}$, is to find $\frac{3}{4}$ of $\frac{2}{5}$. By multiplying the denominator of $\frac{2}{5}$ by 4, we obtain one fourth of $\frac{2}{5}$, or $\frac{2}{20}$. By multiplying the numerator (of $\frac{2}{20}$) by 3, we obtain three fourths of $\frac{2}{5}$, or $\frac{6}{20}$. $\frac{6}{20} = \frac{3}{10}$. *Illustrative Problem:* At $\frac{2}{5}$ of a dollar a yard, how much will $\frac{3}{4}$ of a yard of ribbon cost? *Ans.* $\frac{3}{4}$ of $\frac{2}{5}$ of a dollar, or $\frac{3}{10}$ of a dollar. |

NOTE. Observe that $\frac{3}{4}$ of $\frac{2}{5}$ is exactly equal to $\frac{2}{5}$ of $\frac{3}{4}$; one fifth of $\frac{3}{4}$ is $\frac{3}{20}$, and two fifths of $\frac{3}{4}$ is $\frac{6}{20}$, or $\frac{3}{10}$. Hence in the multiplication of one fraction by another, either fraction may be thought of as the multiplicand, the other fraction being the multiplier.

RULE. *Multiply the numerators together for the numerator of the product and the denominators together for the denominator of the product.*

Find the product:

*a.* $\frac{7}{16} \times \frac{1}{2}$ *b.* $\frac{5}{8} \times \frac{1}{3}$ *c.* $\frac{5}{8} \times \frac{1}{4}$
*d.* $\frac{7}{16} \times \frac{3}{10}$ *e.* $\frac{5}{8} \times \frac{3}{4}$ *f.* $\frac{1}{2} \times \frac{3}{8}$
*g.* $\frac{7}{16} \times \frac{1}{5}$ *h.* $\frac{5}{8} \times \frac{2}{3}$ *i.* $\frac{1}{2} \times \frac{3}{4}$

**2.** What is the cost of:

| | |
|---|---|
| 3 tons of coal at \$6 per ton? | $\$6 \times 3 =$ |
| 2 tons of coal at \$6 per ton? | $\$6 \times 2 =$ |
| 1 ton of coal at \$6 per ton? | $\$6 \times 1 =$ |
| $\frac{1}{2}$ ton of coal at \$6 per ton? | $\$6 \times \frac{1}{2} =$ |

**3.** What is the cost of:

| | |
|---|---|
| 3 yd. ribbon at $\frac{1}{2}$ a dollar a yard? | $\$\frac{1}{2} \times 3 =$ |
| 2 yd. ribbon at $\frac{1}{2}$ a dollar a yard? | $\$\frac{1}{2} \times 2 =$ |
| 1 yd. ribbon at $\frac{1}{2}$ a dollar a yard? | $\$\frac{1}{2} \times 1 =$ |
| $\frac{1}{2}$ yd. ribbon at $\frac{1}{2}$ a dollar a yard? | $\$\frac{1}{2} \times \frac{1}{2} =$ |
| $\frac{1}{3}$ yd. ribbon at $\frac{1}{2}$ a dollar a yard? | $\$\frac{1}{2} \times \frac{1}{3} =$ |

**To multiply a mixed number by a mixed number:**

1. Multiply $346\frac{2}{5}$ by $2\frac{1}{2}$.

OPERATION

$$\begin{array}{r} 346\frac{2}{5} \\ \underline{2\frac{1}{2}} \\ 692\frac{4}{5} \\ \underline{173\frac{1}{5}} \\ 866 \end{array}$$

866 = two and a half times the multiplicand.

NOTE 1. Remember that to multiply a number by $2\frac{1}{2}$ is to take 2 times the number plus $\frac{1}{2}$ of the number.

RULE. *Find the product of the multiplicand and the integral part of the multiplier, to which add such part of the multiplicand as is indicated by the fractional part of the multiplier.*

Find the product:

*a.* $462\frac{2}{3} \times 2\frac{1}{10}$ *b.* $346\frac{1}{6} \times 3\frac{2}{3}$
*c.* $462\frac{2}{3} \times 3\frac{7}{10}$ *d.* $346\frac{1}{6} \times 2\frac{1}{4}$
*e.* $462\frac{2}{3} \times 2\frac{1}{5}$ *f.* $275\frac{1}{2} \times 4\frac{1}{2}$
*g.* $346\frac{1}{6} \times 2\frac{1}{3}$ *h.* $275\frac{1}{2} \times 3\frac{1}{4}$
*i.* $346\frac{1}{6} \times 3\frac{3}{4}$ *j.* $275\frac{1}{2} \times 2\frac{1}{4}$
*k.* Find the sum of the ten products, *a* to *j*.

NOTE 2. One mixed number may be multiplied by another mixed number by reducing both to improper fractions and then multiplying as directed in the rule on page 65.

Solve the following as suggested by Note 2. Prove, by solving a second time, following the rule given at the top of this page:

*l.* $1\frac{1}{2} \times 2\frac{1}{2}$ *m.* $2\frac{1}{4} \times 1\frac{1}{2}$ *n.* $3\frac{1}{2} \times 2\frac{1}{2}$
*o.* $2\frac{1}{3} \times 2\frac{1}{2}$ *p.* $2\frac{1}{2} \times 1\frac{1}{4}$ *q.* $4\frac{1}{2} \times 2\frac{1}{2}$
*r.* $5\frac{1}{2} \times 2\frac{1}{4}$ *s.* $4\frac{1}{3} \times 2\frac{1}{4}$ *t.* $3\frac{1}{2} \times 2\frac{1}{3}$

## To reduce compound fractions to simple fractions:

NOTE. The reduction of the so-called compound fraction is in no way unlike multiplication of fractions — it *is* multiplication of fractions. The finding of $\frac{2}{3}$ of $\frac{4}{5}$ (the reducing of this "compound fraction" to a simple fraction) is simply multiplying $\frac{4}{5}$ by $\frac{2}{3}$. No new rule is required. The rule for multiplying one fraction by another fraction is all that is necessary. See page 65.

Reduce the following to simple fractions:

*a.* $\frac{3}{4}$ of $\frac{2}{3}$. *b.* $\frac{1}{3}$ of $\frac{3}{4}$. *c.* $\frac{1}{2}$ of $\frac{5}{6}$. *d.* $\frac{1}{6}$ of $\frac{1}{2}$.
*e.* $\frac{2}{3}$ of $\frac{5}{8}$. *f.* $\frac{1}{3}$ of $\frac{1}{8}$. *g.* $\frac{5}{8}$ of $\frac{1}{3}$. *h.* $\frac{3}{8}$ of $\frac{2}{3}$.

*i.* Find the sum of the eight results, *a* to *h*.

*j.* James received $\frac{1}{4}$ of $\frac{2}{3}$ of his father's farm. Express James's share as a simple fraction of the entire farm.

*k.* Henry received $\frac{1}{4}$ of $\frac{2}{3}$ of his father's farm and afterward purchased his mother's share, $\frac{1}{3}$ of the farm. What part of the entire farm did he then own?

*l.* Change $\frac{2}{3}$ to 144ths.

*m.* George rode his wheel $24\frac{3}{4}$ miles Monday, $26\frac{1}{5}$ miles Tuesday, and returned over the same course Wednesday. How far did he ride Wednesday?

*n.* Henry lives $16\frac{1}{4}$ miles north of Chicago. His cousin lives $33\frac{3}{4}$ miles south of Chicago. If Henry can go on his wheel at the rate of 8 miles an hour, how long will he be in going from his own home to his cousin's?

*o.* At $\frac{3}{4}$ of a cent a pound, how much will 1 ton (2000 lb.) of oats cost?

*p.* At $\frac{3}{4}$ of a cent a pound, how much will 1 bushel (32 lb.) of oats cost?

**To divide a fraction by an integer:**

**1.** Divide $\frac{6}{7}$ by 3.

OPERATION NO. 1

$$\frac{6}{7} \div 3 = \frac{2}{7}$$

OPERATION NO. 2

$$\frac{6}{7} \div 3 = \frac{6}{21} = \frac{2}{7}$$

NOTE 1. Observe that by the first operation we obtain $\frac{2}{7}$; that in $\frac{2}{7}$ there are 1 third as many parts as there are in $\frac{6}{7}$, and that the parts are of the same size as those in $\frac{6}{7}$.

NOTE 2. Observe that by the second operation we obtain $\frac{6}{21}$; that in $\frac{6}{21}$ there are the same number of parts as there are in $\frac{6}{7}$, and that the parts are 1 third as great as those in $\frac{6}{7}$.

RULE. *Divide its numerator, or multiply its denominator, by the integer.*

Find the quotient:

*a.* $\frac{7}{12} \div 4$   *b.* $\frac{5}{6} \div 4$   *c.* $\frac{17}{24} \div 4$

*d.* $\frac{7}{12} \div 5$   *e.* $\frac{5}{6} \div 5$   *f.* $\frac{17}{24} \div 5$

*g.* $\frac{7}{12} \div 20$   *h.* $\frac{5}{6} \div 20$   *i.* $\frac{17}{24} \div 20$

*j.* Find the sum of the nine quotients, *a* to *i*.

Find the quotient:

*k.* $17\frac{1}{5} \div 3$   *l.* $18\frac{3}{10} \div 3$   *m.* $16\frac{1}{2} \div 3$

*n.* $17\frac{1}{5} \div 6$   *o.* $18\frac{3}{10} \div 6$   *p.* $16\frac{1}{2} \div 6$

*q.* Find the sum of the six quotients, *k* to *p*.

*r.* If a farm of $277\frac{1}{2}$ acres is divided into 3 equal fields, how many acres in each field?

*s.* A railroad train moved $178\frac{1}{4}$ miles in 4 hours. What was its average rate per hour?

*t.* If $\frac{9}{10}$ of an acre of land is divided into 4 equal lots, what part of an acre in each lot? *u.* How many square rods in each lot? *v.* If each lot is 4 rods wide, how long is it?

**To divide by a fraction or by a mixed number:**

**1.** Divide 345 by $\frac{5}{12}$.

OPERATION

$$\begin{array}{r|r} \frac{5}{12} & 345 \\ & 12 \\ \hline 5 & 4140 \\ \cline{2-2} & 828 \end{array}$$

EXPLANATION

345 = 4140 twelfths. 5 twelfths are contained in 4140 twelfths, 828 times.

RULE. *Express the divisor and the dividend in like fractional units; then find how many times the former is contained in the latter.*

Find the quotient:

*a.* $702 \div \frac{3}{5}$

*b.* $336 \div \frac{3}{5}$

*c.* $1680 \div \frac{3}{5}$

*d.* $702 \div \frac{3}{4}$

*e.* $336 \div \frac{3}{4}$

*f.* $1680 \div \frac{3}{4}$

*g.* $702 \div \frac{6}{7}$

*h.* $336 \div \frac{6}{7}$

*i.* $1680 \div \frac{6}{7}$

*j.* $702 \div \frac{6}{5}$

*k.* $336 \div \frac{6}{5}$

*l.* $1680 \div \frac{6}{5}$

*m.* Find the sum of the twelve quotients, *a* to *l*.

**2.** Divide 341 by $2\frac{3}{4}$.

OPERATION

$$\begin{array}{r|r} 2\frac{3}{4} & 341 \\ 4 & 4 \\ \hline 11 & 1364 \\ \cline{2-2} & 124 \end{array}$$

EXPLANATION

$2\frac{3}{4}$ = 11 fourths. 341 = 1364 fourths. 11 fourths are contained in 1364 fourths, 124 times.

Find the quotient:

*n.* $84625 \div 3\frac{1}{3}$

*o.* $12573 \div 4\frac{1}{2}$

*p.* $2538 \div 3\frac{3}{4}$

*q.* $84625 \div 1\frac{3}{7}$

*r.* $12573 \div 1\frac{2}{7}$

*s.* $2538 \div 1\frac{4}{11}$

*t.* $25638 \div 2\frac{1}{2}$

*u.* $33472 \div 2\frac{2}{3}$

*v.* $4216 \div 8\frac{1}{2}$

*w.* $25638 \div 1\frac{2}{3}$

*x.* $33472 \div 1\frac{3}{5}$

*y.* $4216 \div 1\frac{2}{15}$

*z.* Find the sum of the twelve quotients, *n* to *y*.

**To divide a fraction by a fraction:**

1. Divide $\frac{3}{5}$ by $\frac{1}{4}$.

OPERATION

| | |
|---|---|
| $\frac{1}{4}$ | $\frac{3}{5}$ |
| 5 20ths | 12 20ths * |
| | $2\frac{2}{5}$ times |

EXPLANATION

I can change 4ths and 5ths to twentieths. $\frac{1}{4} = 5$ twentieths, $\frac{3}{5} = 12$ twentieths. 5 twentieths are contained in 12 twentieths, $2\frac{2}{5}$ times.

NOTE 1. The rule given on page 69 will apply in this case. *Express the divisor and the dividend in like fractional units; that is, reduce them to equivalent fractions having a common denominator; then find how many times the numerator of the divisor is contained in the numerator of the dividend.*

Find the quotients:

*a.* $\frac{5}{8} \div \frac{1}{6}$ *b.* $\frac{7}{9} \div \frac{2}{3}$ *c.* $\frac{6}{7} \div \frac{1}{4}$

*d.* $\frac{5}{8} \div \frac{1}{4}$ *e.* $\frac{7}{9} \div \frac{1}{3}$ *f.* $\frac{6}{7} \div \frac{3}{4}$

*g.* Find the sum of the six quotients, *a* to *f*.

---

*h.* If a small furnace consumes $\frac{1}{10}$ of a ton of coke in a day, in how many days will $\frac{5}{8}$ of a ton be consumed?

*i.* If a yard of ribbon is worth $\frac{1}{2}$ of a dollar, how much is $\frac{3}{4}$ of a yard worth?

*j.* Change $\frac{5}{8}$ to 40ths and add it to $\frac{11}{40}$; then reduce the sum to its lowest terms.

*k.* Change $\frac{7}{12}$ to 60ths and subtract it from $\frac{47}{60}$; then reduce the difference to its lowest terms.

*l.* If eggs are worth $1\frac{1}{3}$ cents each, how many can be bought for $10 (1000 cents)?

---

* In practice "20ths" may be omitted — may be simply understood. It must be thought of as the name of the unit.

There is another method of dividing by a fraction. It is to multiply the dividend by the reciprocal of the divisor,* that is, by the "inverted" divisor; or,

RULE. *Invert the divisor and proceed as in multiplication.*

1. Divide $\frac{3}{4}$ by $\frac{2}{3}$.

| OPERATION | EXPLANATION |
|---|---|
| $\frac{3}{4} \div \frac{2}{3} = \frac{3}{4} \times \frac{3}{2} = \frac{9}{8} = 1\frac{1}{8}$ | $1 \div \frac{2}{3} = \frac{3}{2}$;† then $\frac{3}{4} \div \frac{2}{3} = \frac{3}{4}$ of $\frac{3}{2} = \frac{9}{8} = 1\frac{1}{8}$. $\frac{2}{3}$ is contained in $\frac{3}{4}$, $1\frac{1}{8}$ times. |

2. Divide 4 by $\frac{2}{3}$.

| | |
|---|---|
| $\frac{4}{1} \div \frac{2}{3} = \frac{4}{1} \times \frac{3}{2} = \frac{12}{2} = 6$ | $1 \div \frac{2}{3} = \frac{3}{2}$; then $4 \div \frac{2}{3} = 4$ times $\frac{3}{2} = \frac{12}{2} = 6$. $\frac{2}{3}$ is contained in 4, 6 times. |

3. Divide $5\frac{1}{2}$ by $\frac{2}{3}$.

| | |
|---|---|
| $5\frac{1}{2} = \frac{11}{2}$<br>$\frac{11}{2} \div \frac{2}{3} = \frac{11}{2} \times \frac{3}{2} = \frac{33}{4} = 8\frac{1}{4}$ | $1 \div \frac{2}{3} = \frac{3}{2}$; then $5\frac{1}{2} \div \frac{2}{3} = 5\frac{1}{2}$ times $\frac{3}{2} = \frac{33}{4} = 8\frac{1}{4}$. $\frac{2}{3}$ is contained in $5\frac{1}{2}$, $8\frac{1}{4}$ times. |

NOTE 1. The pupil should now prove each of the above solutions by solving as directed on page 69.

Find the quotient by "inverting the divisor," etc., then prove by "expressing the dividend and the divisor in like fractional units," etc.:

*a.* $\frac{7}{10} \div \frac{2}{5}$ *b.* $7 \div \frac{2}{5}$ *c.* $3\frac{1}{2} \div \frac{2}{5}$
*d.* $\frac{7}{10} \div \frac{2}{7}$ *e.* $7 \div \frac{2}{7}$ *f.* $3\frac{1}{2} \div \frac{2}{7}$
*g.* $\frac{7}{10} \div \frac{3}{4}$ *h.* $7 \div \frac{3}{4}$ *i.* $3\frac{1}{2} \div \frac{3}{4}$
*j.* $\frac{7}{10} \div \frac{3}{8}$ *k.* $7 \div \frac{3}{8}$ *l.* $3\frac{1}{2} \div \frac{3}{8}$

*m.* Find the sum of the twelve quotients, *a* to *l*.

---

* The reciprocal of a fraction is the "inverted" fraction; it is the quotient obtained by dividing 1 by the fraction.

† $\frac{2}{3}$ are contained in 1 (3 thirds) $1\frac{1}{2}$ or $\frac{3}{2}$ times.

Find the quotient:

*a.* $46 \div \frac{1}{2}$ *b.* $375 \div \frac{1}{2}$ *c.* $196 \div \frac{1}{2}$

*d.* $46 \div \frac{3}{4}$ *e.* $375 \div \frac{3}{4}$ *f.* $196 \div \frac{3}{4}$

*g.* $46 \div \frac{3}{5}$ *h.* $375 \div \frac{3}{5}$ *i.* $196 \div \frac{3}{5}$

*j.* Find the sum of the nine quotients, *a* to *i*.*

Find the quotient:

*k.* $\frac{7}{8} \div \frac{2}{3}$ *l.* $\frac{5}{12} \div \frac{2}{3}$ *m.* $\frac{11}{24} \div \frac{2}{3}$

*n.* $\frac{7}{8} \div \frac{5}{6}$ *o.* $\frac{5}{12} \div \frac{5}{6}$ *p.* $\frac{11}{24} \div \frac{5}{6}$

*q.* $\frac{7}{8} \div \frac{10}{13}$ *r.* $\frac{5}{12} \div \frac{10}{13}$ *s.* $\frac{11}{24} \div \frac{10}{13}$

*t.* Find the sum of the nine quotients, *k* to *s*.†

Find the quotient:

*u.* $5\frac{1}{2} \div \frac{3}{4}$ *v.* $24\frac{2}{3} \div \frac{3}{4}$ *w.* $19\frac{5}{6} \div \frac{3}{4}$

*x.* $5\frac{1}{2} \div \frac{6}{7}$ *y.* $24\frac{2}{3} \div \frac{6}{7}$ *z.* $19\frac{5}{6} \div \frac{6}{7}$

*aa.* $5\frac{1}{2} \div \frac{2}{5}$ *bb.* $24\frac{2}{3} \div \frac{2}{5}$ *cc.* $19\frac{5}{6} \div \frac{2}{5}$

*dd.* Find the sum of the nine quotients, *u* to *cc*.‡

Find the quotient:

*ee.* $325\frac{1}{2} \div 2\frac{1}{2}$ *ff.* $174\frac{1}{2} \div 2\frac{1}{2}$

*gg.* $325\frac{1}{2} \div 2\frac{2}{5}$ *hh.* $174\frac{1}{2} \div 2\frac{2}{5}$

*ii.* $325\frac{1}{2} \div 3\frac{1}{3}$ *jj.* $174\frac{1}{2} \div 3\frac{1}{3}$

*kk.* $325\frac{1}{2} \div 7\frac{1}{2}$ *ll.* $174\frac{1}{2} \div 7\frac{1}{2}$

*mm.* $325\frac{1}{2} \div 1\frac{1}{3}$ *nn.* $174\frac{1}{2} \div 1\frac{1}{3}$

*oo.* Find the sum of the ten quotients, *ee* to *nn*.

---

* Because $\frac{2}{1} + \frac{4}{3} + \frac{5}{3} = 5$, the sum of the nine quotients is equal to $(46 + 375 + 196) \times 5$.

† Because $\frac{3}{2} + \frac{6}{5} + \frac{13}{10} = 4$, the sum of the nine quotients is equal to $(\frac{7}{8} + \frac{5}{12} + \frac{11}{24}) \times 4$.

‡ Because $\frac{4}{3} + \frac{7}{6} + \frac{5}{2} = 5$, the sum of the nine quotients is equal to $(5\frac{1}{2} + 24\frac{2}{3} + 19\frac{5}{6}) \times 5$.

### To reduce complex fractions to simple fractions:

NOTE 1. Observe that every complex fraction may be regarded as a problem in division.

$\frac{\frac{3}{4}}{\frac{7}{8}}$ is read, $\frac{3}{4}$ divided by $\frac{7}{8}$. $\frac{3}{4} \div \frac{7}{8} = \frac{3}{4}$ of $\frac{8}{7} = \frac{24}{28} = \frac{6}{7}$

Reduce the following to their simplest forms:

*a.* $\frac{\frac{5}{6}}{\frac{1}{2}}$ *b.* $\frac{\frac{5}{6}}{\frac{3}{4}}$ *c.* $\frac{\frac{5}{6}}{\frac{3}{5}}$ *d.* $\frac{\frac{5}{6}}{\frac{1}{5}}$

*e.* Find the sum of the four fractions, *a* to *d*.*

NOTE 2. Observe that a complex fraction may be reduced to a simple fraction by multiplying or dividing its numerator and its denominator by some number that will in each case give an integral result. When this number can be easily discovered, this is a convenient method of reduction; thus,

$$\frac{\frac{1}{2}}{7} = \frac{\frac{1}{2} \times 2}{7 \times 2} = \frac{1}{14} \qquad \frac{2\frac{1}{2}}{5\frac{1}{2}} = \frac{2\frac{1}{2} \div \frac{1}{2}}{5\frac{1}{2} \div \frac{1}{2}} = \frac{5}{11}$$

Reduce the following to their simplest forms:

*f.* $\frac{\frac{5}{6}}{5}$ *g.* $\frac{\frac{1}{3}}{4}$ *h.* $\frac{5}{\frac{1}{2}}$ *i.* $\frac{1\frac{1}{2}}{2}$

*j.* Find the sum of the four fractions, *f* to *i*.

Reduce the following to their simplest forms:

*k.* $\frac{8}{\frac{1}{2}}$ *l.* $\frac{8}{\frac{3}{4}}$ *m.* $\frac{8}{\frac{3}{5}}$ *n.* $\frac{8}{\frac{1}{5}}$

*o.* Find the sum of the four fractions, *k* to *n*.

Reduce the following to their simplest forms:

*p.* $\frac{12\frac{1}{2}}{100}$ *q.* $\frac{16\frac{2}{3}}{100}$ *r.* $\frac{66\frac{2}{3}}{100}$ *s.* $\frac{87\frac{1}{2}}{100}$

*t.* Find the sum of the four fractions, *p* to *s*.

---

* Because $\frac{2}{1} + \frac{4}{3} + \frac{5}{3} + \frac{5}{1} = 10$, the sum of the four fractions is equal to $\frac{5}{6}$ multiplied by 10.

## CANCELLATION

When one number is to be divided by another, the striking out (canceling) of one or more factors common to both dividend and divisor will not change the quotient. In the problem, Divide 84 by 21, we know that the quotient is 4. But we may think of 84 as 7 × 12, and of 21 as 3 × 7. Putting this in the form of a fraction with the dividend above the line and the divisor below it, we have $\frac{7 \times 12}{3 \times 7}$. Canceling the common factor, 7, $\left(\frac{\cancel{7} \times 12}{3 \times \cancel{7}}\right)$, we have 12 divided by 3 = 4. This process for shortening a division problem is called **cancellation.**

1. Divide 8 × 36 × 20 by 4 × 10 × 24.

**OPERATION**

$$\frac{\overset{\cancel{2}}{\cancel{8}} \times \overset{3}{\cancel{36}} \times \overset{2}{\cancel{20}}}{\cancel{4} \times \cancel{10} \times \underset{\cancel{2}}{\cancel{24}}} = \frac{2 \times 3}{1} = 6$$

**EXPLANATION**

Canceling the 4 and 8 and writing 2 above the 8 takes the factor 4 out of both divisor and dividend.

Canceling the 10 and 20 and writing 2 above the 20 takes the factor 10 out of both divisor and dividend.

Canceling the 24 and 36 and writing 2 below the 24 and 3 above the 36 takes the factor 12 out of both the divisor and dividend.

Canceling the 2 in the divisor and one of the 2's in the dividend takes the factor 2 out of both divisor and dividend. There remain 3 × 2 in the dividend and 1 in the divisor. Hence the quotient is 6.

Prove by solving without cancellation and comparing the result with that given above.

NOTE. If all the factors in the dividend or in the divisor are canceled, the unwritten factor 1 remains.

*a.* Divide 12 × 15 × 18 by 6 × 20 × 9.

*b.* Divide 11 × 42 × 12 by 8 × 21 × 15.

*c.* Divide 15 × 36 × 72 by 9 × 14 × 18.

In the multiplication of fractions, page 65, since the numerator of the answer is the product of the several numerators, and the denominator of the answer the product of the several denominators, cancellation may be employed.

1. Multiply $\frac{5}{8}$ by $\frac{2}{7}$.

OPERATION

$$\frac{5}{\underset{4}{\cancel{8}}}\times\frac{\cancel{2}}{7}=\frac{5}{4\times 7}=\frac{5}{28}$$

EXPLANATION

Canceling the factor 2 in both the numerator and the denominator, there remains 5 over 4 × 7 = $\frac{5}{28}$.

Employing cancellation whenever possible, solve the following:

*a.* Multiply $\frac{6}{11}$ by $\frac{2}{3}$. *b.* Multiply $\frac{6}{11}$ by $\frac{1}{3}$.

*c.* Find the sum of the answers to *a* and *b.**

*d.* Multiply $\frac{5}{12}$ by $\frac{3}{10}$. *e.* Multiply $\frac{5}{12}$ by $\frac{7}{10}$.

*f.* Find the sum of the answers to *d* and *e.*

Observe that cancellation may sometimes be employed in reducing compound fractions to simple fractions. See page 67.

Employing cancellation whenever possible, solve the following:

*g.* Find $\frac{3}{4}$ of $\frac{6}{11}$. *h.* Find $\frac{1}{4}$ of $\frac{6}{11}$.

*i.* Find the sum of the answers to *g* and *h.*†

*j.* Find $\frac{2}{3}$ of $\frac{7}{8}$ of $\frac{9}{10}$. *k.* Find $\frac{1}{3}$ of $\frac{7}{8}$ of $\frac{9}{10}$.

*l.* Find the sum of the answers to *j* and *k.*

*m.* Find $\frac{1}{2}$ of $\frac{3}{4}$ of 56. *n.* Find $\frac{1}{2}$ of $\frac{1}{4}$ of 56.

*o.* Find the sum of the answers to *m* and *n.*

---

* Why is the answer to *c* the same as the first fraction in *a* and *b*?

† Why is the answer to *i* the same as the second fraction in *g* and *h*?

In the division of fractions, page 71, *after the inversion of the divisor*, cancellation may be employed.

1. Divide $\frac{7}{8}$ by $\frac{3}{4}$.

OPERATION

$$\frac{7}{8} \div \frac{3}{4} = \frac{7}{\underset{2}{\cancel{8}}} \times \frac{\cancel{4}}{3} = \frac{7}{2 \times 3} = \frac{7}{6} = 1\frac{1}{6}$$

This means that $\frac{3}{4}$ is contained in $\frac{7}{8}$, $1\frac{1}{6}$ times. Prove by reducing the fractions to equivalent fractions having a common denominator, and dividing the numerator of the dividend by the numerator of the divisor. (In this instance the latter method is the shorter; since 4ths can be reduced to 8ths and the solution completed by dividing the numerator 7 by the numerator 6.)

Employing cancellation whenever possible, solve the following:

*a.* Divide $\frac{8}{9}$ by $\frac{2}{3}$.
*b.* Divide $\frac{8}{9}$ by $\frac{2}{5}$.
*c.* Find the sum of the answers to *a* and *b*.*
*d.* Divide $\frac{7}{12}$ by $\frac{3}{4}$.
*e.* Divide $\frac{7}{12}$ by $\frac{3}{8}$.
*f.* Find the sum of the answers to *d* and *e*.†

Observe that cancellation may sometimes be employed in reducing complex fractions to simple fractions. See page 73.

Employing cancellation whenever possible, reduce the following to simple fractions in their lowest terms.

*g.* $\dfrac{\frac{5}{6}}{\frac{3}{10}}$ *h.* $\dfrac{\frac{5}{6}}{\frac{3}{8}}$ *i.* $\dfrac{\frac{5}{6}}{\frac{3}{5}}$ *j.* $\dfrac{\frac{5}{6}}{\frac{3}{7}}$

*k.* Find the sum of the four fractions, *g* to *j*.‡

---

* This answer should be exactly 4 times $\frac{8}{9}$, because the answer to *a* is $\frac{3}{2}$ of $\frac{8}{9}$, and the answer to *b* is $\frac{5}{2}$ of $\frac{8}{9}$; and $\frac{3}{2} + \frac{5}{2}$ are $\frac{8}{2}$, or 4.

† The answer to *f* is how many times $\frac{7}{12}$?

‡ The answer to *k* is exactly 10 times $\frac{5}{6}$. Can you see why?

In the multiplication of a fraction by an integer, or of an integer by a fraction, cancellation may sometimes be advantageously employed. In such cases the integer may be thought of as the numerator of a fraction whose denominator is 1.

1. Find the product of $\frac{2}{3}$ of 42 multiplied by $\frac{3}{7}$.

OPERATION

$$\frac{2}{\cancel{3}} \times \frac{\overset{6}{\cancel{42}}}{1} \times \frac{\cancel{3}}{\cancel{7}} = \frac{2 \times 6}{1} = 12$$

Solve the following, using cancellation whenever possible:

*a.* Find the product of $\frac{2}{5}$ of 75 multiplied by $\frac{3}{8}$.
*b.* Find the product of $\frac{3}{5}$ of 75 multiplied by $\frac{3}{8}$.
*c.* Find the product of $\frac{2}{5}$ of 75 multiplied by $\frac{5}{8}$.
*d.* Find the product of $\frac{3}{5}$ of 75 multiplied by $\frac{5}{8}$.
*e.* Find the sum of the four answers, *a* to *d*.
*f.* Find the product of 215 multiplied by $\frac{3}{5}$ of $\frac{5}{7}$.
*g.* Find the product of 215 multiplied by $\frac{2}{5}$ of $\frac{5}{7}$.
*h.* Find the product of 215 multiplied by $\frac{3}{5}$ of $\frac{2}{7}$.
*i.* Find the product of 215 multiplied by $\frac{2}{5}$ of $\frac{2}{7}$.
*j.* Find the sum of the four answers, *f* to *i*.
*k.* Four ninths of $\frac{2}{3}$ of 99 is what number?
*l.* Five ninths of $\frac{2}{3}$ of 99 is what number?
*m.* Four ninths of $\frac{1}{3}$ of 99 is what number?
*n.* Five ninths of $\frac{1}{3}$ of 99 is what number?
*o.* Find the sum of the four answers, *k* to *n*.

**To add two fractions, each of whose numerators is 1:**

1. Add $\frac{1}{5}$ and $\frac{1}{7}$.

OPERATION

$$\frac{1}{5}+\frac{1}{7},\quad \frac{5+7=12}{5\times 7=35}\ \textit{Ans.}$$

EXPLANATION

Reducing these fractions to equivalent fractions having a common denominator by the method given on page 58, the common denominator is 5 × 7 (35), and the numerator of the first fraction is 7, and of the second is 5; the sum of the fractions is therefore (7 + 5) 12 35ths.

RULE. *Take the sum of the denominators for the numerator of the sum of the fractions, and the product of the denominators for the denominator of the sum of the fractions.*

Without using a pencil, give the answer to each of the following:

$\frac{1}{2}+\frac{1}{3}$
$\frac{1}{2}+\frac{1}{7}$
$\frac{1}{3}+\frac{1}{8}$
$\frac{1}{3}+\frac{1}{4}$
$\frac{1}{3}+\frac{1}{7}$
$\frac{1}{5}+\frac{1}{8}$
$\frac{1}{3}+\frac{1}{5}$
$\frac{1}{4}+\frac{1}{7}$
$\frac{1}{7}+\frac{1}{8}$
$\frac{1}{2}+\frac{1}{5}$
$\frac{1}{5}+\frac{1}{7}$
$\frac{1}{2}+\frac{1}{9}$

**To find the difference of two fractions, each of whose numerators is 1:**

RULE. *Take the difference of the denominators for the numerator of the difference of the fractions, and the product of the denominators for the denominator of the difference of the fractions.*

Without using a pencil, give the answer to each of the following:

$\frac{1}{2}-\frac{1}{8}$
$\frac{1}{2}-\frac{1}{7}$
$\frac{1}{3}-\frac{1}{8}$
$\frac{1}{3}-\frac{1}{4}$
$\frac{1}{3}-\frac{1}{7}$
$\frac{1}{5}-\frac{1}{8}$
$\frac{1}{3}-\frac{1}{5}$
$\frac{1}{4}-\frac{1}{7}$
$\frac{1}{7}-\frac{1}{8}$
$\frac{1}{2}-\frac{1}{5}$
$\frac{1}{5}-\frac{1}{7}$
$\frac{1}{2}-\frac{1}{9}$

**To multiply a mixed number whose fraction is ½ by itself**; as, 8½ by 8½, 5½ by 5½, etc.:

1. Multiply 8½ by 8½.

OPERATION

8½ × 8½ ; . 9 × 8 = 72
72 + ¼ = 72¼

EXPLANATION

To multiply 8½ by 8½, is to find 8 eights and 1 half of 8 and 1 half of 8, that is 9 eights, or 72, plus 1 half multiplied by 1 half, or 72¼. Prove by taking 8 times 8½ plus 1 half of 8½.

RULE. *Multiply the integral part of the number plus 1 by the integral part of the number, and to the product add* ¼.

Without using a pencil, give the answer to the following:

2½ × 2½ 3½ × 3½ 4½ × 4½
5½ × 5½ 6½ × 6½ 7½ × 7½

NOTE 1. Observe that the foregoing principle will apply when the fraction, 1 half, is expressed decimally. Thus 2.5 multiplied by 2.5 = 2 times 3, + .25 = 6.25; 3.5 × 3.5 = 12.25; 4.5 × 4.5 = 20.25, etc.

Without using a pencil, give the answer to each of the following:

5.5 × 5.5 6.5 × 6.5 7.5 × 7.5
8.5 × 8.5 9.5 × 9.5 10.5 × 10.5

NOTE 2. Observe that the foregoing principle will apply in the multiplication of 25 by 25, 35 by 35, etc., since 25 may be thought of as 2½ tens; 35, as 3½ tens, etc.

25 × 25 35 × 35 45 × 45
55 × 55 65 × 65 75 × 75
85 × 85 95 × 95 15 × 15

**To change a decimal to a common fraction:**

**1.** Change .36 to a common fraction in its lowest terms.

OPERATION

$$.36 = \frac{36}{100} = \frac{9}{25}$$

EXPLANATION

The numerator of .36 is 36, the denominator is 100. Reducing $\frac{36}{100}$ to its lowest terms, we have $\frac{9}{25}$.

RULE. *Omitting the decimal point, write the numerator over the denominator and reduce the fraction to its lowest terms.*

Reduce the following to common fractions in their lowest terms:

*a.* .45 *b.* .375 *c.* .55 *d.* .625

*e.* .045 *f.* .0375 *g.* .055 *h.* .0625

*i.* Find the sum of the eight decimals.

*j.* Find the sum of the eight common fractions.

**To reduce a complex decimal to a common fraction.**

NOTE. No new rule is required. Follow the direction given above.

**2.** Change $.27\frac{3}{11}$ to a common fraction in its lowest terms.

OPERATION

$$.27\tfrac{3}{11} = \frac{27\frac{3}{11}}{100}$$

$$27\tfrac{3}{11} = \frac{300}{11}$$

$$\frac{300}{11} \div 100 = \frac{300}{1100} = \frac{3}{11}$$

EXPLANATION

The numerator of $.27\frac{3}{11}$ is $27\frac{3}{11}$; the denominator is 100. Changing the numerator to 11ths and dividing by 100, we have $\frac{3}{11}$.

Reduce the following to common fractions in their lowest terms:

*k.* $.38\frac{1}{3}$ *l.* $.24\frac{2}{11}$ *m.* $.61\frac{2}{3}$

### To change a common fraction to a decimal:

1. Change $\frac{3}{8}$ to a decimal.

OPERATION

$$\begin{array}{r} 8\overline{)3.000} \\ .375 \end{array}$$

EXPLANATION

3 over 8 means 3 divided by 8. We therefore perform the division indicated. One eighth of 30 tenths is 3 tenths with a remainder of 6 tenths. 6 tenths equal 60 hundredths; 1 eighth of 60 hundredths is 7 hundredths, with a remainder of 4 hundredths. 4 hundredths equal 40 thousandths, 1 eighth of 40 thousandths is 5 thousandths. So 1 eighth of 3 is 3 tenths, 7 hundredths, and 5 thousandths, or 375 thousandths.

RULE. *Write the numerator of the fraction with a decimal point to the right of it; annex one or more zeros and divide by the denominator.*

Reduce the following to decimals:

*a.* $\frac{3}{4}$ *b.* $\frac{3}{5}$ *c.* $\frac{1}{4}$ *d.* $\frac{2}{5}$

*e.* $\frac{3}{40}$ *f.* $\frac{3}{50}$ *g.* $\frac{37}{40}$ *h.* $\frac{47}{50}$

NOTE 1. A fraction in its lowest terms whose denominator contains any prime factor other than 2 and 5 cannot be reduced to an exact or terminating decimal. If the division is carried sufficiently far in any non-terminating decimal, there will be found a figure or set of figures that is continually repeated; thus we may have .366666 or .27272727 or .5236236. The part repeated is called a **repetend**, and may be written thus: $.3\dot{6}$, $.\dot{2}\dot{7}$, $.5\dot{2}3\dot{6}$. It is a curious fact that the real denominator of any repetend is as many 9's as there are figures in the repetend; $.3\dot{6} = .3\frac{6}{9}$, $.\dot{2}\dot{7} = \frac{27}{99}$, $.5\dot{2}3\dot{6} = .5\frac{236}{999}$.

Attempt to reduce each of the following to an exact decimal, and observe the repeating figures:

*i.* $\frac{2}{3}$ *j.* $\frac{1}{3}$ *k.* $\frac{1}{6}$ *l.* $\frac{5}{6}$

*m.* $\frac{2}{7}$ *n.* $\frac{5}{7}$ *o.* $\frac{4}{11}$ *p.* $\frac{5}{11}$

A common fraction that cannot be changed to an exact decimal may be changed to a decimal that is correct so far as the quotient figures obtained are concerned. Such a decimal is said to be true to tenths, hundredths, or thousandths, etc., according to the number of decimal figures given.

NOTE. No new rule is required. Follow the direction given on page 81, and continue the division until the required figure is obtained.

1. Change $\frac{2}{7}$ to a decimal.

OPERATION

$7\overline{)2.00}$
$.28+$

$7\overline{)2.000}$
$.285+$

$7\overline{)2.0000}$
$.2857+$

EXPLANATION

It will be observed that, however far this division may be carried, there is always a remainder. The fact that there is a remainder is indicated by writing the plus sign after the last figure of the decimal. The first quotient may be read 28 *hundredths, plus.*

Observe, too, that the error in the first answer is less than 1 hundredth, since the true quotient is more than 28 hundredths and less than 29 hundredths. We may therefore say that the first result is *true to hundredths;* the second, *true to thousandths,* etc.

Reduce to decimals, true to thousandths:

*a.* $\frac{3}{7}$ *b.* $\frac{2}{11}$ *c.* $\frac{1}{6}$ *d.* $\frac{5}{9}$
*e.* $\frac{5}{7}$ *f.* $\frac{3}{11}$ *g.* $\frac{5}{6}$ *h.* $\frac{2}{3}$
*i.* $\frac{6}{7}$ *j.* $\frac{6}{11}$ *k.* $\frac{4}{9}$ *l.* $\frac{1}{3}$

*m.* Find the sum of the twelve common fractions.

*n.* Find the sum of the twelve decimals.*

---

* Observe that the answers to *m* and *n* are not alike. Why? Observe that the difference between *m* and *n* is less than 12 thousandths. Why?

### To reduce a fraction to hundredths:

NOTE. This work is in no way different from that given on page 81, except that the number of zeros to be annexed to the numerator is *always two*. Why?

1. Change $\frac{3}{4}$ to hundredths.

| OPERATION | EXPLANATION |
|---|---|
| $\frac{3}{4} = 4\overline{)3.00}$<br>$.75$ | Three equals three hundred hundredths (written 3.00). One fourth of three hundred hundredths is 75 hundredths (written .75). |

Reduce the following to hundredths:

*a.* $\frac{1}{2}$ *b.* $\frac{1}{4}$ *c.* $\frac{1}{10}$ *d.* $\frac{3}{20}$
*e.* $\frac{1}{3}$ *f.* $\frac{1}{6}$ *g.* $\frac{2}{3}$ *h.* $\frac{5}{6}$
*i.* $\frac{1}{8}$ *j.* $\frac{3}{8}$ *k.* $\frac{11}{40}$ *l.* $\frac{9}{40}$
*m.* $\frac{3}{10}$ *n.* $\frac{2}{5}$ *o.* $\frac{7}{25}$ *p.* $\frac{1}{50}$
*q.* $\frac{1}{7}$ *r.* $\frac{2}{7}$ *s.* $\frac{3}{14}$ *t.* $\frac{5}{14}$

*u.* Find the sum of the twenty decimals.

*v.* Find the sum of the twenty common fractions.

Reduce the following to hundredths:

*w.* $\frac{11}{175}$ *x.* $\frac{18}{175}$ *y.* $\frac{24}{175}$
*z.* $\frac{15}{175}$ *aa.* $\frac{21}{175}$ *bb.* $\frac{16}{175}$
*cc.* $\frac{17}{175}$ *dd.* $\frac{35}{175}$ *ee.* $\frac{18}{175}$

*ff.* Find the sum of the nine decimals.

*gg.* Find the sum of the nine common fractions.

NOTE. The work on this page and on page 84 is especially important as preparation for the work in percentage. Pupils should now learn to reduce any common fraction whatsoever to hundredths. They should learn to do this without hesitation and accurately.

**To reduce a fraction whose denominator is an integral number of hundreds to hundredths:**

RULE. *Divide the numerator and the denominator by that number which will change the denominator to* 100.*

Reduce the following to hundredths :

*a.* $\frac{37}{500}$ *b.* $\frac{45}{400}$ *c.* $\frac{36}{300}$

*d.* $\frac{46}{500}$ *e.* $\frac{92}{400}$ *f.* $\frac{40}{300}$

*g.* $\frac{94}{500}$ *h.* $\frac{120}{400}$ *i.* $\frac{85}{300}$

*j.* $\frac{323}{500}$ *k.* $\frac{143}{400}$ *l.* $\frac{139}{300}$

*m.* Find the sum of the twelve results, *a* to *l.*

**To reduce a fraction whose denominator is an aliquot part of one hundred to hundredths:**

RULE. *Multiply the numerator and the denominator by that number which will change the denominator to* 100.

Reduce the following to hundredths:

*n.* $\frac{7}{20}$ *o.* $\frac{8}{12\frac{1}{2}}$ *p.* $\frac{13}{20}$

*q.* $\frac{14}{25}$ *r.* $\frac{7}{10}$ *s.* $\frac{11}{25}$

*t.* $\frac{27}{50}$ *u.* $\frac{4\frac{1}{2}}{12\frac{1}{2}}$ *v.* $\frac{23}{50}$

Reduce the following to hundredths:

*w.* $\frac{115}{250}$ *x.* $\frac{6}{40}$ *y.* $\frac{135}{250}$

*z.* $\frac{69}{150}$ *aa.* $\frac{18}{40}$ *bb.* $\frac{81}{150}$

*cc.* $\frac{154}{350}$ *dd.* $\frac{16}{40}$ *ee.* $\frac{196}{350}$

---

* Every common fraction can be changed to hundredths by dividing its numerator, followed by a decimal point and two zeros, by its denominator; but this method of reduction is not always the most simple.

See page 54, problems *a* and *b*.

*a.* A man owned a farm of 375 acres; he gave to his eldest son 275 acres of his farm. What part of the farm did the son receive?

*b.* Peter Lundy earned $650 in one year; of this sum he expended $520. What part of his earnings did he expend?

See page 57, problems *a* and *b*.

*c.* A lady owned $\frac{5}{8}$ of an acre of land. How many 80ths of an acre did she own?

*d.* Benton walked $\frac{11}{16}$ of a mile. Express the distance he walked in 80ths of a mile.

See page 60, problems *a* and *b*.

*e.* In a certain furnace $\frac{5}{16}$ of a ton of coal was consumed in one day, and $\frac{7}{20}$ of a ton the next day. What part of a ton was consumed in the two days?

*f.* Mr. Luker has three lots of land; in the first lot there are $\frac{11}{16}$ of an acre, in the second, $\frac{1}{2}$ of an acre, and in the third, $\frac{13}{20}$ of an acre. How many acres in all?

See page 61, problems *b* and *e*.

*g.* Of $\frac{9}{20}$ of a mile of board fence $\frac{1}{3}$ of a mile was burned. What part of a mile remained?

*h.* Mr. Reynolds had put into the bank $\frac{11}{20}$ of his annual salary; he drew from this money a sum equal to $\frac{1}{6}$ of his salary. What part of his salary remained in the bank?

See page 62, problems *g* and *i*.

*i.* From $52\frac{5}{8}$ tons of hay were sold and delivered $31\frac{3}{5}$ tons. How many tons remained of the unsold hay?

*j.* From $47\frac{3}{8}$ tons take $18\frac{4}{5}$ tons.

**See page 63, problems *a* and *b*.**

*a.* If a street car makes a round trip in $\frac{5}{12}$ of an hour, in how long a time can it make 4 such trips?

*b.* If $\frac{5}{4}$ yd. of ribbon are required to trim a hat, how much ribbon will be required to trim 8 such hats?

**See page 63, problems *n* and *o*.**

*c.* At $3\frac{1}{2}$ dollars a cord, what is the cost of 7 cords of wood?

*d.* If Henry rides his wheel at the rate of $4\frac{2}{5}$ miles an hour, how far does he ride in 5 hours?

**See page 68, problems *a* and *b*.**

*e.* If $\frac{7}{12}$ of a yard of ribbon is cut into 4 equal pieces, what part of a yard is each piece?

*f.* John hoes 4 rows of corn in $\frac{5}{6}$ of an hour. In what part of an hour does he hoe one row?

**See page 68, problems *k* and *o*.**

*g.* A horse traveled $17\frac{1}{5}$ miles in 3 hours. What was its rate per hour?

*h.* A farmer divided a field containing $18\frac{3}{10}$ acres into 6 equal lots. How many acres in each lot?

**See page 64, problems *a* and *e*.**

*i.* At $345 an acre, what is the cost of $\frac{1}{2}$ acre?

*j.* At $263 an acre, what is the cost of $\frac{3}{4}$ acre?

**See page 65, problems *a* and *i*.**

*k.* A piece of land in the form of a rectangle is $\frac{7}{16}$ of a mile long and $\frac{1}{2}$ of a mile wide. The piece is what part of a square mile?

*l.* At \$$\frac{1}{2}$ a yard, what is the cost of $\frac{3}{4}$ of a yard?

See page 64, problems *k* and *p*.

*a.* A strip of land 372¼ rods long and ½ a rod wide contains how many square rods?

*b.* At $153½ an acre, find the cost of ⅜ of an acre.

See page 66, problems *e* and *f*.

*c.* How many square feet in a piece of land 462¾ ft. by 2$\frac{1}{10}$ ft.?

*d.* At $275½ a mile, what is the cost of grading 4½ miles of road?

See page 72, problems *a* and *e*.

*e.* At ½ a dollar a bushel, how many bushels of potatoes can be bought for 46 dollars?

*f.* At ¾ of a dollar a bushel, how many bushels of apples can be bought for 375 dollars?

See page 72, problems *k* and *o*.

*g.* How many times can a line ⅜ of an inch long be measured off on a line ⅞ of an inch long?

*h.* What part of a line ⅚ of an inch long is a line $\frac{5}{12}$ of an inch long?

See page 72, problems *u* and *aa*.

*i.* If a rectangular diagram on the blackboard contains 5½ square feet and is ¾ of a foot wide, how long is it? Make the diagram.

*j.* At $⅖ a bushel, how many bushels of meal can be bought for $5½?

See page 72, problems *ee* and *ll*.

*k.* A strip of land contains 325½ square rods, and is 2½ rods wide. How long is it?

*l.* At $7½ per A., how many acres for $174½?

**1.** The difference of two numbers is 5; the smaller number is 11. What is the larger number?

*a.* The difference of two numbers is $374\frac{11}{40}$; the smaller number is $243\frac{17}{30}$. What is the larger number?

**2.** A piece of land in the form of an equilateral triangle measures $4\frac{1}{2}$ rods on one side. What is the distance around it?

*b.* A piece of land in the form of an equilateral triangle measures $46\frac{3}{11}$ rods on one side. What is the distance around it?

**3.** The perimeter of a piece of land that is an exact square is 24 feet. How far across one side?

*c.* The perimeter of a piece of land that is an exact square is $246\frac{3}{4}$ feet. How far across one side?

**4.** If $\frac{3}{7}$ of a certain number is 24, what is the number? What is $\frac{7}{8}$ of the number?

*d.* If $\frac{3}{7}$ of an acre of land is worth $96, what is $\frac{7}{8}$ of an acre worth at the same rate?

*e.* If $\frac{3}{7}$ of the value of a farm is $2154, what is $\frac{7}{8}$ of the value of the farm?

**5.** I spent $\frac{3}{5}$ of my money and had 10¢ remaining. How much did I spend? What I had remaining equals what part of what I spent?

*f.* I spent $\frac{3}{5}$ of my money and had $3.60 remaining. How much did I spend? *g.* What I had remaining equals what part of what I spent?

*h.* Change $\frac{5}{12}$ to an equivalent fraction whose denominator is 30.

*i.* One tenth of $\frac{1}{2}$ is what part of $\frac{3}{4}$?

*a.* Employing cancellation, multiply $\frac{9}{40}$ by $\frac{7}{20}$ by 25. Prove by solving the problem without canceling.

**1.** If $\frac{3}{4}$ of a yard of cloth is worth $1.50, how much are 8 yards worth at the same rate?

*b.* If $\frac{3}{4}$ of an acre of land is worth $36, how much are $37\frac{1}{2}$ acres worth at the same rate?

**2.** Multiplying both terms of a fraction by the same number does not change the value of the fraction. Does adding the same number to both terms of a fraction change its value?

*c.* If the number 3 is added to each of the terms of the fraction $\frac{2}{5}$, will the fraction thus obtained be more or less than $\frac{2}{5}$?

*d.* If the number 3 is added to each of the terms of the fraction $\frac{5}{2}$, will the fraction thus obtained be more or less that $\frac{5}{2}$?[*]

**3.** Dividing both terms of a fraction by the same number does not change the value of the fraction. Does subtracting the same number from both terms of a fraction change its value?

*e.* If the number 3 is subtracted from each of the terms of the fraction $\frac{5}{6}$, will the fraction thus obtained be more or less than $\frac{5}{6}$?[†]

*f.* Multiply $\frac{11}{20}$ by 150 and divide the product by $\frac{3}{5}$.

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[*] Observe that if the same number is added to each of the terms of a proper fraction, its value will be increased; if the same number is added to each of the terms of an improper fraction, its value will be diminished.

[†] Observe that if the same number is subtracted from each of the terms of a proper fraction, its value will be diminished; if the same number is subtracted from each of the terms of an improper fraction, its value will be increased.

**1.** The product of two numbers is $6\frac{1}{2}$; one of the numbers is 2. What is the other number?

*a.* The product of two numbers is $6\frac{1}{2}$; one of the numbers is $2\frac{1}{2}$. What is the other number?

**2.** The product of two numbers is 8; one of the numbers is $\frac{1}{2}$. What is the other number?

*b.* The product of two numbers is 8; one of the numbers is $\frac{5}{6}$. What is the other number?

**3.** The sum of two numbers is $8\frac{1}{2}$; one of the numbers is $2\frac{1}{4}$. What is the other number?

*c.* The sum of two numbers is $12\frac{4}{7}$; one of the numbers is $6\frac{1}{4}$. What is the other number?

**4.** The difference of two numbers is $3\frac{1}{2}$; the smaller number is 8. What is the larger number?

*d.* The difference of two numbers is $25\frac{5}{18}$; the smaller number is $43\frac{2}{7}$. What is the larger number?

**5.** The difference of two numbers is $4\frac{1}{4}$; the larger number is $18\frac{1}{2}$. What is the smaller number?

*e.* The difference of two numbers is $7\frac{1}{3}$; the larger number is $24\frac{3}{4}$. What is the smaller number?

**6.** What part of $\frac{3}{4}$ of an acre is $\frac{1}{2}$ of an acre?

*f.* What part of $\frac{7}{10}$ of an acre is $\frac{1}{4}$ of an acre?

*g.* How much bran at $16 per ton can be purchased for $2\frac{2}{5}$ tons of straw at $6 per ton?

*h.* How much sugar at 5¢ a pound can be purchased for $23\frac{1}{2}$ lb. butter at 30¢ a pound?

*i.* How much coal at $4\frac{1}{2}$ dollars a ton can be purchased for 5 cords of wood at 6 dollars per cord?

*j.* What part of $\frac{4}{5}$ of a cord is $\frac{1}{8}$ of a cord?

**1.** The expense of building 2 miles of road was $15000. How much did the road cost per mile?

**2.** The expense of building $2\frac{1}{2}$ miles of road was $15000. How much did the road cost per mile?

**3.** The expense of building $3\frac{1}{4}$ miles of road was $18000. Without making any figures, estimate the cost of this road per mile. Was it more or less than $6000?

*a.* Divide $18000 by $3\frac{1}{4}$, obtaining an answer true to units of dollars.

**4.** A bale of cotton weighed 534 lb. Without making any figures, estimate the value of this cotton at $8\frac{1}{2}$ ¢ a pound. Is it more or less than $40? Is it more or less than $50? At 1 ¢ a pound, how much would the bale cost?

*b.* Find the product of $5.34 multiplied by $8\frac{1}{2}$.

*c.* Find the product of 534 multiplied by $.08\frac{1}{2}$.

*d.* Find the product of $8\frac{1}{2}$ ¢ multiplied by 534.

**5.** Find $\frac{2}{5}$ of 80. Prove by adding $\frac{3}{5}$ of 80 to the answer. The sum should be ——.

*e.* Find $\frac{3}{7}$ of 246. Prove by adding $\frac{4}{7}$ of 246 to the answer.

*f.* Multiply 397 by $\frac{6}{7}$. Prove by subtracting $\frac{1}{7}$ of 397 from 397.

*g.* Change .56 to a common fraction in its lowest terms.

*h.* Change $\frac{7}{8}$ to a decimal fraction.

*i.* Change $\frac{5}{9}$ to a decimal fraction true to thousandths.

**1.** Change $\frac{3}{5}$ of an acre to hundredths of an acre.

**2.** Change $\frac{40}{500}$ of an acre to hundredths of an acre.

**3.** Change $\frac{11\frac{1}{2}}{50}$ of an acre to hundredths of an acre.

**4.** Change $\frac{17}{25}$ of an acre to hundredths of an acre.

*a.* Find the sum of the answers to problems 1 to 4.

**5.** Which is the greater fraction, $\frac{1}{3}$ or $\frac{1}{4}$?

*b.* Which is the greater fraction, $\frac{7}{8}$ or $\frac{8}{9}$?

**6.** If a furnace consumes $\frac{3}{16}$ of a ton of coal per day, in how many days will it consume a ton?

*c.* If a furnace consumes $\frac{3}{16}$ of a ton of coal per day, in how many days will it consume $6\frac{1}{2}$ tons?

**7.** The product of two fractions is $\frac{15}{63}$; one of the fractions is $\frac{3}{7}$. What is the other fraction?

*d.* The product of two fractions is $\frac{56}{99}$; one of the fractions is $\frac{8}{9}$. What is the other fraction?

*e.* If $\frac{3}{4}$ of a cord of wood cost $4.50, how much will 27 cords cost at the same rate?

*f.* If $2\frac{1}{4}$ tons of coal cost $12.60, how much will $17\frac{3}{5}$ tons cost at the same rate?

*g.* A man owned $7\frac{5}{10}$ acres of land; he sold $2\frac{3}{8}$ acres. What fractional part of his land did he sell?

*h.* How many pounds of coffee at $33\frac{1}{3}$¢ a pound must be given for $27\frac{1}{2}$ lb. of butter at 23¢ a pound?

**8.** One half an inch is what part of a foot?

**9.** Two and $\frac{1}{2}$ inches are what part of a foot?

**10.** Five and $\frac{1}{2}$ inches are what part of a foot?

**11.** Eleven and $\frac{1}{2}$ inches are what part of a foot?

# PERCENTAGE

**Per cent** means *hundredth* or *hundredths.* Per cent may be expressed as a common fraction whose denominator is 100, or it may be expressed decimally; thus 6 per cent $= \frac{6}{100}$, or .06; $28\frac{1}{2}$ per cent $= \frac{28\frac{1}{2}}{100}$, or $.28\frac{1}{2}$; $\frac{1}{2}$ per cent $= \frac{\frac{1}{2}}{100}$, or $.00\frac{1}{2}$.

**Note.** Instead of the words *per cent* the sign (%) is often used; thus 6 per cent may be written 6%.

The **base** in percentage is the number of which hundredths are taken; thus in the problem, *Find* 11% *of* 600, the base is 600; in the problem, 16 *is what per cent of* 800? the base is 800; in the problem, 18 *is* 3% *of what?* the base is not given, but is to be found by the student.

There are three cases in percentage and only three.

Case I. **To find some per cent (hundredths) of a number**; as, Find 15% of 600.

Case II. **To find a number when some per cent of it is given**; as, 24 is 8% of what number?

Case III. **To find what per cent one number is of another**; as, 12 is what % of 400?

Observe that a thorough knowledge of fractions is the necessary preparation for percentage. The work in percentage is work in fractions, *the denominator employed being* 100.

**1.** Find 17 per cent (.17) of 8460.*

**OPERATION**

$$\begin{array}{r} 84^{\vee}60 \\ .17 \\ \hline 592.20 \\ 846.0 \\ \hline 1438.20 \end{array}$$

**EXPLANATION**

**One per cent (1 hundredth) of 8460 is 84.60; 17 per cent (17 hundredths) of 8460 is 17 times 8460, or 1438.20.**

*a.* Find 17% of 6420; of 5252; of 31.40.

*b.* Find 35% of 6420; of 5252; of 31.40.

*c.* Find 43% of 6420; of 5252; of 31.40.

*d.* Find 25% of 6420; of 5252; of 31.40.

*e.* Find 50% of 6420; of 5252; of 31.40.

*f.* Find 14% of 6420; of 5252; of 31.40.

*g.* Find 16% of 6420; of 5252; of 31.40.

*h.* Find 35% of 6420; of 5252; of 31.40.

*i.* Find 65% of 6420; of 5252; of 31.40.

*j.* Find the sum of the twenty-seven results.

*k.* A sold goods for B. As remuneration for his services he received a sum equal to 12% of the sales. He sold $2146 worth of goods. How much did he receive for his services?

*l.* C is a collector of money. For this service he charges a commission of 6%; that is, his pay is 6% of the amount collected. He collected for D $375. How much should he pay over to D, and how much should he retain as pay for collecting?

---

*** No new rule is necessary for performing this operation. We may find ¾ of a number by finding 3 times 1 fourth of it; that is, by multiplying it by ¾. So we may find .17 of a number by finding 17 times 1 hundredth of the number; that is, by multiplying by .17.**

**1.** A sold goods for B. As remuneration for his services he received a sum equal to 15% of the sales. His sales for the week were as follows:

Monday, $34.60; Tuesday, $27.30; Wednesday, $49.50; Thursday, $16.20; Friday, $21.40; Saturday, $11.90.

*a.* Find each day's earnings. *b.* Find the total earnings.

*c.* Find the total sales for the week.

**2.** C is a collector of money. For this service he usually retains 8% of the amount collected. In reckoning his commission he omits from the amount due him any fraction of a dollar that is less than one cent. He collected for D, $75.62; for E, $21.43; for F, $60.75; for G, $84.50; for H, $91.25; for I, $15.80; for J, $67.25; for K, $15.50; for L, $39.90; for M, $57.45.

*d.* Find the amount of his commission in each case.

*e.* Find the total amount collected. *f.* Find the total amount to be paid to those for whom he made the collections. *g.* Find the total amount of his commission.

**3.** A fruit dealer purchased 840 barrels of apples for $2 a barrel. He lost 25% of them by decay and sold the remainder at $3 a barrel. *h.* How many barrels did he lose? *i.* How many barrels did he sell? *j.* Did he gain or lose by the transaction, and how much?

**4.** From a bill of $475 there is a discount of 6%. *k.* How much is the discount? *l.* How much must be paid?

*m.* How much must be paid on a bill of $396 if the discount is 5%?

1. 673.20 is 17 per cent (.17) of what number?

OPERATION NO. 1

```
17)673.20(39.60
   51           100
   163       3960.
   153
    102
    102
```

EXPLANATION

Since 673.20 is 17 hundredths of the number, 1 hundredth of the number is 1 seventeenth of 673.20, or 39.60; and 100 hundredths = 100 times 39.60, or 3960.

OPERATION NO. 2

```
.17)673.20'(3960
    51
    163
    153
     102
     102
```

EXPLANATION

We may find 100 seventeenths of a number by finding 1 seventeenth of a hundred times the number. 100 times 673.20 is 67320. 1 seventeenth of 67320 is 3960.

Observe that in the second operation we simply divide the number by 17 hundredths. Dividing by .17 is finding $\frac{100}{17}$ of the dividend, just as dividing by $\frac{3}{4}$ is finding $\frac{4}{3}$ of the dividend, and dividing by $\frac{1}{2}$ is finding $\frac{2}{1}$ of the dividend.*

*a.* 360 is 15% of what number?

*b.* 360 is 25% of what number?

*c.* 360 is 50% of what number?

*d.* 360 is 75% of what number?

*e.* 360 is 40% of what number?

*f.* 360 is 3% of what number?

*g.* Find the sum of the six results, *a* to *f*.

---

* Sometimes the process may be shortened by writing the per cent as a common fraction and reducing it to its lowest terms; then using the reduced fraction instead of the one whose denominator is 100.

*a.* $34.32 is 13 per cent of what?
*b.* $34.32 is 15 per cent of what?
*c.* $34.32 is 25 per cent of what?
*d.* $34.32 is 33⅓ per cent of what?
*e.* $34.32 is 50 per cent of what?
*f.* $64.98 is 19 per cent of what?
*g.* $64.98 is 12 per cent of what?
*h.* $64.98 is 20 per cent of what?
*i.* $64.98 is 24 per cent of what?
*j.* $64.98 is 25 per cent of what?
*k.* Find the sum of the ten results, *a* to *j*.

**1.** A sold goods for B. As remuneration for his services A received a sum equal to 12% of the sales. He received $33.06. *l.* What was the amount of his sales? *m.* How much money did B have left of what he received for the goods after paying out of it A's commission?

**2.** C is a collector of money. For this service he charges a commission of 6%; that is, his pay is 6% of the amount collected. His commission on a certain collection was $74.40. *n.* What was the amount collected? *o.* How much should the man for whom he collected the money receive?

**3.** C collected a sum of money for D, deducted his commission of 6%, and paid the remainder of the sum collected to D. He paid D $350.15.* *p.* What was the sum collected? *q.* How much money did C retain as his commission for collecting?

---

* $350.15 is what per cent of the amount collected?

**1.** A sold goods for B. As remuneration for his services he received a sum equal to 15% of the sales. His earnings for a week were as follows:

| | | | |
|---|---|---|---|
| Monday, | $3.15; | Thursday, | $4.71; |
| Tuesday, | $4.50; | Friday, | $5.22; |
| Wednesday, | $3.75; | Saturday, | $2.13. |

*a.* Find the amount of each day's sales. *b.* Find the amount of the week's sales. *c.* Find the amount A received. *d.* Find the amount B received.

**2.** Jones spent 24% of his money and had $57.76 left. *e.* How much had he before spending any? *f.* How much did he spend?

**3.** 36 is $4\frac{1}{2}$% of what number?

EXPLANATION. $4\frac{1}{2} = 9$ halves. Then 36 is 9 halves of 1 per cent of the required number. So 1 ninth of 36, or 4, is 1 half of one per cent of the number; and 8 is one per cent of the number; and the required number is ——.

Again: $4\frac{1}{2}\% = 4\frac{1}{2}$ hundredths, and $4\frac{1}{2}$ hundredths equals 9 two-hundredths. Hence this problem is equivalent to the following: 36 is 9 two-hundredths of what number?

Again: This problem may be solved as in "Operation No. 2," page 96—by dividing 36 by $.04\frac{1}{2}$ or .045. Dividing by $.04\frac{1}{2}$ is finding 200 ninths of the dividend; just as dividing by $\frac{3}{4}$ is finding $\frac{4}{3}$ of the dividend; dividing by $\frac{2}{1}$, finding 2 ($\frac{2}{1}$) times the dividend, etc.

*g.* 72 is $4\frac{1}{2}$ per cent of what number?

*h.* 105 is $3\frac{1}{2}$ per cent of what number?

*i.* 132 is $5\frac{1}{2}$ per cent of what number?

*j.* 210 is $2\frac{1}{3}$ per cent of what number?

*k.* 148 is $\frac{1}{2}$ of 1 per cent of what number?

*l.* 120 is $\frac{1}{4}$ of 1 per cent of what number?

1. 625 is what per cent of 900?

OPERATION NO. 1

EXPLANATION

900)625.00(.69$\frac{4}{9}$
5400
8500
8100
$\frac{400}{900}=\frac{4}{9}$

625 is $\frac{625}{900}$ of 900.

This fraction may be changed to hundredths in the usual manner, viz., by performing the division indicated, carrying out to hundredths' place.

OPERATION NO. 2

EXPLANATION

$\frac{625}{900}=\frac{25}{36}$
36)25.00(.69$\frac{4}{9}$ = 69$\frac{4}{9}$ %
216
340
324
$\frac{16}{36}=\frac{4}{9}$

625 is $\frac{625}{900}$ or $\frac{25}{36}$ of 900.

The fraction, $\frac{25}{36}$, may be changed to hundredths in the usual manner.

OPERATION AND EXPLANATION NO. 3

$$625 \text{ is } \frac{625}{900} \text{ of } 900. \quad \frac{625 \div 9}{900 \div 9}=\frac{69\frac{4}{9}}{100}=.69\tfrac{4}{9}=69\tfrac{4}{9}\%$$

OPERATION AND EXPLANATION NO. 4

One hundredth of 900 is 9; then 625 is as many hundredths of 900 as 9 is contained times in 625. 9 is contained in 625, 69$\frac{4}{9}$ times; so 625 is 69$\frac{4}{9}$% (hundredths) of 900.

Solve each of the following problems by each of the methods given above.

2. What per cent (how many hundredths) of 800 is:

*a.* 250? *b.* 375? *c.* 475? *d.* 350? *e.* 150?

*f.* Find the sum of the five results, *a* to *e*.

*g.* What per cent of 700 is 91?

*h.* What per cent of 900 is 252?

**1.** What per cent is:

*a.* 32 of 600? *b.* 54 of 600? *c.* 75 of 600?

*d.* 95 of 600? *e.* 344 of 600? *f.* 50 of 750?

*g.* 90 of 750? *h.* 85 of 750? *i.* 80 of 750?

*j.* 445 of 750? *k.* 40 of 325? *l.* 50 of 325?

*m.* 62 of 325? *n.* 78 of 325? *o.* 95 of 325?

*p.* Find the sum of the first five results, *a* to *e*.

*q.* Find the sum of the second five results, *f* to *j*.

*r.* Find the sum of the third five results, *k* to *o*.

**2.** What per cent is:

*s.* \$.76 of \$38? (What is 1 % of \$38?)

*t.* \$9.02 of \$225.50? (What is 1 % of \$225.50?)

*u.* \$17.13 of \$342.60? (What is 1 % of \$342.60?)

*v.* \$58.45 of \$835? (What is 1 % of \$835?)

*w.* \$1.29 of \$6.45? (What is 1 % of \$6.45?)

*x.* \$113.10 of \$754? (What is 1 % of \$754?)

*y.* \$21 of \$175? (What is 1 % of \$175?)

*z.* A sold goods for B to the amount of \$346.25; he received as his commission for selling the goods \$41.55. His commission was what per cent of the sales?

*aa.* C collected for D \$643.50; he retained as his commission for collecting the money \$38.61. His commission was what per cent of the amount collected?

*bb.* F purchased 256 barrels of apples; he lost by decay a quantity equal to 16 barrels. What per cent of his apples did he lose?

*cc.* The discount on a bill of \$840 was \$25.20. What was the per cent of the discount?

1. A sold goods for B to the amount of $724.35. He received as his commission for selling the goods $144.87. *a.* His commission was what per cent of the sales?

2. C collected for D $836.25. He retained as his commission for collecting the money $66.90. *b.* His commission was what per cent of the amount collected?

3. E purchased 430 barrels of apples. He lost by decay a quantity equal to 64½ barrels. *c.* What per cent of his apples did he lose?

4. On the first day of September, Farmer John found in an 80-rod row of corn 965 stalks. Upon each of 124 stalks there were 2 ears. Upon each of 711 stalks there was 1 ear. The remainder of the stalks were barren. *d.* What per cent of the stalks was barren? *e.* What per cent had 1 ear each? *f.* What per cent had 2 ears each?*

5. In the year 1890 the population of a certain city was 94320. In the year 1900 it was 105430. *g.* What was the per cent of increase in the ten years?†

6. James has $850. Arthur has $975. *h.* The difference equals what per cent of James's money? *i.* The difference equals what per cent of Arthur's money?

7. A clerk's salary for 1905 was $800 per year; for 1906 it was $900; for 1907 it was $1000. *j.* What per cent was his salary increased at the beginning of 1906? *k.* What was the per cent of increase for 1907?

---

* One per cent of the stalks is 9.65; then 124 stalks are as many per cent of the 965 stalks as 9.65 is contained times in 124.

† First find the increase in population. Then find what per cent this increase is of the population in 1890. One per cent of the population in 1890 is 943.20.

*a.* If I lose $\frac{3}{25}$ of my money, what per cent of my money do I lose?

*b.* Mr. Button spent $5 of the $12 which he had earned. What per cent of what he earned did he spend?

*c.* Mr. Thomas earns $150 per month. The monthly rent of the house in which he lives is equal to 15 % of what he earns. How much is his rent per year?

*d.* Mr. Jones purchased 500 barrels of apples. He lost by decay a quantity equal to 75 barrels. What per cent of the apples purchased remained sound?

*e.* Mr. Brown borrowed $625. He used 87 % of this money to pay debts. How much of the money borrowed did he have left?

*f.* Mr. Green paid $74.60 for a suit of clothes. If this was exactly 15 % of his monthly earnings, how much does he earn per month?

*g.* Mr. White earns $1500 per year. He spends $312 for board, $200 for clothes, $75 for books and papers, $80 for traveling and amusements; gives to his church $40, for charitable purposes, $35, and all his other expenses amount to $23. The remainder of his salary he puts into a savings bank. What per cent of his salary does he save?

*h.* If to 50 % of my money you add 20 % of my money, the sum will be $630. How much money have I?

1. What per cent of 25 is 4? 7? 15? $\frac{1}{2}$?
2. What per cent of 50 is 6? 11? 17? $\frac{1}{2}$?
3. What per cent of $33\frac{1}{3}$ is 5? 7? 9? 11?
4. What per cent of 20 is 6? 8? 11? 12?

*a.* If I lose $\frac{5}{11}$ of my money, what per cent of my money do I lose?

*b.* If I earn $954 and spend $236, the money spent equals what per cent of the money earned?*

*c.* In three months a physician charged for services $1242.50. Of this amount he collected $936.20. The amount collected is what per cent of the amount charged?

*d.* A merchant on going out of business found that his accounts receivable amounted to $3531.28. Of this amount he was able to collect only $2974.28. He lost what per cent of the amount represented in the accounts receivable?

*e.* The monthly rent of the house in which Mr. Brown lives is $24. This is exactly 18 per cent of his monthly salary. How much does he earn per year?

*f.* If to $\frac{1}{2}$ of my money you add 10 per cent of it, the sum will be $792.36. How much money have I?†

*g.* A lot of corn when "cribbed" in November weighed 48350 lb. When taken out of the crib in May, it weighed 43660 lb. What was the per cent of shrinkage?

*h.* An ear of corn weighed 14.3 ounces. The shelled corn from this ear weighed 12.1 ounces. The cob was what per cent of the ear?

*i.* A man owed $4000. In one year he paid 25 per cent of his debt. In the next year he paid 30 per cent of what he owed at the beginning of the year. How much did he owe at the end of the second year?

---

* One per cent of the money earned is how much?

† $\frac{1}{2}$ is 50%. 50% + 10% = 60%. If $792.36 is 60%, what is 1 per cent? 100%?

NOTE. Before attempting the percentage part of these problems it is necessary to find how many one of the given numbers is *more than* or *less than* the other number — the base. In such problems as these, from the very nature of the language employed, the base must follow the word *than.*

**1.** 75 is how many per cent more than 60?

75 is 15 more than 60; so the percentage part of this problem is, 15 *is what per cent of* 60? *Ans.* 25 per cent. Therefore 75 is 25 per cent more than 60; that is, 75 is 25 hundredths of 60 more than 60.

**2.** 60 is how many per cent less than 75?

60 is 15 less than 75; so the percentage part of this problem is, 15 *is what per cent of* 75? *Ans.* 20 per cent. Therefore 60 is 20 per cent less than 75; that is, 60 is 20 hundredths of 75 less than 75.

*a.* 60 is how many per cent more than 45?

*b.* 45 is how many per cent less than 60?

*c.* 150 is how many per cent more than 125?

*d.* 125 is how many per cent less than 150?

*e.* 225 is how many per cent more than 200?

*f.* 200 is how many per cent less than 225?

*g.* James has $345; Peter has $414. Peter has how many per cent more than James?* *h.* James has how many per cent less than Peter?†

*i.* My expenses in 1906 were $850; in 1907 they were $1000. My expenses were how many per cent more in 1907 than in 1906?‡ *j.* My expenses were how many per cent less in 1906 than in 1907? §

* James's money is the base.

† Peter's money is the base.

‡ My expenses in 1906 are the base.

§ My expenses in 1907 are the base.

NOTE. In speaking of the per cent of loss or of gain the *cost* is regarded as the *base* unless otherwise specified.

1. Find the per cent of loss or gain in each of the following:

*a.* Bought for 25¢ and sold for 30¢.
*b.* Bought for 25¢ and sold for 23¢.
*c.* Bought for 25¢ and sold for 27¢.
*d.* Bought for 25¢ and sold for 21¢.
*e.* Bought for $40 and sold for $48.
*f.* Bought for $40 and sold for $35.
*g.* Bought for $40 and sold for $52.
*h.* Bought for $40 and sold for $36.
*i.* Sold for 65¢ that which cost 50¢.
*j.* Sold for 18¢ that which cost 20¢.
*k.* Sold for 30¢ that which cost 36¢.
*l.* Sold for 35¢ that which cost 30¢.

2. Mr. Watson bought 60 lb. of tea at 32¢ a pound and sold it at 50¢. *m.* What was his per cent of gain if he sold as many pounds as he bought? *n.* If he lost 4 lb. by "down weights" and wastage, how much money did he gain? *o.* What was his real per cent of gain?

3. Mr. Jenkins bought gloves at $4.50 per dozen and sold them at 50¢ a pair. *p.* What was his per cent of gain?

4. Mr. Warner bought apples at 40¢ a bushel. He lost 25% of them by decay and sold the remainder at 50¢ a bushel. *q.* Did he gain or lose by the transaction? *r.* What was his per cent of gain or loss?

**Note.** The problems on this page are similar to those on pages 94 and 95. The difference is in the fact that in the problems given below the per cent is more than 100.

1. Find 175% ($\frac{175}{100}$, or 1.75) of \$632.60.

OPERATION NO. 1

$$\begin{array}{r} \$\,6^{v}32.60 \\ 1.75 \\ \hline 31.6300 \\ 442.820 \\ 632.60 \\ \hline \$\,1107.0500 \end{array}$$

EXPLANATION

One % of \$632.60 is \$6.326.

175% of \$632.60 is 175 times \$6.326, or \$1107.05.

Observe that 5% of \$632.60 is \$31.63; that 70% of \$632.60 is \$442.82; that 100% of \$632.60 is \$632.60.

OPERATION NO. 2

$$175\% = \tfrac{175}{100} = \tfrac{7}{4}. \quad \tfrac{1}{4} \text{ of } \$632.60 = \$158.15.$$
$$\tfrac{7}{4} \text{ of } \$632.60 = 7 \text{ times } \$158.15, \text{ or } \$1107.05.$$

*a.* Find 175% of 356; of 276; of 540.20.

*b.* Find 155% of 356; of 276; of 540.20.

*c.* Find 145% of 356; of 276; of 540.20.

*d.* Find 125% of 356; of 276; of 540.20.

*e.* Find 200% of 356; of 276; of 540.20.

*f.* Find the sum of the fifteen results, *a* to *f*.

*g.* David's money is equal to 150% of Henry's money. Henry has \$240. How much has David?

*h.* If goods cost \$260, and the profit on them is 125%, what is the selling price?

*i.* If a Chicago lot at the beginning of a certain year was worth \$8000 and during the year increased in value 50%, what was it worth at the end of the year? *j.* What would have been its worth at the end of the year had it increased 250%? *k.* How much would it have been worth had it decreased in value 100%?

**Note.** The problems on this page are similar to those on pages 96, 97, and 98. The difference is in the fact that in the problems given below the per cent is more than 100.

1. \$1107.05 is 175% of how much money?

OPERATION NO. 1

$$\begin{array}{lr}
175)\$1107^{\vee}05(\$6.326 & \\
\underline{\;1050\;} & \underline{\quad 100} \\
\;\;570 & \$632.60 \\
\;\;\underline{525} & \\
\;\;\;455 & \\
\;\;\;\underline{350} & \\
\;\;\;1050 & \\
\;\;\;\underline{1050} &
\end{array}$$

EXPLANATION

Since \$1107.05 is 175 hundredths of the money, 1 hundredth of the money is one 175th of \$1107.05, or \$6.326; and 100 hundredths, 100 times \$6.326, or \$632.60.

OPERATION NO. 2

$$\begin{array}{l}
1.75)\$1107.05^{\vee}(\$632.60 \\
\quad\;\underline{1050} \\
\quad\;\;570 \\
\quad\;\;\underline{525} \\
\quad\;\;\;455 \\
\quad\;\;\;\underline{350} \\
\quad\;\;\;1050 \\
\quad\;\;\;\underline{1050}
\end{array}$$

EXPLANATION

We may find one hundred 175ths of a number by finding one 175th of 100 times the number. One hundred times \$1107.05 is \$110705. One 175th of \$110705 is \$632.60.

OPERATION AND EXPLANATION NO. 3

$175\% = \frac{175}{100} = \frac{7}{4}$. If \$1107.05 is 7 fourths of the money, 1 seventh of \$1107.05, or \$158.15, is 1 fourth of the money, and 4 fourths, or the whole of it, is 4 times \$158.15, or \$632.60.

*a*. 43.50 is 125% of what number?

*b*. 65.10 is 150% of what number?

*c*. 48.50 is 200% of what number?

*d*. 59.20 is 250% of what number?

*e*. 29.44 is 115% of what number?

*f*. Find the sum of the five answers, *a* to *e*.

**Note.** The problems on this page are similar to those on pages 99, 100, and 101. The difference is in the fact that in the problems given below the per cent is more than 100.

1. \$1107.05 is what per cent of \$632.60?

OPERATION

```
$6.326)$1107.050‸(175
        6326
        -----
        47445
        44282
        ------
         31630
         31630
         -----
```

EXPLANATION

One per cent of \$632.60 is \$6.326; then \$1107.05 is as many per cent of \$632.60 as \$6.326 is contained times in \$1107.05. It is contained 175 times, so \$1107.05 is 175 % of \$632.60.

**Note.** The above problem may be solved as a similar problem is solved on page 99, Operation No. 1. \$1107.05 is $\frac{110705}{63260}$ of \$632.60. Perform the division indicated, carrying out to hundredths only. The quotient will be 1.75 or $\frac{175}{100}$ = 175 %.

*a.* What per cent of 845 is 2112.5?

*b.* What per cent of 845 is 1056.25?

*c.* What per cent of 845 is 1352?

*d.* What per cent of 845 is 1183?

*e.* What per cent of 845 is 2746.25?

*f.* Find the sum of the five answers, *a* to *e*.

*g.* Reuben has \$2420; Bernie has \$4961. Bernie's money equals how many per cent (hundredths) of Reuben's money?

*h.* A certain house costs \$6425. The lot upon which it stands costs \$2325. The cost of the house equals how many per cent (hundredths) of the cost of the lot? *i.* The cost of the lot equals how many per cent (hundredths) of the cost of the house? *j.* The cost of the lot equals how many per cent of the cost of the house and lot together?

*a.* Find $\frac{1}{2}$ %* of 632; of 356; of 272.

*b.* Find $\frac{1}{4}$ % of 632; of 356; of 272.

*c.* Find .25 %† of 632; of 356; of 272.

*d.* Find the sum of the nine results, *a* to *c*.

*e.* Find $3\frac{1}{2}$ % of 496; of 532; of 720.

*f.* Find $4\frac{3}{4}$ % of 496; of 532; of 720.

*g.* Find 1.75 % of 496; of 532; of 720.

*h.* Find the sum of the nine results, *e* to *g*.

*i.* What part of 94 is 11?‡ What per cent?

*j.* What part of 94 is 36? What per cent?

*k.* What part of 94 is 47? What per cent?

*l.* Find the sum of the three "per cents," *i* to *k*.

*m.* 15 is 3 % of what number? 4 %? 5 %?

*n.* 24 is 3 % of what number? 4 %? 5 %?

*o.* 42 is 3 % of what number? 4 %? 5 %?

*p.* Find the sum of the nine answers, *m* to *o*.

*q.* 375 is 125 % ($\frac{125}{100}$) of what number?

*r.* 436 is 109 % ($\frac{109}{100}$) of what number?

*s.* 598 is 115 % ($\frac{115}{100}$) of what number?

*t.* Find the sum of the three answers, *q* to *s*.

*u.* 544 is 15 % less than what number? §

*v.* 545 is 25 % more than what number?

---

* This means $\frac{1}{2}$ of 1 per cent.

† This means 25 hundredths of 1 per cent.

‡ Answer with a common fraction in its lowest terms.

§ If it is 15 per cent less than the number, it is 85 per cent of the number.

Remembering that in speaking of the per cent of loss or gain the ***cost is the base*** unless otherwise specified, tell the per cent of loss or gain in each of the following:

*a.* Bought for 2 and sold for 3.

*b.* Bought for 3 and sold for 2.

*c.* Bought for 4 and sold for 5.

*d.* Bought for 5 and sold for 4.

*e.* Bought for 5 and sold for 6.

*f.* Bought for 6 and sold for 5.

*g.* Bought for 8 and sold for 10; for 12.

*h.* Bought for 8 and sold for 14; for 16.

*i.* Bought for 8 and sold for 18; for 20.

*j.* Bought for 8 and sold for 4; for 2.

*k.* Mr. Parker sold goods at a profit of 25%; the amount of his sales on a certain day was $24.60. How much was his profit? *

*l.* Mr. Jewell sold goods at a loss of 25%; the amount of his sales on a certain day was $24.60. How much was his loss?*

*m.* By selling a horse for $156 there was a loss to the seller of 20%. What would have been his gain per cent if he had sold the horse for $234? †

*n.* A bill was made for wood that was supposed to be 4 feet long. It was afterward found to be only 46 inches long. What per cent should be deducted from the bill?

---

* $24.60 is what per cent of the cost?

† First find the cost of the horse.

NOTE. Many kinds of goods are usually sold "on time"; that is, the buyer may have 30, 60, or 90 days in which to pay for them. If he pays for such goods at the time of purchase or within ten days from the time of purchase, his bill is "discounted" from 1% to 6%, according to agreement; that is, a certain part of the amount of the bill is deducted from the amount.

1. Mr. Smith bought of Marshall Field & Co. a bill of goods amounting to $350.20. The discount for immediate payment ("spot cash") was 1%. How much must he pay for the goods?

1% of $350.20 is $3.50. $350.20 – $3.50 = $346.70

"Figure the discounts" on the following bills:*

*a.* Bill of $324.37, discounted at 2%.

*b.* Bill of $276.45, discounted at 1%.

*c.* Bill of $356.50, discounted at 3%.

*d.* Bill of $536.50, discounted at 6%.

*e.* Bill of $561.80, discounted at 4%.

*f.* Bill of $384.25, discounted at 3%.

*g.* Find the sum of the six bills before they are discounted, *a* to *f*.

*h.* Find the sum of the discounts, *a* to *f*.

*i.* Find the sum of the six bills after discounting, *a* to *f*.

*j.* Mr. Albert Carter, a retail merchant, buys goods upon which there is an average discount of 2½ per cent. In one year he buys goods amounting to $48250. His discounts for the year amount to how much?

*k.* How much money is needed to pay for the goods?

---

* Fractions of cents in the discount on each bill may be ignored.

NOTE. Dealers in hardware, rubber boots and shoes, belting, rubber hose, and many other kinds of goods, sell from a list price agreed upon by the manufacturers. The actual price is usually less than the list price. "20% off" means that the list price is to be discounted 20%. "20 and 10 off" means that the list price is to be discounted 20%, and what remains is to be discounted 10%. Sometimes as many as nine successive discounts are allowed. Observe that in computing these the base changes with each discount.

Find the actual cost of:

*a.* 500 ft. $\frac{3}{4}$-inch gas pipe (list, 7¢ per foot) at 50 and 10 off.

*b.* 350 ft. $\frac{1}{2}$-inch gas pipe (list, 8¢ per foot) at 50 and 10 off.

*c.* 200 ft. $1\frac{1}{2}$-inch gas pipe (list, 26¢ per foot) at 55 and 10 off.

*d.* 260 ft. 2-inch gas pipe (list, 35¢ per foot) at 55 and 10 off.

*e.* 48 $\frac{1}{2}$-inch elbows (list, 7¢ each) at 65 and 20 off.

*f.* 86 $\frac{3}{4}$-inch elbows (list, 9¢ each) at 65 and 20 off.

*g.* Find the entire cost of the six items, *a* to *f*.

*h.* Find the cost of 12 pairs men's rubber boots (list, $3 per pair) at 25 and 10 off. Find the cost of the same at 35 off. Why are the results unlike?

*i.* Which is the lower price, 50 and 10 off, or 60 off, the list being the same?

*j.* Bought for 40 off from list price and sold for 10 off from list price. What was my gain per cent?

*k.* Bought for 70 off from list and sold for 50 and 20 off from list. Did I lose or gain and how many per cent?

**Commission** is the sum paid an agent, or commission merchant, for transacting business.

NOTE. When goods are sold "on commission," *the selling price is the base;* that is, the seller receives a certain per cent of the selling price as remuneration for services.

*a.* At 40%, what is the commission for selling $275 worth of books? *b.* If the salesman sells and collects for 40% of the selling price, how much of the $275 will he retain and how much "pay over" to the man for whom he sells the books?

*c.* While selling books on a commission of 40% my commission amounted to $56. What was the selling price of the books? *d.* If I not only sold but collected for 40% of the selling price, how much money should I "pay over" to my employer?

*e.* A real estate agent sold a house and lot for $4250. If his commission is 5%, how much should he receive for his services?

*f.* A real estate agent sold a piece of property upon which his commission at 5% amounted to $275. What was the selling price of the property? *g.* How much should the owner receive for the property after deducting the commission?

*h.* A commission merchant sold 2140 lb. of butter at 23¢ a pound. After deducting his commission of 5% and paying freight charges of $36.50, and storage charges of $21.40, how much should he send to the man for whom he made the sale? *i.* The freight, storage charges, and commission are together what per cent of the selling price of the butter?

NOTE. When goods are bought "on commission," *the buying price is the base;* that is, the buyer receives as remuneration for his services an amount equal to a certain per cent of the buying price.

1. A Boston merchant buys through a Chicago commission house 2456 lb. butter at 23¢ a pound. *a.* If the rate of commission is 2%, how much does the butter cost the Boston merchant at the warehouse in Chicago? *b.* If cartage in Chicago, freight from Chicago to Boston, and cartage in Boston amount to $26.40, how much does the butter cost the Boston merchant laid down in his own warehouse? *c.* How much does the butter cost the Boston merchant per pound?

2. A commission house charges 2% for buying grain. If wheat is bought at 90¢ per bushel, the commission adds how much to the cost per bushel? *d.* If oats are bought at 40¢ per bushel, the commission adds how much to the cost per bushel? *e.* If corn is bought at 50¢ per bushel, the commission adds how much to the price per bushel?

3. Find the cost, including the commission at 5%, for the buying of:

*f.* 254 lb. butter @ 27¢.
*g.* 526 lb. cheese @ 12½¢.
*h.* 300 doz. eggs @ 19¢.
*i.* 600 doz. eggs @ 21¢.

*j.* Find the entire cost of the butter, cheese, and eggs, including commission as above, cartage $2.25, and freight $2.40.

*k.* Find the cost of a car load of wheat, 1000 bushels at 86¢, commission ½¢ per bushel.

*a.* Twenty per cent of the heads of oats in a certain field were destroyed by smut, and yet there were harvested 944 bushels. What would have been the yield had the smutted heads produced as well as the others ?[*]

*b.* A crib of corn "shelled out" 7465 bushels of 56 pounds each. If 12 per cent of the ear corn was cob, what was the weight of the corn before shelling ?[†]

*c.* After a number of bills had been discounted 5 per cent they were paid. The amount paid was $7833.13. What was the amount of the bills before they were discounted ?[‡]

*d.* Mr. Johnson sold goods at a profit of 20 per cent. The amount of a week's sales was $1120.20. How much did the goods sold cost Mr. Johnson ?[§]

*e.* Mr. Leet sold goods at a loss of 20 per cent. The amount of a week's sales was $1120.20. How much did the goods sold cost Mr. Leet ?[‖]

*f.* How much was Mr. Johnson's profit in the sales described in problem *d* ?

*g.* How much was Mr. Leet's loss in the sales described in problem *e* ?

*h.* If by selling goods at a profit of 18 per cent a man gains $153.36, what was the cost of the goods ?

*i.* If by selling goods at a loss of 18 per cent a man loses $95.85, what was the cost of the goods ?

---

[*] 944 bushels is 80% ($\frac{4}{5}$) of what the yield should have been.

[†] 7465 bushels is 88% ($\frac{22}{25}$) of the weight before shelling.

[‡] \$7833.13 is 95% ($\frac{19}{20}$) of the amount of the bills before they were discounted.

[§] \$1120.20 is 120% ($\frac{6}{5}$) of the cost. [‖] \$1120.20 is 80% ($\frac{4}{5}$) of the cost.

*a.* Bought books which, according to the list price, amounted to $236.45. If a discount of 30%, and 6% off for cash, is allowed, how much will the books cost?

*b.* Bought 1324 ft. 1-inch black pipe, list 16½ cents a foot. What will be the actual cost if it is sold at 20, 10, and 5 off?

*c.* Bought 36 pairs men's rubber boots, list $3 per pair. Find cost at 25, 10, and 5 off.

*d.* Bought for 50 off from list and sold at 40 off from list. What was the per cent of gain? *

*e.* Bought for 40 off from list and sold at 50 off from list. What was the per cent of loss?

*f.* A salesman receives $1000 per year, plus 2 per cent of the amount of his sales. If during the year his sales amount to $25000, what is his average monthly salary?

*g.* The commissions of a real estate agent, at 5 per cent, amounted for the month of May to $346.25. Find the amount of his sales for the same period.

*h.* Find the amount paid over to those who employed the agent referred to in problem *g.*

*i.* What per cent does an agent receive who is paid $21.60 for selling $864 worth of goods? †

*j.* An agent who sells $756.80 worth of merchandise, pays $24.30 for freight, storage, and insurance, and remits to the shipper of the merchandise $694.66, charges what per cent commission?

---

* If the list price is $1, what is the cost? The selling price?

† It is well to estimate the result before attempting a written solution. Observe that 1% of $864 is $8.64; that 2% of $864 is more than $16, and that 3% of $864 is more than $24.

A **tax** is a sum of money paid for public purposes. A tax on property is reckoned at a certain per cent of the **assessed value** of the property. The assessed value may or may not be the real value. It is often much below the real value.

*a.* Mr. Hardy has a farm of 240 acres which he values at $24000. Its assessed value is $22 per acre. If his state tax is ½%, his county tax 1¼%, his town tax ¼%, his school tax 2%, and his special road and bridge tax 1%, how much money must he pay as taxes on his farm?

*b.* The assessed value of the taxable property of a certain school district is $176242.25. If the school tax is 2½% and the collector receives 2% of the amount collected as his commission, and collects the entire amount of the tax, how much should the district officers receive from this source for school purposes?

*c.* The assessed value of Mr. Randall's property is $3400. At the rate of 15 mills on a dollar,* how much tax must he pay?

*d.* The assessed value of the property of a district of a certain city is $250000. What must be the per cent of taxation to raise $10000? *e.* What will be the net sum realized for public purposes if the collector is able to collect only 95% of this tax, and he receives for his services 2% of the amount collected?

*f.* If in a certain city the assessed value of property is one fifth its actual value, and if the rate of taxation is 4½%, how much tax must the man pay whose property is worth $12000?

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* "15 mills on a dollar" is the same as 1½%.

**1.** Mr. Arnold has property as follows:
A farm of 320 acres valued at $75 per acre.
Stock and farm implements valued at $4270.
Hay, straw, and grain valued at $1850.
Household goods, watch, etc., valued at $1240.
Taxable bonds and notes valued at $3560.

*a.* If in the state in which Mr. Arnold lives, property is usually assessed at about one fifth its actual value, and the rate of taxation is from 5 to 6 per cent of the assessed value, about how much money will be required to pay his taxes?

*b.* If Mr. Arnold's property is assessed at exactly one fifth the above valuation, and if his school tax is 2.3 per cent, how much will he be required to pay toward the support of the schools of his district?

*c.* If this property is assessed at $\frac{9}{40}$ of the above valuation, and if the entire tax is at the rate of $5.20 on each hundred dollars of assessed value, how much money will be required to pay the taxes?

*d.* A tax of $5.20 on each hundred dollars of $\frac{9}{40}$ of the actual value is equivalent to a tax of how much on each hundred dollars of actual value?

**2.** The assessed valuation of the property of a certain school district is as follows: Real estate, $1640400; personal property, $542600. A tax of 2.1 per cent is levied.

*e.* If the tax gatherer collects only 95 per cent of this tax, and receives 2 per cent of the amount collected for his services, how much money will thus be secured for school purposes? *f.* How much will the collector receive for collecting the school tax?

**Insurance** is a guaranty by one party to pay a certain sum to another party in the event of loss or damage.

The **policy** is the written contract given by the underwriter to the insured. The **premium** is the sum paid for insurance.

**1.** A store valued at $7500 was insured for $5000 for 1 year. The rate of insurance was 2%. *a.* What was the amount of the premium?

**2.** A stock of goods valued at $10000 was insured for $5000. A fire occurred, but some of the goods were saved. It was found that the entire loss to the owner of the goods was $4750. *b.* How much should he receive? *c.* How much should he receive if the loss were $5750?*

**3.** An insurance agent offers to insure my farm buildings for $3500 for 1 year at 1%, or for 5 years at 3%; the entire premium in either case to be paid in advance. *d.* If I accept the first proposition, how much is the premium to be paid? *e.* How much if I accept the second?

**4.** What is the rate of insurance on the nearest store and stock of goods? On farm property? On village or city property other than stores?†

**5.** A large building was insured in one company for $25000, in another company for $15000. It was damaged by fire to the extent of $12800. *f.* How much of the damage should each company pay?

---

* In case of total loss the owner would receive $5000. In case of partial loss the owner would receive the full amount of the loss, provided it did not exceed $5000.

† Any insurance agent will be willing to answer these questions for you.

*a.* A dealer who had marked goods 50 % above the cost, sold them after deducting 10 % from the marked price. His profit on that sale was what per cent of the cost of the goods?

*b.* By selling a suit of clothes for $7.20 I would lose an amount equal to 10 % of the cost. For what must I sell the suit to gain a sum equal to 10 % of the cost?

*c.* I sold goods at a loss of 7 %. My actual loss was $3.50. What was the cost of the goods?

*d.* The real value of a stock of goods was $8250. They were insured for $5500. A fire occurred, and the salvage amounted to only $575. If the insurance company promptly settled in accordance with the above facts, what was my actual loss by the fire?

*e.* If I sell goods on a commission of 12½ %, what must be the amount of my sales in order that I may receive an annual salary of $2500?

*f.* A school numbers 140 pupils. The absence for one week was as follows: Mon., 3 days; Tues., 5 days; Wed., 4 days; Thurs., 5 days; Fri., 3 days. What was the per cent of absence? *g.* What was the per cent of attendance?

*h.* Sold a horse for $120 and gained 25 %. How much did the horse cost me?

*i.* When the cost is ¾ of the selling price, what is the gain per cent?

*j.* When the selling price is ¾ of the cost, what is the loss per cent?

*k.* The sum collected was $2321.50; commission for collecting was 6 %. What was the cost of collecting?

*a.* A man buys goods for $60 and sells them for $75. He gains —— dollars. The gain equals what part of the cost? *b.* What per cent? *c.* The gain equals what part of the selling price? *d.* What per cent? *e.* The cost equals what part of the selling price? *f.* What per cent?

*g.* When the cost is 2 thirds of the selling price, what is the per cent of gain?

*h.* When the selling price is 2 thirds of the cost, what is the per cent of loss?

*i.* Bought for $200 and sold for $300. What was the per cent of gain?

*j.* Bought for $300 and sold for $200. What was the per cent of loss?

*k.* A tax of 15 mills on a dollar was levied in a certain town, the assessed value of the taxable property being $475250. If 5 % of the tax proves to be noncollectible, and if the collector is allowed 2 % of the amount collected for his services, how much will be realized from the levy?

*l.* Which is the greater discount, "20 and 10 and 5 off" or "35 off"?

*m.* A sold goods for B on a commission of 15 %. His sales for a certain period amounted to $780. If the goods cost B $600, was B's net profit more or less than 10 %?

*n.* A offers rubber boots at "50 and 20 off"; B offers them at "20 and 50 off." The quality and list price being the same, which offer shall I accept?

*a*. When the cost is $\frac{4}{5}$ of the selling price, what is the per cent of gain?

*b*. When the selling price is $\frac{4}{5}$ of the cost, what is the per cent of loss?

*c*. Arthur sells books on commission and receives for his services 40 per cent of the amount of his sales. If the price of the book he sells is $2.40, how many copies must he sell in order to earn $14.40?

*d*. Arthur sold books at the price and upon the terms given in problem *c*, collected the pay for them, and remitted to his employers what remained after he had deducted his commission. When he had remitted exactly $24 more than he had retained, how many copies had he sold? *e*. How much money had he remitted? *f*. How much had he retained?

How much cash will settle the following bills:

*g*. Bill of $624, 6 % off, and 2 % off for cash?

*h*. Bill of $125.34, 20 and 10 off, and 2 off for cash?

*i*. Bill of $3428, 50 and 40 and 10 off?

*j*. Bill of $47.62, 30 and 5 off, and 2 off for cash?

*k*. The value of a stock of goods as estimated by the owner was $9650. These goods were insured for $6500. A fire occurred and the salvage amounted to $1260. The officers of the insurance company insisted that the owner's estimate of the value of the stock was 25 % above its real value. If this contention is at length accepted by the owner, and settlement made in accordance with the facts as stated, how much will the owner receive from the insurance company?

# INTEREST

**Interest** is compensation for the use of money.

The money for which interest is paid is called the **principal.**

The principal and interest together are called the **amount.**

NOTE. Interest is usually reckoned in per cent, the principal being the base; that is, the borrower pays for the use of money a sum equal to a certain per cent of the principal. "The rate of interest" is the per cent per annum which the borrower agrees to pay. When a man loans money "at 6%" he expects to receive back the principal, and a sum equal to 6% of the principal for every year the money is loaned and at that rate for fractions of years.

1. Find the interest of $ 257 for 2 yr. at 6 %.

OPERATION

$$\begin{array}{r} \$\,2^{\vee}57 \\ .12 \\ \hline 5.14 \\ 25.7\phantom{0} \\ \hline \$\,30.84 \end{array}$$

EXPLANATION

The interest of any sum for 2 yr. at 6% is 12 hundredths of the principal. One hundredth of $257 is $2.57, and 12 hundredths of $257 is 12 times $ 2.57, or $30.84.

*a.* Find the interest of $ 242 for 3 yr. at 7 %.

*b.* Find the interest of $ 375 for 2 yr. at 6 %.

*c.* Find the interest of $ 146 for 1 yr. at 5 %.

*d.* Find the interest of $ 274 for 3 yr. at 5 %.

*e.* Find the sum of the four results, *a* to *d*.

NOTE. The interest for 1 month is 1 twelfth as much as it is for 1 year; for 2 months, 2 twelfths, or 1 sixth, etc.

**1.** Find the interest of $324.50 for 2 yr. 5 mo. at 6%.

OPERATION AND EXPLANATION NO. 1

| | | |
|---|---|---|
| Interest of $324.50 for 1 yr. at 1% = | $3.245 | |
| Interest of $324.50 for 1 yr. at 6% = | $19.470 | |
| Interest of $324.50 for 2 yr. at 6% = | | $38.940 |
| Interest of $324.50 for 1 mo. at 6% = | $1.6225 | |
| Interest of $324.50 for 5 mo. at 6% = | | 8.1125 |
| Interest of $324.50 for 2 yr. 5 mo. at 6% = | | $47.0525 |

OPERATION NO. 2

2 yr. 5 mo. = $2\frac{5}{12}$ yr.

$2\frac{5}{12}$ times .06 = .14½

$$\begin{array}{r} \$3'24.50 \\ \underline{.14\frac{1}{2}} \\ 1.6225 \\ 12.9800 \\ \underline{32.450} \\ \$47.0525^{*} \end{array}$$

EXPLANATION

The interest of any sum for 2 yr. 5 mo. at 6% is 14½ hundredths of the principal.

| | |
|---|---|
| 1 hundredth of $324.50 is | $3.2450 |
| ½ hundredth of $324.50 is | $1.6225 |
| 4 hundredths of $324.50 is | 12.9800 |
| 10 hundredths of $324.50 is | 32.450 |
| 14½ hundredths of $324.50 is | $47.0525 |

*a.* Find the interest of $325.40 for 1 yr. 6 mo. at 7%.
*b.* Find the interest of $420.38 for 2 yr. 10 mo. at 6%.
*c.* Find the interest of $221.60 for 2 yr. 3 mo. at 6%.
*d.* Find the interest of $145.20 for 1 yr. 9 mo. at 5%.
*e.* Find the interest of $340.10 for 3 yr. 1 mo. at 4%.
*f.* Find the interest of $275.30 for 2 yr. 4 mo. at 6%.
*g.* Find the interest of $350.20 for 3 yr. 8 mo. at 6%.
*h.* Find the sum of the seven results, *a* to *g*.

---

* In an interest problem, if there is a fraction of a cent in the answer and this fraction is not greater than ½, it may be disregarded; if it is more than ½, it may be regarded as 1 cent.

**Note.** In computing interest, 30 days is usually regarded as 1 month, and 360 days as 1 year; so each day is $\frac{1}{30}$ of a month or $\frac{1}{360}$ of a year.

**1.** Find the interest of $256.20 for 2 yr. 7 mo. 13 da. at 6%.

OPERATION AND EXPLANATION NO. 1

| | | |
|---|---|---|
| Interest of $256.20 for 1 yr. at 1% = | $2.5620 | |
| Interest of $256.20 for 1 yr. at 6% = | $15.3720 | |
| Interest of $256.20 for 2 yr. at 6% = | | $30.7440 |
| Interest of $256.20 for 1 mo. at 6% = | $1.2810 | |
| Interest of $256.20 for 7 mo. at 6% = | | 8.9670 |
| Interest of $256.20 for 1 da. at 6% = | $.04270 | |
| Interest of $256.20 for 13 da. at 6% = | | .5551 |
| Interest of $256.20 for 2 yr. 7 mo. 13 da. at 6% = | | $40.2661 |

OPERATION NO. 2

2 yr. 7 mo. 13 da. = $2\frac{223}{360}$ yr.

$2\frac{223}{360}$ times .06 = $.15\frac{43}{60}$

$$\begin{array}{r} \$2^{\vee}56.20 \\ .15\frac{43}{60} \\ \hline 1.8361 \\ 12.8100 \\ 25.620 \\ \hline \$40.2661 \end{array}$$

EXPLANATION

The interest of any sum for 2 yr. 7 mo. 13 da. at 6% is $.15\frac{43}{60}$ of the principal.

| | |
|---|---|
| 1 hundredth of $256.20 is | $2.5620 |
| $\frac{43}{60}$ of 1 hundredth of $256.20 is | $1.8361 |
| 5 hundredths of $256.20 is | 12.81 |
| 10 hundredths of $256.20 is | 25.62 |
| $15\frac{43}{60}$ hundredths of $256.20 is | $40.2661 |

*a.* Find the interest of $350.40 for 2 yr. 5 mo. 7 da. at 6%.

*b.* Find the interest of $145.30 for 1 yr. 7 mo. 10 da. at 8%.

*c.* Find the interest of $174.20 for 2 yr. 3 mo. 15 da. at 7%.

*d.* Find the sum of the three results, *a* to *c*.

NOTE. If the teacher so prefers, the problems on the three preceding pages, as well as those that follow, may be solved by the "six per cent method."

The interest at 6 % for 1 yr. = .06 of the principal.
The interest at 6 % for 1 mo. = 1/12 of .06, or .005 of the principal.
The interest at 6 % for 1 da. = 1/30 of .005, or .000 1/6 of the principal.

Or

The interest of $1 for 1 yr. is 6 cents.
The interest of $1 for 1 mo. is 5 mills.
The interest of $1 for 1 day is 1/6 of a mill.
The interest of $1 for 6 days is 1 mill.

1. Interest for 1 yr. at 6 % = —— 100ths of the principal;

for 2 yr. = —— 100ths; for 3 yr. = —— 100ths;
for 1 mo. = —— 1000ths; for 2 mo. = —— 1000ths;
for 3 mo. = —— 1000ths; for 4 mo. = —— 1000ths;
for 5 mo. = —— 1000ths; for 6 mo. = —— 1000ths;
for 1 da. = —— 1000th; for 3 da. = —— 1000th;
for 6 da. = —— 1000th; for 12 da. = —— 1000ths;
for 18 da. = —— 1000ths; for 24 da. = —— 1000ths;
for 25 da. = —— 1000ths; for 27 da. = —— 1000ths.

2. Find the interest of $243.25 for 2 yr. 5 mo. 18 da. at 6 %.

OPERATION AND EXPLANATION

Interest for 2 yr. = .12 of the principal.
Interest for 5 mo. = .025 of the principal.
Interest for 18 da. = .003 of the principal.
Total interest = .148 of the principal.
$243.25 × .148 = $36.001*

* $36.001 plus 1/6 of $36.001 = int. of same prin. for the same time at 7 %.

$36.001 less 1/6 of $36.001 = int. of same prin. for the same time at 5 %.

How can you find the interest at 8 %? at 9 %? at 4 %? at 3 %?

Find the interest of:

*a.* \$265 for 1 yr. 3 mo. 13 da. at 6%.*

*b.* \$346 for 2 yr. 5 mo. 20 da. at 6%.†

*c.* \$537 for 1 yr. 7 mo. 10 da. at 6%.‡

*d.* \$428 for 3 yr. 3 mo. 14 da. at 6%.

*e.* \$150 for 1 yr. 6 mo. 15 da. at 6%.

*f.* Find the sum of the five results, *a* to *e*.

Find the interest of:

*g.* \$245.30 for 6 mo. 18 da. at 7%.§

*h.* \$136.25 for 8 mo. 10 da. at 7%.

*i.* \$321.42 for 5 mo. 15 da. at 7%.

*j.* \$108.00 for 10 mo. 8 da. at 7%.

*k.* \$210.80 for 7 mo. 21 da. at 7%.

*l.* Find the sum of the five results, *g* to *k*.

Find the amount of:

*m.* \$55.36 for 2 yr. 8 mo. 12 da. at 6%.

*n.* \$82.75 for 2 yr. 10 mo. 8 da. at 6%.

*o.* \$27.35 for 1 yr. 1 mo. 1 da. at 6%.

*p.* \$875 for 3 yr. 8 mo. 15 da. at 5%.

*q.* \$346 for 2 yr. 6 mo. 12 da. at 4%.

*r.* \$500 for 1 yr. 7 mo. 18 da. at 3%.

*s.* \$600 for 2 yr. 5 mo. 21 da. at 8%.

*t.* \$800 for 1 yr. 8 mo. 24 da. at 7%.

*u.* Find the sum of the eight results, *m* to *t*.

---

| | | | | | |
|---|---|---|---|---|---|
| * 1 yr. | .06 | † 2 yr. | .12 | ‡ 1 yr. | .06 |
| 3 mo. | .015 | 5 mo. | .025 | 7 mo. | .035 |
| 13 da. | .002⅙ | 20 da. | .003⅓ | 10 da. | .001⅔ |
| Total | .077⅙ | Total | .148⅓ | Total | .096⅔ |

§ First find the interest at 6%; then add ⅙ of that sum to itself.

**1.** How many years, months, and days from Sept. 25, 1892 to June 10, 1896?

### THE USUAL METHOD

1896–6–10
1892–9–25
3–8–15

From the 1896th yr., the 6th mo., and the 10th da. subtract the 1892d yr., the 9th mo., and the 25th da. Regard a month as 30 da.

### A BETTER METHOD

From Sept. 25, 1892 to Sept. 25, 1895 is 3 years.
From Sept. 25, 1895 to May 25, 1896 is 8 months.
From May 25, 1896 to June 10, 1896 is 16 days.

NOTE. The two methods will not always produce the same results. The greatest possible variation is two days. Find the time from Jan. 22, 1895 to March 10, 1897 by each method, and compare results. The difference arises from the fact that the month as a measure of time is a variable unit — sometimes 28 days, sometimes 31. The "usual method" regards each month as 30 days; the "better method" counts first the whole years, then the whole months, then the days remaining. By the "usual method," the time from Feb. 28, 1897 to March 1, 1897 is 3 days; by the "better method," it is 1 day.

Find the time by both methods and compare the results:

*a.* From March 15, 1894 to Sept. 10, 1897.
*b.* From March 15, 1894 to Sept. 20, 1897.
*c.* From May 25, 1895 to Sept. 4, 1898.
*d.* From May 25, 1895 to Oct. 4, 1898.
*e.* From June 28, 1894 to March 1, 1898.
*f.* From June 28, 1894 to May 1, 1898.
*g.* From Jan. 10, 1892 to Jan. 25, 1898.
*h.* From Jan. 10, 1892 to Dec. 25, 1898.
*i.* From April 15, 1893 to Aug. 15, 1898.

Find the interest of:

*a.* $230.60 for 2 yr. 7 mo. 16 da. at 6 %.
*b.* $230.60 for 1 yr. 4 mo. 14 da. at 6 %.
*c.* $274.28 for 1 yr. 3 mo. 10 da. at 7 %.
*d.* $274.28 for 1 yr. 8 mo. 20 da. at 7 %.
*e.* $136.45 for 3 yr. 3 mo. 12 da. at 8 %.
*f.* $136.45 for 2 yr. 8 mo. 18 da. at 8 %.
*g.* $350.42 for 1 yr. 1 mo. 5 da. at 5 %.
*h.* $350.42 for 1 yr. 10 mo. 25 da. at 5 %.
*i.* $546.25 for 2 yr. 9 mo. 8 da. at 6 %.
*j.* $546.25 for 1 yr. 2 mo. 22 da. at 6 %.
*k.* $432.21 for 3 yr. 4 mo. 17 da. at 7 %.
*l.* $432.21 for 2 yr. 7 mo. 13 da. at 7 %.

Find the time between the dates given below, using for this purpose the "better method," as described on the preceding page:

*m.* From March 15, 1902 to Aug. 10, 1903.
*n.* From Jan. 10, 1902 to Sept. 5, 1904.
*o.* From April 20, 1901 to Oct. 5, 1903.
*p.* From June 18, 1900 to Nov. 11, 1903.
*q.* From Feb. 13, 1901 to Dec. 6, 1904.
*r.* From May 19, 1901 to Jan. 12, 1904.
*s.* From July 30, 1900 to Feb. 1, 1903.
*t.* From Sept. 12, 1889 to March 5, 1902.
*u.* From Aug. 24, 1902 to March 20, 1904.
*v.* From Nov. 15, 1900 to April 10, 1903.
*w.* From Oct. 26, 1902 to May 15, 1905.
*x.* From Dec. 30, 1901 to June 18, 1904.

*a.* Find the interest of $375.24 from April 10, 1900 to Aug. 25, 1902, at 6%.

*b.* Find the interest of $247.37 from Jan. 15, 1901 to March 20, 1902, at 6%.

*c.* Find the interest of $312.25 from Oct. 15, 1902 to June 5, 1904, at 6%.*

*d.* Find the interest of $23.20 from Aug. 12, 1902 to Sept. 18, 1904, at 6%.

*e.* Find the interest of $125.50 from April 15, 1903 to Dec. 18, 1903, at 6%.

*f.* Find the interest of $1250 from March 20, 1902 to Jan. 10, 1903, at 7%.*

*g.* Find the interest of $8.34 from Dec. 10, 1900 to May 15, 1901, at 7%.

*h.* Find the interest of $54684.50 from Feb. 28, 1903 to March 1, 1903, at 6%.*

*i.* Find the interest of $3250 from Jan. 1, 1902 to Jan. 1, 1903, at 5%.

*j.* Find the amount of $145.23 from April 1, 1900 to Dec. 1, 1900, at 5%.

*k.* Find the amount of $3256 from July 10, 1901 to June 13, 1902, at 7%.

*l.* Find the amount of $78.36 from Nov. 12, 1901 to June 1, 1903, at 5%.*

*m.* Find the amount of $275.40 from July 1, 1903 to Feb. 1, 1904, at 7%.

*n.* Find the amount of $1000 from May 12, 1903 to Dec. 21, 1903, at 5%.

---

* Find the time by the "*better method*," described in example 1, on page 128 of this book.

A written promise to pay a sum of money at a specified time is called a **promissory note.**

The person who promises to pay and whose name is signed to the note is the **maker.** The person named in the body of the note as the one to whom the money is to be paid is the **payee.**

When a man borrows money at a bank, he gives his note for a specified sum to be paid at a specified time and he receives therefor, not the sum named in the note, but that sum less the interest upon it from the time the note is given to the bank officials to the time the note is due.

The acceptance of a note by the bank officials and the payment of a sum less than it will be worth at maturity is called **discounting the note.**

The **proceeds** of a note is the sum paid for it. As a rule, the notes discounted at a bank are those which bear no interest, and the date of discounting is usually the same as the date of the note, though not necessarily so.

1. Find the discount and proceeds of the following:

$800. JACKSONVILLE, ILL., Jan. 10, 1907.

Sixty days after date I promise to pay James Rice or order, eight hundred dollars, value received.

ARTHUR WILLIAMS.

Discounted at 6%, Jan. 10, 1907.

From the date of discount to the date of maturity it is 60 days. Interest of $800 for 60 days = $8.

Proceeds of note = $800 − $8 = $792.

Observe that the bank receives the interest on $800 for 2 months at 6% for the use of $792 for 2 months. The actual rate of interest is therefore a little more than the rate named.

*a.* $375.00. MINNEAPOLIS, MINN., April 5, 1907.

Thirty days after date I promise to pay to the order of John Smith, three hundred seventy-five dollars, at the Union National Bank of Chicago. Value received.

JAMES WHITE.

Discounted April 5, at 6%. Find proceeds.

*b.* $450.00. AURORA, ILL., March 10, 1907.

Sixty days after date I promise to pay to Wm. George, or order, four hundred fifty dollars, at the Old Second National Bank. Value received.

FAY D. WINSLOW.

Discounted March 10, at 7%. Find proceeds.

*c.* $2300.00. WAUKEGAN, ILL., Feb. 9, 1907.

Ninety days after date I promise to pay to the order of John Mulhall, two thousand three hundred dollars, at the Security Savings Bank. Value received.

CHAS. WHITNEY.

Discounted Feb. 9, at 7%. Find proceeds.

*d.* $5000.00. LANSING, MICH., Jan. 20, 1907.

Sixty days after date I promise to pay to John Parr, or order, five thousand dollars at the First National Bank of Detroit. Value received.

HARRY BROWN.

Discounted Jan. 20, at 7%. Find proceeds.

*e.* $3500. BOSTON, MASS., April 12, 1907.

Sixty days after date I promise to pay to W. J. Button, or order, three thousand five hundred dollars. Value received.

HARRY WILSON.

Discounted April 24, at 6%. Find proceeds.

*a.* $452.25. BOSTON, MASS., April 25, 1908.

Thirty days after date I promise to pay to the order of George W. Libbey, four hundred fifty-two and $\frac{25}{100}$ dollars. Value received.

MARTIN LUTHER.

Discounted April 25, at 7%. Find proceeds.

*b.* $974.28. SPRINGFIELD, ILL., May 10, 1908.

Sixty days after date, for value received, I promise to pay to the order of Alfred Bayliss, nine hundred seventy-four and $\frac{28}{100}$ dollars, at the First National Bank.

J. H. COLLINS.

Discounted May 10, at 6%. Find proceeds.

*c.* $2463.45. ROCK ISLAND, ILL., May 23, 1908.

Thirty days after date, for value received, I promise to pay to the order of S. J. Ferguson, two thousand four hundred sixty-three and $\frac{45}{100}$ dollars.

PETER PETERS.

Discounted May 23, at 5%. Find proceeds.

*d.* $145.50. ST. PAUL, MINN., June 20, 1908.

Ninety days after date, for value received, I promise to pay to the order of S. C. Rutherford, one hundred forty-five and $\frac{50}{100}$ dollars.

RICHARD S. TUTHILL.

Discounted June 20, at 6%. Find proceeds.

*e.* $48.25. URBANA, ILL., July 1, 1908.

Sixty days after date I promise to pay to the order of Eugene Davenport, forty-eight and $\frac{25}{100}$ dollars, at the Champaign National Bank. Value received.

W. A. HENRY.

Discounted July 20, at 6%. Find proceeds.

Note. Whenever a note is presented at a bank to be discounted, *its value at maturity* is regarded as the sum upon which the discount is to be reckoned. When the note is not interest-bearing, its face value * is its value at maturity. In the discounting of short-time notes at banks, it is customary to find the exact number of days from the date of discounting to the date of maturity, and to regard each day as $\frac{1}{360}$ of a year.

1. $750.00. De Kalb, Ill., Jan. 1, 1907.

Six months after date I promise to pay to the order of N. D. Gilbert, seven hundred fifty dollars, with interest at the rate of six per cent per annum. O. T. Bright.

Discounted at a bank, May 10, 1907, at 7 %.

| | |
|---|---|
| Value of the note at maturity, | $772.50 |
| Discount 52 days, at 7 % | 7.81 |
| Proceeds, | $764.69 |

*a.* $540.00. Topeka, Kans., Dec. 10, 1907.

Five months after date I promise to pay to the order of J. C. Thomas, five hundred forty dollars, with interest at the rate of six per cent per annum. Hiram Baker.

Discounted at a bank, April 1, 1908, at 7 %.

*b.* $325.00. Earlville, Ill., Feb. 1, 1907.

Six months after date I promise to pay to the order of Wm. R. Haight, three hundred twenty-five dollars, with interest at the rate of six per cent per annum.

Chas. Hoss.

Discounted at a bank, June 1, 1907, at 7 %.

*c.* Face of note, $500; date, Jan. 1, 1907; time, 3 months; rate of interest, 5 %; discounted, Feb. 1, 1907; rate of discount, 6 %. Find the proceeds.

---

* The *face value*, or "face" of a note, is the value shown upon its face — the sum of money expressed in words in the body of the note.

*a.* $345.00. WOBURN, MASS., April 25, 1907.

Six months after date, for value received, I promise to pay to the order of C. H. Root, three hundred forty-five dollars, with interest at the rate of six per cent per annum.

ALEXANDER HAMILTON.

Discounted August 30, 1907, at 7%. Find proceeds.

*b.* $800.00. EVANSTON, ILL., May 12, 1907.

Three months after date I promise to pay to Homer H. Kingsley, or order, eight hundred dollars, with interest at the rate of 5 per cent per annum. Value received.

PATRICK HENRY.

Discounted June 20, 1907, at 6%. Find proceeds.

*c.* $5000.00. DE KALB, ILL., March 1, 1907.

Four months after date, for value received, I promise to pay to the order of S. F. Parson, five thousand dollars, with interest at the rate of four per cent per annum.

ELI WHITNEY.

Discounted June 1, 1907, at 5%. Find proceeds.

*d.* $546.62. CANTON, OHIO, May 21, 1907.

Five months after date, for value received, I promise to pay to the order of R. B. Holmes, five hundred forty-six and $\frac{62}{100}$ dollars, with interest at the rate of five per cent per annum.

ROGER WILLIAMS.

Discounted July 12, 1907, at 6%. Find proceeds.

*e.* $1000.00. QUINCY, ILL., July 5, 1907.

One year after date, for value received, I promise to pay to the order of Leaton Irwin, one thousand dollars, with interest at the rate of five per cent per annum.

Witness: Clyan H. Hall. ALBERT $\underset{\text{mark}}{\overset{\text{his}}{\times}}$ ROGERS.

Discounted May 25, 1908, at 6%. Find proceeds.

A **partial payment** is a part payment made upon a note before the time of final settlement.

There are two methods in common use of finding the value of a note at maturity, upon which one or more partial payments have been made. The first method is often applied to computation of "short-time notes," such as run one year or less. A formal statement of this method is called the —

MERCHANTS' RULE. *Find the amount of the face of the note from the date to maturity. Then find the amount of each payment from the time it was made to the maturity of the note. From the amount of the face of the note subtract the sum of the amounts of the payments.*

1. $450.00. WAUKEGAN, ILL., Jan. 1, 1907.

One year after date I promise to pay to the order of Wm. E. Toll, four hundred fifty dollars, with interest at six per cent. Value received.

JEROME BIDDLECOM.

Part payments were made upon this note as follows: March 1, 1907, $250; May 1, 1907, $150.

| | | |
|---|---|---|
| Amount of the face of the note for one year, | | $477.00 |
| Amount of $250, March 1, '07 to Jan. 1, '08, | $262.50 | |
| Amount of $150, May 1, '07 to Jan. 1, '08, | 156.00 | 418.50 |
| Value at maturity, | | $58.50 |

*a.* $1000.00 SPRINGFIELD, ILL., Jan. 1, 1908.

Eight months after date I promise to pay to the order of Fred H. Wines, one thousand dollars with interest at six per cent. Value received.

J. H. FREEMAN.

Part payments were made upon this note as follows: April 1, 1908, $350; June 15, 1908, $240.

How much was due on this note at maturity, Sept. 1, 1908?

*a.* $500.00. CHICAGO, ILL., April 25, 1907.

One year after date, for value received, I promise to pay to the order of L. M. Dillman, five hundred dollars, with interest at six per cent. CARTER HARRISON.

Part payments were made on this note as follows: Aug. 1, 1907, $125; Jan. 1, 1908, $150.

How much was due on this note at maturity?

*b.* $750.00. DAVENPORT, IOWA, Jan. 1, 1907.

Nine months after date, for value received, I promise to pay to the order of John M. Raymond, seven hundred fifty dollars, with interest at the rate of seven per cent per annum. LEE N. GOODWIN.

Part payments were made on this note as follows: March 1, 1907, $250; May 1, 1907, $175; July 1, 1907, $225.

How much was due on this note at maturity?

*c.* $5000.00. WASHINGTON, PA., Jan. 11, 1907.

On demand, for value received, I promise to pay to the order of F. R. Hall, five thousand dollars, with interest at the rate of six per cent per annum. WM. PENN.

Part payments were made on this note as follows: Feb. 25, 1907, $650; April 10, 1907, $875; June 18, 1907, $1650.

How much was due on this note, Sept. 1, 1907?

*d.* $1175.00. FREEPORT, ILL., Sept. 23, 1906.

On or before November 1, 1907, for value received, I promise to pay to the order of Cyrus Grove, one thousand one hundred seventy-five dollars, with interest at the rate of six per cent per annum after January 1, 1907.

P. O. STIVER.

Part payments were made on this note as follows: Feb. 15, 1907, $500; May 6, 1907, $500.

How much was due on this note, Nov. 1, 1907?

A decision of the United States Supreme Court many years ago led to the very general adoption in the solution of "partial payment problems" of what is now known as the United States Rule. — *Find the amount of the principal at the time of the first payment. Subtract the first payment from this amount. The remainder is a new principal, upon which find the amount at the time of the second payment. Subtract the second payment from this amount. Continue this process to the time of settlement. The last amount is the sum due.*

*An exception to the foregoing rule* is required when there is a payment which is less than the interest due at the time the payment is made. In such case the payment is treated as though made at the time of the next payment; and if the two payments together do not equal the interest due, the sum of both is again carried forward.

$950.00. PETERSBURG, ILL., Jan. 1, 1905.

On or before Jan. 1, 1908, I promise to pay to N. W. Branson, or order, nine hundred fifty dollars, with interest at six per cent. Value received. B. LANING.

Part payments were made on this note as follows: July 1, 1905, $150; Jan. 1, 1906, $200; Jan. 1, 1907, $250. Find the amount due Jan. 1, 1908.

| | |
|---|---|
| Amount of $950, July 1, 1905 (6 mo.) | $978.50 |
| Subtract first payment | 150.00 |
| New principal | 828.50 |
| Amount of $828.50, Jan. 1, 1906 (6 mo.) | 853.35 |
| Subtract second payment | 200.00 |
| New principal | 653.35 |
| Amount of $653.35, Jan. 1, 1907 (1 yr.) | 692.55 |
| Subtract third payment | 250.00 |
| New principal | 442.55 |
| Amount of $442.55, Jan. 1, 1908 (1 yr.) | 469.10 |

*a.* Date of note, Jan. 1, 1906.
Face of note, $800.
Rate of interest, six per cent.
Payment, May 1, 1906, $125.
Find amount due Jan. 1, 1908.

*b.* Date of note, July 10, 1906.
Face of note, $500.
Rate of interest, six per cent.
Payment, Dec. 22, 1906, $200.
Payment, July 15, 1907, $200.
Find amount due Jan. 1, 1908.

*c.* Date of note, Jan. 1, 1907.
Face of note, $600.
Rate of interest, six per cent.
Payment, Sept. 1, 1907, $100.
Find amount due Jan. 1, 1908.

*d.* Date of note, July 1, 1906.
Face of note, $1000.
Rate of interest, six per cent.
Payment, March 1, 1907.
Find amount due Jan. 1, 1908.

NOTE. Solve problem *c* by the Merchants' Rule and by the United States Rule. The answer obtained by the latter will be 48 cents larger than the answer obtained by the former.

Observe that the Merchants' Rule is based upon the supposition that interest is not due until the time of settlement of the note, while the United States Rule is based upon the supposition that interest is due whenever a payment is made. By applying the latter rule to problem *c*, $24 of interest must be paid Sept. 1; by applying the former rule to the same problem the entire $100 paid Sept. 1, applies in payment of principal. The answer, then, by the United States Rule, must be greater than the answer by the Merchants' Rule, by the interest on $24 for 4 months, or 48 cents.

*a.* $550. TUSKEGEE, ALA., June 12, 1905.

On demand, after one year from date, for value received, I promise to pay to the order of J. R. E. Lee, five hundred fifty dollars, with interest at the rate of six per cent per annum from date till paid. HOMER WASHINGTON.

Part payments were made on this note as follows: November 24, 1906, $225; March 1, 1907, $125; October 1, 1907, $100.

How much was due on this note January 1, 1908?

*b.* $960.50. CHICAGO, ILL., January 20, 1906.

On or before January 20, 1908, I promise to pay to A. J. Cheney, or order, nine hundred sixty $\frac{50}{100}$ dollars, with interest at the rate of five per cent per annum. Value received. ALLEN WEBSTER.

Part payments were made on this note as follows: March 17, 1906, $300; December 1, 1906, $250; April 15, 1907, $175.

How much was due on this note January 1, 1908?

*c.* $475.00. HONEY CREEK, WIS., February 25, 1906.

On demand, for value received, I promise to pay to the order of Jonathan Piper, four hundred seventy-five dollars, with interest at the rate of six per cent per annum till paid.

ALBERT CARTER.

Part payments were made on this note as follows: April 13, 1906, $75; April 13, 1907, $15; June 10, 1907, $100.

How much was due on this note January 1, 1908?

*d.* $100.00. AURORA, ILL., July, 1, 1906.

One year after date, for value received, I promise to pay to the order of A. V. Greenman, one hundred dollars, with interest at seven per cent. H. D. JUDSON.

If $50 was paid on the above note January 1, 1907, how much was due July 1, 1907?

*a.* Clarence Marshall wished to borrow some money at a bank. He was told by the president of the bank that they (the bank officials) were "discounting good 30-day paper" at 7%. Mr. Marshall's name being regarded as "good," he drew his note upon one of the forms in use at the bank, for $1000 payable in thirty days without interest. On the presentation of this note to the cashier, how much money should he receive?

*b.* If Mr. Marshall's note described in problem *a* was dated April 10, 1908, when must it be paid? *c.* How much money will he pay when he "takes up" the note? *d.* Does he pay for the use of the money borrowed, at the rate of exactly 7% per annum?

*e.* If a bank is discounting at 7%, how much should be given for a note of $200 due in two months from the time it is discounted and bearing interest at the rate of 6% per annum from the date of discounting?

*f.* $5000. LEWISTON, IDAHO, April 1, 1906.

On or before April 1, 1909, for value received, I promise to pay to the order of Gordon Bowdish, five thousand dollars, with interest annually at the rate of six per cent per annum; and it is mutually agreed that the maker of this note may at his option pay any part or all of the above mentioned sum on any day on which interest is payable, but at no other time.

WILLIAM GEORGE.

Part payments were made on this note as follows: April 1, 1907, $1250; April 1, 1908, $950.

How much was due on this note April 1, 1909?

*a.* A note for $350, bearing no interest, and due in 45 days, is discounted at a bank, at 7%. Find the proceeds.

*b.* A note for $125, which will be due, together with one year's interest at 5%, on the first day of November, is discounted at a bank at 6%, on the first day of September preceding the date of its maturity. Find the proceeds.

*c.* Find the proceeds of a $500 non-interest-bearing note discounted at 7%, 24 days before the date of its maturity.

*d.* What principal at 6% will gain $94.50 in one year?

*e.* What principal at 6% will gain $34.20 in one year and six months? *

*f.* What principal at 6% will gain $19.04 in one year and five months?

*g.* What principal at 6% will gain $48.16 in one year, ten months, and twelve days?

*h.* What principal at 5% will gain $48.75 in one year and six months?

*i.* What principal at 7% will amount to $596.70 in one year and six months?

*j.* What principal at 6% will amount to $346.56 in one year, four months, and eighteen days?

*k.* What principal at 6% will amount to $416.18 in one year, seven months, and twelve days?

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* In 1 year and 6 months 1 dollar at 6% will gain 9 cents. It will take as many dollars to gain $34.20 as 9 cents is contained times in $34.20.

# STOCKS AND BONDS

Some kinds of business require so much capital that many persons combine to provide the necessary money. Such a combination of men organized under the laws of a state, the capital being divided into shares, is known as a **corporation**, or **stock company**. Those who own the shares are called **stockholders**. The stockholders elect from their own number certain men to manage the business. These managers are called **directors**.

The **nominal value** or **par value** of a share is its face value; that is, the sum named on its face. Large corporations usually, though not always, divide their capital into $100 shares.

If the business is prosperous, shares may sell on the market for more than their nominal value. The stock is then said to be *above par*, or *at a premium*.

If the business does not prosper, the shares may sell on the market for less than their nominal value. The stock is then said to be *below par*, or *at a discount*.

If the business is profitable, a part of all of the earnings is periodically divided among the stockholders. The sum divided is called a **dividend**.

Dividends are always reckoned on the par value of the stock. If a corporation declares a 2 % dividend, it pays to each stockholder a sum equal to 2 % of the par value of the stock he owns.

The **capital stock** of a corporation is the amount, at par value, of all the stock issued.

Some corporations issue two kinds of stock known as **preferred stock** and **common stock.** The holders of preferred stock must be paid certain specified dividends before the holders of common stock can participate in the profits of the business.

NOTE. Stocks are sometimes bought and sold on commission. In such cases the commission is usually reckoned upon the par value as a base.

### THE HISTORY OF A STOCK COMPANY

The farmers of a certain community agreed to combine in the building and management of a creamery. It was determined that a capital of $5000 was necessary. This was divided into 50 shares of $100 each. Men then came forward and contributed as follows:

A took 5 shares and paid in $ 500.
B took 3 shares and paid in $ 300.
C took 6 shares and paid in $ 600.
D took 4 shares and paid in $ 400.
E took 7 shares and paid in $ 700.
F took 2 shares and paid in $ 200.
G took 10 shares and paid in $1000.

H, I, J, K, L, M, N, O, P, Q, R, S, & T took 1 share each and paid in $100 each.

*a.* At the end of the first year the directors declared an 8% dividend. How much did A receive? B? C? K?

*b.* What was the entire amount of the money divided among the stockholders at the end of the first year?

*c.* Soon after this dividend was paid, A sold his stock to X at a premium of 10%. How much did A receive for his stock?

*d.* At the end of the second year the profits were found to be comparatively small, and the directors could pay only a 3 % dividend. How much did X receive? B? C? K?

*e.* What was the entire amount of the money divided among the stockholders at the end of the second year?

*f.* Soon after this dividend was paid, B, C, D, E, and F sold their stock to Y at a discount of 10 %. How much did this stock cost Y?

*g.* At the end of the third year the profits were so small that no dividend was declared. The stockholders became disheartened, and many of them offered to sell their stock at a large discount. Z appeared in the market and bought at "50¢ on the dollar" all the stock of the company except that owned by X and Y. How much did this stock cost Z?

*h.* At the end of the fourth year the directors, X, Y, and Z, declared a 10% dividend. How much money was divided, and how much did each receive?

*i.* Before the close of the fifth year the property burned, and the lot upon which it stood was sold. After the insurance money had been received, the book accounts collected, and all debts paid, there remained in the treasury of the company $ 4350. How much of this money should each stockholder, X, Y, and Z, receive?

*j.* Did this creamery enterprise prove a good investment for X? for Y? for Z? for A? for B? for M?

*k.* Mr. Steele owns 20 shares ($ 100) in the C., B. & Q. R.R. He receives as his part of a certain dividend $ 110. What is the per cent of the dividend?

*a.* The directors of a company whose capital is $200000 determine to distribute among the stockholders $70000 of profits. A dividend of what per cent will be declared? *b.* How much will a man receive who owns twenty-five 100-dollar shares?

*c.* A bank, the capital of which is $100000, pays a dividend of 4½%. How much money is divided among the stockholders? *d.* How much does a man receive who owns fifteen 100-dollar shares?

*e.* A man who owns twenty 100-dollar shares in the Illinois Central Railroad receives as his part of a certain dividend $140. The dividend is what per cent of the par value of the stock?

*f.* Stock that was bought at 20 per cent below par yielded an annual dividend of 4%. This was equivalent to what rate of interest on the money invested?*

*g.* Stock that was bought at 20 per cent above par yielded an annual dividend of 7%. This was equivalent to what rate of interest on the money invested?

*h.* The present and prospective annual dividend on certain stock is 8%. If the present and prospective rate for money is 5%, what is the actual value of the stock per 100-dollar share? †

*i.* The present and prospective annual dividend on certain stock is 4%. If the present and prospective rate for money is 5%, what is the actual value of the stock per 100-dollar share?

---

* Observe that the stock paid $4 on an investment of $80. On $1 it pays $\frac{1}{80}$ of $4.

† This stock yields $8 per share. $8 is 5% on $160. Hence the stock is worth $160 per share.

A **bond** is a very formal promissory note given by a government or other corporate body, as a railway or a gas company, for money borrowed. Bonds usually have attached to them small certificates called **coupons**. These are really little notes for the interest that will be due at different times. Thus a 10-year bond for $1000 with interest at 6% payable semiannually will have 20 coupons attached, each calling for $30 of interest.

Money invested in bonds yields a specified income; but the income from money invested in stocks depends upon the profits of the company.

Bonds, like stocks, are sometimes sold for more than their face value. They are then said to be *above par* or *at a premium*. Like stocks, too, they are sometimes *below par*, or *at a discount*.

*a*. What is the semiannual interest on two 1000-dollar U.S. 5% bonds?

*b*. What sum should be named on each coupon of a 1000-dollar city bond if the interest is payable annually at the rate of 7%?

*c*. To raise the money to build a courthouse, a certain county issued $50000 worth of 6% 10-year bonds. These sold upon the market at 2% premium.* How much money was received for the bonds? *d*. How much did A pay, who bought three 1000-dollar bonds?

*e*. If the interest was payable semiannually, how much should A receive each 6 months on this investment? *f*. Did A receive exactly 6% on the money invested?

---

*** Premium, discount, interest, and commission are all reckoned on the par value of the bond.**

*a.* Has the county or city in which you live any "bonded indebtedness"? *b.* If so, how much, and what is the rate of interest? *c.* Did the bonds sell at a premium, at par, or at a discount?

*d.* If the rate is 4½% per annum, what is the half-yearly interest on a $5000 bond?

*e.* What is the face value of 5% bonds sufficient to yield a half-yearly income of $625?

*f.* If the face of a bond is $1000 and the rate 3½%, what sum should be written in each half-yearly coupon?

*g.* If the face of a bond is $5000 and the rate 5½%, what sum should be written in each quarter-yearly coupon?

*h.* How much must I invest in 5% bonds at 2% premium to secure an annual income of $250?

*i.* How much must I invest in 3½% bonds at 8% discount to secure an annual income of $700?

*j.* How much must be paid for ten $1000 bonds at 6% premium if they are purchased through a broker who charges ⅛ of 1% for doing the business?†

*k.* If 4% bonds are sold at 20% discount, what rate of interest will the purchaser receive on his investment?*

*l.* If 6% bonds are sold at 20% premium, what rate of interest will the purchaser receive on his investment?

*m.* How much must be paid for $5000 of U. S. 4% bonds at 6% premium, brokerage ⅛ of 1%?†

---

* He will receive $4 on each $80 invested. How much on $1?

† Brokerage (commission) is reckoned on the par value.

*a.* The rate of interest on certain bonds is 5% per annum payable half yearly on the first day of January and the first day of July. If the actual value of the bonds is the same as their nominal value, how much, including accrued interest, should be paid for five $1000 bonds on the first day of April?

*b.* The rate of interest on certain bonds is 6% per annum payable half yearly on the first day of March and the first day of September. If the bonds are worth 4% premium, how much should be paid, including accrued interest, for ten $1000 bonds on the fifteenth day of July?*

*c.* Paid $2090 for two $1000 bonds. What per cent premium was paid?

*d.* Paid $5220 for six $1000 bonds. At what per cent below par were the bonds purchased?

*e.* A man bought stock at 6% below par and sold it at 5% premium. He gained $1045. How much money did he invest?

*f.* A man bought stock at 35% below par and sold it at 22% below par. He gained $1495. How much did he receive for the stock?

*g.* How much must a man invest in 4% stock at 12% discount to yield an annual income of $560?

*h.* A stock company, capitalized at $200,000, makes in one year a net gain of $18,250. If an 8% dividend is declared, how much may be "carried to surplus"? *i.* The amount carried to surplus is what per cent of the capital?

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* Each $1000 bond will cost $1000 plus 4% of $1000 plus the interest on $1000 from March 1 to July 15 at 6%.

*a.* Mr. Leet purchased 40 shares of Illinois Central Railroad stock at 35% premium. How much did he pay for the stock? *b.* When a 3½% dividend is declared and paid, how much does he receive?

*c.* Mr. Love bought 50 shares of stock in a gas manufactory at 42% below par and sold it at 13% below par. If he bought and sold this stock through a broker, paying ⅛ of 1% for buying and ⅛ of 1% for selling, how much was his gain?

*d.* What must be the nominal value of 6% bonds that will yield to their owner an annual income of $396?

*e.* What is the actual value of 6% bonds that will yield to their owner an annual income of $738, provided they will bring on the market a premium of 8%?

*f.* What is the actual value of 3½% bonds that will yield to their owner an annual income of $196, provided they are selling on the market at 17% below par?

*g.* A broker paid $8925 for some stock at 5% premium. What was the par value of the stock?*

*h.* A broker paid $5208 for some stock, buying it at 16% below par. What was the par value of the stock?

*i.* Which is the better investment, 4% stock at 20% below par, or 6% stock at 20% above par?

*j.* If the par value of certain stock is $50, how much above par is it when a share sells for $60? *k.* How much below par when a share sells for $45?

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* This question is equivalent to the following: $8925 is 105% of what?

# DENOMINATE NUMBERS

## LINEAR MEASURE

The English and United States standard unit of length is the **Imperial yard** arbitrarily fixed by Act of Parliament and afterward adopted in the United States. It is about $\frac{360000}{391393}$ of the length of a pendulum that vibrates once a second at the level of the sea in the latitude of London. It is $\frac{3600}{3937}$ of a meter.

### Linear Measure

12 inches (in.) = 1 foot (ft.).
3 feet = 1 yard (yd.).
$5\frac{1}{2}$ yards = 1 rod (rd.).
$16\frac{1}{2}$ feet = 1 rod.*
320 rods = 1 mile (mi.).
1760 yards = 1 mile.
5280 feet = 1 mile.
1 fathom (used in measuring the depth of the sea) = 6 feet.
1 knot (used in navigation) = 1.15 + miles.
1 league (used in navigation) = 3 knots.
1 hand (used in measuring the heights of horses) = 4 inches.
1 chain (used by civil engineers) = 100 feet.
1 chain (used by land surveyors) = 66 feet.
1 pace (used in measuring approximately) = $\frac{1}{5}$ of a rod.
1 barleycorn (used in grading length of shoes) = $\frac{1}{3}$ of an inch.
1 furlong (a term nearly obsolete) = $\frac{1}{8}$ of a mile.

* The term *pole* is sometimes used instead of *rod*.

**1.** Mont Blanc is 15810 feet, or about —— miles high.

**2.** Mt. Everest is 29000 feet, or about —— miles high.

**3.** Commodore Dewey opened fire on the enemy at a distance of 5000 yards, or about —— miles.

**4.** My horse, measured over the front feet, is 16¼ hands, or —— feet —— inches high.

**5.** The vessel seemed to be about three leagues, or —— miles distant.

**6.** On sounding, they found the depth of the water to be 15 fathoms, or —— feet.

**7.** The cruiser made 20 knots, or about —— miles, an hour.

**8.** The length of the lot was 36 paces, or about —— rods.

**9.** 10000 feet is nearly —— miles.

**10.** 15000 feet is nearly —— miles.

**11.** 1000 yards is about —— —— of a mile.

**12.** 100 feet is —— rods —— foot.

**13.** 200 feet is —— rods —— feet.

**14.** 300 feet is —— rods —— feet.

**15.** 400 feet is —— rods —— feet.

**16.** A civil engineer's chain is —— rods —— foot.

*a.* A 7-foot drive wheel of a locomotive makes how many revolutions to the mile?

*b.* Which is the longest distance, 5 miles 319 rods 16 feet 6 inches, 5 miles 319 rods 5 yards 1 foot 6 inches, or 6 miles?

*c.* Reduce 40 rd. 4 ft. 5 in. to inches.

*d.* Reduce 1100 in. to yards, feet, and inches.

1. One half of a foot is what part of a rod?

2. One foot is what part of a rod?

3. Three and one half feet is what part of a rod?

4. Ten and one half feet is what part of a rod?

*a.* Express 64 rods as a fraction of a mile, in its lowest terms.

*b.* Express 96 rods as a fraction of a mile, in its lowest terms.

*c.* One hundred eighty rods is what part of a mile?

*d.* Change 80 rods to the fraction of a mile, and express the fraction decimally.

*e.* Change 40 rods to the fraction of a mile, and express the fraction decimally.

*f.* Change 1200 feet to the fraction of a mile, in its lowest terms.

*g.* Change 3000 feet to the fraction of a mile, in its lowest terms.

*h.* Change $\frac{1}{8}$ of a mile to feet.

*i.* Change $\frac{5}{8}$ of a mile to feet.

*j.* Change $\frac{5}{8}$ of a mile to rods.

*k.* Change $\frac{5}{8}$ of a mile to yards.

*l.* A hundred yards is what part of a mile?

*m.* A hundred feet is how many yards?

*n.* A hundred yards is how many feet?

*o.* A hundred rods is how many feet?

*p.* A hundred feet is how many rods?

*q.* A hundred inches is how many feet?

1. One mile is how many rods?

*a.* One mile and 56 rods are how many rods?

*b.* Change 2 miles 84 rods to rods.

*c.* Reduce 5 miles 63 rods to rods.

*d.* Reduce 7 miles 230 rods to rods.

2. One mile is how many yards?

*e.* One mile and 500 yards are how many yards?

*f.* Change 2 miles 300 yards to yards.

*g.* Reduce 4 miles 800 yards to yards.

*h.* Reduce 3 miles 1000 yards to yards.

3. One mile is how many feet?

*i.* One mile and 720 feet are how many feet?

*j.* Change 2 miles 750 feet to feet.

*k.* Reduce 3 miles 375 feet to feet.

*l.* Reduce 4 miles 3000 feet to feet.

4. One rod is how many feet?

5. One rod and 7½ feet are how many feet?

*m.* Five rods and 8½ feet are how many feet?

*n.* Change 6 rods 1 foot to feet.

*o.* Reduce 12 rods 2 feet to feet.

*p.* Reduce 15 rods 6½ feet to feet.

6. One rod is how many yards?

7. One rod and 3½ yards are how many yards?

*q.* Five rods and 2½ yards are how many yards?

*r.* Change 3 rods 4 yards to yards.

*s.* Reduce 5 rods 2½ yards to yards.

1. Change 96 inches to feet.
*a.* Change 1000 inches to feet and inches.
*b.* Reduce 4240 inches to feet and inches.
*c.* Reduce 5550 inches to feet and inches.

2. Change 33 feet to rods.

3. Change 38 feet to rods and feet.
*d.* Change 500 feet to rods and feet.
*e.* Reduce 800 feet to rods and feet.
*f.* Reduce 1250 feet to rods and feet.

4. Change 36 feet to yards.

5. Change 40 feet to yards and feet.
*g.* Change 200 feet to yards and feet.
*h.* Reduce 530 feet to yards and feet.
*i.* Reduce 725 feet to yards and feet.

6. Change 11 yards to rods.

7. Change 15 yards to rods and yards.
*j.* Change 75 yards to rods and yards.
*k.* Reduce 100 yards to rods and yards.
*l.* Reduce 1000 yards to rods and yards.

8. Change 640 rods to miles.

9. Change 650 rods to miles and rods.
*m.* Change 1200 rods to miles and rods.
*n.* Reduce 2000 rods to miles and rods.
*o.* Reduce 3000 rods to miles and rods.

10. Change 6000 feet to miles and feet.
*p.* Change 12000 feet to miles and feet.

**1.** Multiply 3 feet 5 inches by 4.

*a.* Multiply 27 feet 9 inches by 4.

**2.** Multiply 6 yards 2 feet by 2.

*b.* Multiply 15 yards 1 foot by 7.

**3.** Multiply 5 miles 200 rods by 2.

*c.* Multiply 15 miles 300 rods by 6.

**4.** Find ½ of 3 miles 100 rods.

*d.* Find ½ of 11 miles 200 rods.

**5.** Find ¼ of 5 miles 40 rods.

*e.* Find ¼ of 37 miles 120 rods.

**6.** Find how many times 1 foot 8 inches is contained in 10 feet.

*f.* Find how many times 2 feet 3 inches is contained in 75 feet.

**7.** Find how many times 1 yard 2 feet is contained in 10 yards 1 foot.

*g.* Find how many times 2 yards 1 foot is contained in 46 yards 2 feet.

**8.** Find the sum of 3 ft. 7 in. and 4 ft. 8 in.

*h.* Find the sum of 27 ft. 9 in. and 43 ft. 7 in.

**9.** Find the sum of 3 rd. 10 ft. and 4 rd. 8 ft.

*i.* Find the sum of 84 rd. 9 ft. and 31 rd. 10½ ft.

**10.** From 8 ft. 2 in. subtract 4 ft. 10 in.

*j.* From 43 ft. 5 in. subtract 20 ft. 11 in.

**11.** From 1 mi. subtract 100 rd.

*k.* From 6 mi. subtract 2 mi. 100 rd.

**12.** From 1 mi. subtract 1000 ft.

The standard unit of surface measure is a **square yard**, which is the equivalent of a 1-yard square. This unit, like the square foot, square inch, square rod, and square mile, is derived from the corresponding unit of linear measure.

### Square Measure

144 square inches (sq. in.) = 1 square foot (sq. ft.).
9 square feet = 1 square yard (sq. yd.).
30¼ square yards = 1 square rod (sq. rd.).
272¼ square feet = 1 square rod.
160 square rods = 1 acre (A.).
4840 square yards = 1 acre.
43560 square feet = 1 acre.
640 acres = 1 square mile (sq. mi.).

1. Show by a drawing that there are 144 square inches in a 1-foot square.

2. Show by a drawing that there are 9 square feet in a 1-yard square.

3. Show by a drawing that there are 30¼ square yards in a 1-rod square.

4. Estimate the number of square yards of blackboard in the room; the number of square feet of blackboard.

5. Estimate the number of square feet in the floor of the schoolroom; the number of square yards.

6. Estimate the square yards of plastering on the walls of the schoolroom.

7. Estimate the number of square rods in the schoolhouse lot. Is the lot more or less than ½ of an acre?

8. Estimate the area of your flower-bed.

Find how many acres in:

1. A piece of land 1 rod by 160 rods.
2. A piece of land 7 rods by 160 rods.
3. A piece of land 13 rods by 160 rods.
4. A piece of land 22 feet* by 160 rods.
5. A piece of land 8¼ yards* by 160 rods.

*a.* Find the sum of the five results, **1** to **5**.

6. A piece of land 8 rods by 80 rods.
7. A piece of land 17 rods by 80 rods.
8. A piece of land 37½ rods by 80 rods.
9. A piece of land 618¾ feet* by 80 rods.
10. A piece of land 550 yards* by 80 rods.

*b.* Find the sum of the five results, **6** to **10**.

11. A piece of land 12 rods by 40 rods.
12. A piece of land 27 rods by 40 rods.
13. A piece of land 46 rods by 20 rods.
14. A piece of land 36 rods by 20 rods.
15. A piece of land 264 feet* by 20 rods.

*c.* Find the sum of the five results, **11** to **15**.

16. A piece of land 1 rod by 1 mile.
17. A piece of land 11 rods by 1 mile.
18. A piece of land 66 feet* by 1 mile.
19. A piece of land 99 yards* by 1 mile.
20. A piece of land 198 feet* by ½ of a mile.

*d.* Find the sum of the five results, **16** to **20**.

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* Change to rods.

*a.* A piece of land 1 foot wide and 43560 feet long is how many acres?

*b.* Change 43560 feet to miles.

*c.* A piece of land 1 foot wide must be how many miles in length to contain 1 acre?

*d.* Some country roads are 66 feet wide. How many acres in 8¼ miles of such road?

*e.* How many acres in 1 mile of road that is 4 rods wide?

*f.* A farmer walking behind a plow that makes a furrow 1 foot wide will travel how far in plowing 1 acre?

*g.* A farmer walking behind a plow that makes a furrow 16 inches wide will travel how far in plowing 1 acre?

*h.* If a mowing machine cuts a swath that averages 4 feet in width, how far does it move in cutting 1 acre?

*i.* If potatoes are planted in rows that are 3 feet apart, how many miles of row to each acre? *j.* How many rods of row to each acre? *k.* If 4 rods of row on the average yield 1 bushel, what is the yield per acre?

*l.* Strawberry plants are set in rows that are 2 feet apart. How many miles of row to the acre? *m.* How many rods of row to the acre? *n.* How many feet of row to the acre?

*o.* If corn is planted in rows 3½ feet apart and if the "hills" are 3½ feet apart in the row, how many hills to each acre? *

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* Think of each "hill" as occupying a piece of land 3½ ft. by 3½ ft.

In the measurement of land it is more convenient to use a decimal scale; hence the invention of the **Gunter Chain.** This chain is 4 rods long and is divided into 100 links.

Observe that links are hundredths of chains.
Observe that square chains are tenths of acres.

*a.* Land, 3 chains by 4 chains contains —— acres.*
*b.* Land, 5 chains by 4 chains contains —— acres.
*c.* Land, 3 chains by 8 chains contains —— acres.
*d.* Land, 5 chains by 7 chains contains —— acres.
*e.* Land, 8 chains by 6 chains contains —— acres.
*f.* Two chains 20 links equals —— chains.†
*g.* Two chains 35 links equals —— chains.
*h.* Two chains 75 links equals —— chains.
*i.* Two chains 5 links equals —— chains.
*j.* Two chains 9 links equals —— chains.
*k.* Land, 4 ch. by 4.50 ch. contains —— acres.‡
*l.* Land, 5 ch. by 3.20 ch. contains —— acres.

Find the number of acres in each of the following:

*m.* Land, 12 chains 35 links by 9 chains 50 links.
*n.* Land, 21 chains 8 links by 12 chains 30 links.
*o.* Land, 32 chains 25 links by 15 chains 6 links.

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* Land 3 ch. by 4 ch. contains 12 sq. ch. = 1.2 acres.

† 2 ch. 20 l. = 2.20 chains.

‡ Divide the number of square chains by 10; that is, after "multiplying the length by the breadth," remove the decimal point one place to the left.

To change rods and feet to chains and links: Observe that a rod is exactly 25 links; that a foot is about 1½ links. Hence each 4 rods is 1 chain, and each additional rod is 25 links, and each additional foot is 1½ links. The error resulting from regarding the foot as 1½ links never exceeds 2 inches in any one measurement in which the number of feet is less than 1 rod.

12 rods = 3 chains.
13 rods = 3 chains 25 links.
14 rods = 3 chains 50 links.
15 rods = 3 chains 75 links.
16 rods = 4 chains.
16 rods 6 feet = 4 chains 9 links = 4.09 ch.
16 rods 10 feet = 4 chains 15 links = 4.15 ch.
17 rods 6 feet = 4 chains 34 links = 4.34 ch.
18 rods 10 feet = 4 chains 65 links = 4.65 ch.

Change each of the following to chains:

*a.* 20 rods
*b.* 30 rods
*c.* 21 rods
*d.* 31 rods
*e.* 22 rods
*f.* 32 rods
*g.* 23 rods
*h.* 33 rods
*i.* 24 rd. 6 ft.
*j.* 34 rd. 6 ft.
*k.* 25 rd. 10 ft.
*l.* 35 rd. 10 ft.
*m.* 26 rd. 12 ft.
*n.* 36 rd. 12 ft.
*o.* 27 rd. 14 ft.
*p.* 37 rd. 14 ft.
*q.* 28 rd. 2 ft.
*r.* 38 rd. 2 ft.
*s.* 29 rd. 3 ft.
*t.* 39 rd. 3 ft.
*u.* 30 rd. 7 ft.
*v.* 40 rd. 7 ft.
*w.* 31 rd. 9 ft.
*x.* 41 rd. 9 ft.

*y.* Find the sum of the 24 results, *a* to *x*.

To find the area in acres, of a rectangular piece of land whose dimensions are expressed in rods and feet: *Reduce the given dimensions to chains; "multiply the length by the breadth" and divide by* 10.[*]

Find the number of acres in each of the following rectangular lots:

*a.* 12 rods 6 feet by 9 rods 14 feet.[†]
*b.* 21 rods 10 feet by 18 rods 12 feet.
*c.* 36 rods 2 feet by 23 rods 8 feet.
*d.* 82 rods 7 feet by 31 rods 13 feet.
*e.* 75 rods 15 feet by 19 rods 3 feet.
*f.* 42 rods 6 feet by 37 rods 10 feet.
*g.* 55 rods 9 feet by 28 rods 12 feet.
*h.* 94 rods 11 feet by 18 rods 4 feet.
*i.* 46 rods 5 feet by 22 rods 8 feet.
*j.* 12 rods 12 feet by 16 rods 4 feet.
*k.* 15 rods 2 feet by 24 rods 14 feet.
*l.* 86 rods 3 feet by 53 rods 13 feet.
*m.* 1000 feet by 1 rod 4 feet 8 inches.
*n.* Half a mile by 2 rods 8 feet.
*o.* One mile by 3 rods 7 feet 4 inches.

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[*] The expression "multiply the length by the breadth" is a convenient absurdity. In this instance the length is not the multiplicand, neither is it possible to multiply by breadth. Numbers may be multiplied by numbers, and one of the factors may be thought of as concrete. The concrete factor in this multiplication (if there is one) is the number of square chains in a piece of land as long (or as broad) as the given dimension, and 1 chain wide.

[†] The error in the answer to this problem resulting from regarding the link as 8 inches, when it is really 7.92 inches, is less than one hundredth of an acre.

To determine the amount of **carpet** necessary for a given room several minor problems must be solved which can be best studied by means of an example:

1. How many yards of carpet must be purchased for a room 16 ft. by 20 ft. if the carpet is 1 yd. wide?

*a.* How many breadths will be necessary if the carpet is put down lengthwise of the room? *b.* How much must be cut off or turned under from one breadth in this case?

*c.* How many breadths will be necessary if the carpet is put down crosswise of the room? *d.* How much must be cut off or turned under from one breadth in this case?

*e.* Make two diagrams of the room on a scale of 1 inch to the foot and show the breadths of carpet in each case.

*f.* How many yards must be purchased in each case?

*g.* If in the first case there is no waste in matching the figure and in the second case there is a waste of 8 inches on each breadth except the first,* which plan of putting down the carpet will require the greater number of yards?

*h.* If the carpet costs 90¢ a yard and the conditions are as stated in *g*, what is the cost of the carpet in each case?

*i.* How many yards of carpet must be purchased for a room 16 ft. by 20 ft. if the carpet is $\frac{3}{4}$ of a yard wide and there is no waste in matching the figure?

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* Why should the first breadth be excepted?

*a.* How many square yards of plastering in a room (walls and ceiling) that is 15 ft. by 18 ft. and 12 ft. high, an allowance of 12 square yards being made for openings?

NOTE. In estimating the cost of plastering, allowance is made for "openings" (windows and doors) only when they are very large in proportion to the wall to be covered. Why are plasterers unwilling to deduct the entire area of all the openings?

*b.* At 34¢ a square yard how much will it cost to plaster a room that is 17 ft. by 20 ft. and 10 feet from the floor to the ceiling, deducting 16 square yards for openings?

*c.* How many "double rolls" of paper will be required for the walls of a room that is 14 ft. by 16 ft. and 11 ft. high above the baseboards, if an allowance of 1 full "double roll" is made for openings?

NOTE. Wall paper is usually 18 inches wide. A "single roll" is 24 ft. long. A "double roll" is 48 ft. long. In papering a room 11 ft. high it would be safe to count on 4 full strips from each "double roll." The remnant would be valueless unless it could be used over windows or doors. Since each strip is 18 inches wide, a "double roll" will cover 72 inches (6 ft.) of wall measured horizontally.

*d.* At 12¢ a "single roll," how much will the paper cost for the walls of a room that is 12 ft. by 14 ft. and 7 ft. above the baseboards, if the area of the openings is equivalent to the surface of 2 "single rolls" of paper?

*e.* Find the cost, at 35¢ a square yard, of plastering the walls of a room that is 48 ft. by 60 ft. and 18 ft. high, deducting 30 square yards for openings. *f.* Find the cost if no allowance is made for openings.

*a.* How many yards of carpet must be purchased for a room that is 17 ft. 6 in. by 20 ft. 6 in. if the carpet is 1 yard wide, is to be put down lengthwise of the room, and if there is a waste of 15 inches on each breadth except the first in matching the figure?

*b.* How many yards of carpet must be purchased for a room that is 14 ft. 4 in. by 16 ft. 8 in. if the carpet is $\frac{3}{4}$ of a yard wide, is to be put down lengthwise of the room, and if there is a waste of 10 inches on each breadth except the first in matching the figure? *c.* How many yards required if the carpet is put down crosswise with the same waste in matching?

*d.* How many yards of plastering in a room (walls and ceiling) that is 15 ft. 8 in. by 17 ft. 4 in. and 11 ft. 6 in. high, an allowance of 10 square yards being made for openings?

*e.* The sum of the perimeters of all the rooms on the first floor of a house is 346 feet. The rooms are 10 ft. 6 in. high. Making no allowance for openings, how much will it cost, at 25 cents a yard, to plaster the walls of all the rooms on that floor of the house?

*f.* In problem *e* how many yards would there have been had the rooms been exactly 9 feet high instead of 10 ft. 6 in.? *g.* 10 feet 6 inches is what fraction of 9 feet more than 9 feet? *h.* 346 yards plus $\frac{1}{6}$ of 346 yards equals what?

*i.* If rooms are 10 ft. high, the number of feet in the perimeter must be increased by what fraction of itself to give a number that shall be the same as the number of square yards in the walls? *j.* Perimeter, 60 ft.; height, 10 ft. Find area of walls in square yards.

The standard unit of volume measure is a **cubic yard**, which is the equivalent of a 1-yard cube. This unit, like the cubic foot and the cubic inch, is derived from the corresponding unit of linear measure.

### Cubic Measure

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.).
27 cubic feet = 1 cubic yard (cu. yd.).

1. Show by a drawing that there are 27 cu. ft. in a 1-yard cube.

2. Estimate in cubic feet the amount of air in the schoolroom.

3. Estimate in cubic yards the amount of air in the schoolroom.

4. Estimate in cubic inches the capacity of your dinner box.

**Wood** is usually measured by the **cord**. A cord is a pile 4 feet wide, 4 feet high, and 8 feet long or its equivalent. Hence,

128 cubic feet = 1 cord.

*a.* Estimate the number of cords of wood that could be put upon the floor of the schoolroom if the desks were removed and the wood piled to the depth of four feet.

*b.* If 4-foot wood is piled 6 feet high, what must be the length of the pile to contain 100 cords?

*c.* How many cords of wood in a pile 8 feet wide, 8 feet high, and 16 feet long?

*d.* Compare the amount of wood described in problem *c* with the amount in a pile one half as wide, one half as high, and one half as long.

**Rough stone** is usually measured by the **cord**. A pile 4 feet high, 4 feet wide, and 8 feet long or its equivalent, is one cord.

NOTE. One cord of good stone is sufficient for about 100 cubic feet of wall. Hence in estimates it is customary to use the number 100 instead of 128; that is, as many cords of stone will be required for a given wall as 100 cubic feet is contained times in the number of cubic feet in the wall.

*a.* Estimate the number of cords of stone necessary for a cellar wall 18 inches thick, the inside dimensions of the cellar being 15 feet by 18 feet and 7 feet deep, no allowance being made for openings in the wall.

*b.* What are the outside dimensions of the wall of the cellar described in problem *a*?

*c.* What length of wall 7 feet high and 18 inches thick is equivalent, so far as amount of stone is concerned, to the cellar wall described in problem *a*?

*d.* If $\frac{5}{7}$ of the depth of the cellar described above is to be below the surface of the ground, how many cubic yards of earth must be excavated?

*e.* Estimate the stone necessary for a wall 100 yards long, 11 feet high, and 2 feet thick.

*f.* In estimating stone for a cellar wall masons sometimes figure outside dimensions and make no allowance for corners or openings. Figured in this manner, how many cords of stone will be required for the wall described in problem *a*?

*g.* A cubic foot of water weighs $62\frac{1}{2}$ lb. If stone is $2\frac{1}{2}$ times as heavy as water and a cord of stone is equivalent to 100 solid feet, how much does a cord of stone weigh?

An ordinary **brick** is 2 in. by 4 in. by 8 in. and weighs about 4 pounds.

*a.* How many bricks are equivalent to 1 cubic foot?

NOTE. When bricks are laid in mortar in the usual way, about 22 bricks are required to make a cubic foot of wall.

*b.* Estimate the number of bricks necessary for a cellar wall 12 inches thick, the inside dimensions of the cellar being 15 feet by 18 feet, and 7 feet deep, no allowance being made for openings in the wall.

*c.* What are the outside dimensions of the wall of the cellar described in problem *b*?

*d.* What length of wall 7 feet high and 12 inches thick is equivalent, so far as the number of bricks required is concerned, to the cellar wall described in problem *b*?

*e.* If $\frac{3}{7}$ of the depth of the cellar described above is to be below the surface, how many cubic yards of earth must be excavated?

*f.* Estimate the number of bricks necessary for a wall 100 yards long, 11 feet high, and 1 foot thick.

*g.* Find the approximate weight (in tons) of a pile of bricks as long as your schoolroom, 2 feet wide, and 4 feet high.

*h.* Find the approximate weight of a chimney, outside dimensions 16. in by 16 in., and 20 ft. high, the flue being 8 in. by 8 in., if the brick and mortar weigh 110 lb. to the cubic foot.

*i.* Which will cost the more for a given wall, brick at $6 per M. or stone at $10 per cord?

A **foot of lumber** is a board 1 foot square and 1 inch thick or its equivalent.

NOTE. An exception to the foregoing is made in the measurement of boards less than 1 inch in thickness. A square foot of such boards is regarded as a foot of lumber whatever the thickness.

*Observe that a piece of board* 1 *inch wide,* 1 *inch thick, and* 12 *feet long, is* 1 *foot of lumber.*

Tell the number of feet of lumber in each of the following boards, the thickness in each case being 1 inch (or less):

1 in. wide and 12 ft. long. 2 in. wide and 12 ft. long.
3 in. wide and 12 ft. long. 4 in. wide and 12 ft. long.
7 in. wide and 12 ft. long. 13 in. wide and 12 ft. long.
9 in. wide and 12 ft. long. 12 in. wide and 12 ft. long.

*a.* How many feet (of lumber) in the eight boards?

*b.* How much lumber in 6 12-ft. 1-in. boards whose widths are 11 in., 13 in., 9 in., 10 in., 12 in., and 14 in.?

*c.* How much lumber in 5 12-ft. $\frac{3}{4}$-in. boards whose widths are 10 in., 12 in., 12 in., 11 in., and 14 in?

*d.* How much lumber in 7 12-ft. $\frac{1}{2}$-in. boards whose widths are 9 in., 8 in., 5 in., 7 in., 8 in., 6 in., and 9 in.?

*e.* How much lumber in 8 12-ft. 1-in. boards each of which is 12 in. wide?

*f.* How much lumber in 54 12-ft. 1-in boards each of which is 6 in. wide?

*g.* How much lumber in 36 12-ft. 1-in. boards each of which is 8 in. wide?

*h.* How much lumber in 36 12-ft. $\frac{3}{4}$-in. boards each of which is 8 in. wide?

*i.* Find the sum of the seven results, *b* to *h*.

NOTE 1. **A 14-foot board** contains $\frac{1}{6}$ more lumber than a 12-foot board of the same width and thickness. Hence to find the number of feet of lumber in 14-foot boards, find the number of feet in as many 12-foot boards [*] and add to the result $\frac{1}{6}$ of itself.

*a.* How much lumber in 6 14-ft. 1-in. boards whose widths are 11 in., 12 in., 12 in., 15 in., 10 in., and 13 in.?

*b.* How much lumber in a pile of 14-ft. boards whose united width is 8 feet 7 inches?

*c.* How much lumber in 56 14-ft. boards, each of which is 6 inches wide? [†]

*d.* How much lumber in 24 14-ft. boards, each of which is 12 inches wide? [‡]

*e.* Find the sum of the four results, *a* to *d.*

NOTE 2. **A 16-foot board** contains $\frac{1}{3}$ more lumber than a 12-foot board of the same width and thickness. Make a rule for finding the number of feet of lumber in 16-foot boards.

*f.* How much lumber in 6 16-ft. 1-in. boards whose widths are 12 in., 10 in., 14 in., 13 in., 12 in., and 10 in.?

*g.* How much lumber in a pile of 16-foot boards whose united width is 9 feet 8 inches?

*h.* How much lumber in 48 16-ft. boards, each of which is 6 inches wide?

*i.* How much lumber in 34 16-ft. boards, each of which is 12 inches wide?

*j.* Find the sum of the four results, *f* to *i.*

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[*] Take the nearest integral number of feet.

[†] How much lumber in 1 14-foot board 6 inches wide?

[‡] How much lumber in 1 14-foot board 12 inches wide?

NOTE 1. A **$1\frac{1}{4}$-inch board** contains $\frac{1}{4}$ more lumber than a 1-inch board of the same width and length. A **$1\frac{1}{2}$-inch board** contains $\frac{1}{2}$ more lumber than a 1-inch board of the same width and length.

*a.* How much lumber in 4 12-foot $1\frac{1}{4}$-in. boards whose widths are 12 in., 13 in., 14 in., and 13 in.?

*b.* How much lumber in 4 16-foot $1\frac{1}{2}$-in. boards whose widths are 13 in., 16 in., 12 in., and 13 in.?

*c.* How much lumber in 4 18-foot $1\frac{1}{2}$ in. boards, each of which is 12 inches wide?

*d.* How much lumber in 4 16-ft. $1\frac{1}{2}$-in. boards, each of which is 6 inches wide?

*e.* Find the sum of the four results, *a* to *d*.

NOTE 2. A "**2 by 4, 12**" is a piece of lumber 2 in. thick, 4 in. wide, and 12 feet long.

Find the number of feet of lumber in each of the following items:

*f.* 16 pieces 2 × 4, 12.

*g.* 18 pieces 4 × 4, 12.

*h.* 25 pieces 2 × 8, 12.

*i.* 30 pieces 2 × 6, 12.

*j.* 20 pieces 4 × 6, 12.

*k.* 32 pieces 6 × 6, 12.

*l.* Find the sum of the six results, *f* to *k*.

*Observe that in a 12-foot piece of lumber* there are as many feet as there are square inches in the cross section. A piece of lumber 1 in. by 1 in. and 12 feet long is 1 foot of lumber; a piece 2 in. by 2 in. is 4 feet of lumber; a piece 2 in. by 3 in. is 6 feet of lumber, etc.

NOTE 1. In the measurement of timbers of all sizes it is customary to consider each piece as containing the integral number of feet nearest to the actual content. Thus, a piece of 2 × 4, 14, actually contains $9\frac{1}{3}$ feet, but in all lumber yards it is counted as 9 feet. A piece of 2 × 4, 16, actually contains $10\frac{2}{3}$ feet, but it is counted as 11 feet.

Find the number of feet of lumber in each of the following items :

*a.* 16 pieces 2 × 4, 14.*
*b.* 24 pieces 4 × 4, 14.
*c.* 32 pieces 2 × 8, 14.
*d.* 17 pieces 4 × 6, 14.
*e.* 15 pieces 8 × 8, 16.
*f.* 12 pieces 4 × 10, 16.
*g.* 14 pieces 8 × 12, 16.
*h.* 6 pieces 12 × 12, 24.
*i.* Find the sum of the eight results, *a* to *h*.

NOTE 2. "Lumber at $15 per M." means that the lumber is sold at the rate of $15 per 1000 feet.

Find the cost:

*j.* Of 26 16-foot 6-in. fence boards @ $15 per M.
*k.* Of 34 14-foot 12-in. stock boards @ $18 per M.
*l.* Of 20 pieces 2 × 4, 16, @ $16 per M.
*m.* Of 14 pieces 4 × 6, 18, @ $16 per M.
*n.* Of 25 pieces 4 × 6, 16, @ $15 per M.
*o.* Of 18 pieces 4 × 4, 14, @ $15 per M.
*p.* Find the sum of the six results, *j* to *o*.

---

* Find the number of feet in 1 piece of 2 × 4, 14 (nearest whole number of feet) and multiply by 16.

The standard unit of capacity used in measuring liquids is a **gallon**. *A gallon equals* 231 *cubic inches.*

**Liquid Measure**

4 gills (gi.) = 1 pint (pt.).
2 pints = 1 quart (qt.).
4 quarts = 1 gallon (gal.).
31½ gallons = 1 barrel (bbl.).

Observe that 1 cubic foot = nearly 7½ gallons.

Observe that 4.2 cubic feet = nearly 1 barrel.

A kerosene barrel contains about 50 gallons. It equals nearly 7 cubic feet.

*a.* Find the approximate capacity in gallons of a rectangular tank 3 ft. by 4 ft. by 8 ft.

*b.* Find the approximate capacity in barrels of a rectangular tank 2 ft. by 4 ft. by 12 ft.

*c.* Reduce 7 gal. 3 qt. to pints.

*d.* Reduce 2143 pt. to gallons, quarts, and pints.

*e.* What is the capacity in liquid quarts of a rectangular tank 10 in. by 7 in. by 5 in.?

*f.* At 15¢ per thousand gallons, how much must be paid for 6540 cubic feet of water?

*g.* A ton of water is how many cubic feet? *

*h.* How many gallons in 1 ton of water? How many barrels in 1 ton of water?

*i.* Does the water required to fill 5 kerosene barrels weigh more or less than 1 ton?

*j.* Find the approximate capacity in barrels of a cistern 6 ft. by 6 ft. by 7 ft.

*k.* Find the approximate weight of a pail † of water.

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* See "Miscellaneous Weights," page 176.

† Any pail that can be obtained.

The standard unit of capacity used in measuring grain, fruits, vegetables, lime, coal, etc., is a **bushel**, which equals 2150.4 cubic inches.

NOTE. In measuring large fruits, vegetables, lime, and coal, the unit is the "**heaped bushel.**" A *heaped bushel* equals about 1¼ *stricken bushels.*

### Dry Measure

2 pints (pt.) = 1 quart (qt.).
8 quarts = 1 peck (pk.).
4 pecks = 1 bushel (bu.).

A bushel is nearly 1¼ cubic feet.
A "heaped bushel" is about 1½ cubic feet.

A "dry gallon" (4 quarts dry measure) equals 268.8 cubic inches.

Enough "**ear corn**" to make, when shelled, one bushel, occupies about 2¼ cubic feet. If the corn is inferior in quality, it will occupy more space than this — sometimes 2½ cubic feet.

*a.* Find the capacity in bushels of a wheat bin 8 ft. by 8 ft. by 10 ft.*

*b.* Give the dimensions of the smallest bin in which 1000 bushels of oats may be stored.

*c.* In a bin 12 feet square, there is rye to the depth of 7½ feet. How many bushels?

*d.* How many bushels of potatoes (without heaping the bin) may be stored in a bin that is 8 ft. by 4 ft. by 6 ft?

*e.* If the corn is of excellent quality, how many bushels of "shelled corn" may be expected from a crib of ear corn 8 ft. by 10 ft. by 80 ft.?

---

* For many practical purposes the approximate ratio (1¼) of the bushel to the cubic foot will give, in such problems as these, results sufficiently accurate.

*a.* How many cubic yards of earth must be removed in digging a cellar 22 feet by 28 feet and 4½ feet deep?

*b.* How many cubic yards of crushed stone will be necessary to make a roadway 1 mile long, 8 feet wide, and 8 inches deep, provided 10 per cent must be added to the actual solid content of the roadway to make up for packing and for the lack of a retaining wall at the edges?

*c.* Making the same per cent of allowance for packing, etc., as suggested in problem *b*, find the number of cubic yards of crushed stone necessary for a mile of roadway 9 feet wide and 10 inches deep.

*d.* Find the weight of a load (1 cu. yd.) of wet sand, if it weighs twice as much as the same bulk of water.

*e.* Is the capacity of a box 10 in. by 8 in. by 6¾ in. more or less than 1 pk.?

*f.* What must be the depth of a box 15 in. long and 15 in. wide, so that its capacity may be 1 bushel?

*g.* Is the capacity of a strawberry box 4½ in. by 4½ in. by 3⅖ in. more or less than one dry quart?

*h.* In a bin 10 feet square there is shelled corn to the depth of 4 ft. 5 in. How many bushels?

*i.* Reduce 12 bu. 3 pk. 5 qt. 1 pt. to pints.

*j.* Reduce 26 bu. 1 pk. 7 qt. 1 pt. to pints.

*k.* Reduce 2146 pt. to bushels, pecks, and quarts.

*l.* Reduce 3427 pt. to bushels, pecks, quarts, and pints.

*m.* Reduce 1 pk. 4 qt. to the fraction of a bushel.

*n.* Reduce 1 pk. 4 qt. to the decimal of a bushel.

The standard unit of weight in common use is a **pound Avoirdupois.**

### Avoirdupois Weight

16 ounces (oz.) = 1 pound (lb.).
2000 pounds = 1 ton (T.).

**The abbreviation for 1 hundredweight (100 lb.) is *cwt.***

### Miscellaneous Weights

1 gallon of water = about 8⅓ lb.
1 gallon of milk = about 8.6 lb.
1 gallon of kerosene = about 6½ lb.
1 cubic foot of water = 62½ lb.
1 bushel of wheat = 60 lb.
1 bushel of beans = 60 lb.
1 bushel of clover seed = 60 lb.
1 bushel of potatoes = 60 lb.
1 bushel of shelled corn = 56 lb.
1 bushel of ear corn = 70 lb.*
1 bushel of rye = 56 lb.
1 bushel of barley = 48 lb.
1 bushel of oats = 32 lb.
1 barrel of flour = 196 lb.
1 barrel of beef or pork = 200 lb.

Find the cost:

*a.* Of 2650 lb. coal at $5.50 per ton.
*b.* Of 2650 lb. oats at 24¢ a bushel.
*c.* Of 3330 lb. wheat at 80¢ a bushel.
*d.* Of 4650 lb. potatoes at 42¢ a bushel.
*e.* Find the sum of the four results, *a* to *d*.

---

*** This means the amount of ear corn required to make 1 bushel of shelled corn.**

Find the cost:

*a.* Of 2560 lb. hay at $7.50 per ton.

*b.* Of 1430 lb. straw at 30¢ per cwt.

*c.* Of $2\frac{1}{4}$ tons meal at $\frac{3}{4}$ of a cent a pound.

*d.* Of $1\frac{1}{2}$ tons corn husks at $1\frac{1}{4}$ cents a pound.

*e.* Of 3420 lb. hay at $8.00 per ton.

*f.* Find the sum of the five results, *a* to *e*.

Find the cost:

*g.* Of 2140 lb. oats at 24¢ a bushel.

*h.* Of 2140 lb. corn at 28¢ a bushel.

*i.* Of 2140 lb. wheat at 90¢ a bushel.

*j.* Of 2140 lb. barley at 36¢ a bushel.

*k.* Of 2140 lb. rye at 42¢ a bushel.

*l.* Find the sum of the five results, *g* to *k*.

Find the cost:

*m.* Of 520 lb. clover seed at $6.30 a bushel.

*n.* Of 520 lb. potatoes at 75¢ a bushel.

*o.* Of 520 lb. beans at $2.15 a bushel.

*p.* Of 520 lb. corn at 20¢ a bushel.

*q.* Of 520 lb. ear corn at 35¢ a bushel.

*r.* Find the sum of the five results, *m* to *q*.

Find the approximate weight:

*s.* Of a barrel of kerosene.

*t.* Of 1 quart of milk.

*u.* Of the oats that will fill a bin that is 4 ft. by 4 ft. by 9 ft.

*v.* Of the water that will fill a tank that is 2 ft. by 2 ft. by 12 ft.

*a.* One ton of water is how many gallons?

*b.* One ton of water is how many cubic feet?

*c.* One ton of milk is how many gallons?

*d.* One ton of kerosene is how many gallons?

*e.* One ton of wheat, beans, clover seed, or potatoes is how many bushels?

*f.* One ton of shelled corn is how many bushels?

*g.* One ton of ear corn is how many bushels?

*h.* One ton of rye is how many bushels?

*i.* One ton of barley is how many bushels?

*j.* One ton of oats is how many bushels?

*k.* One ton of flour is how many barrels?

*l.* One ton of beef or pork is how many barrels?

*m.* Which costs the more per ton, oats at 35 cents a bushel or shelled corn at 59 cents a bushel?

*n.* Which is the cheaper feed, "pound for pound," rye at 50 cents a bushel, or barley at 45 cents a bushel?

*o.* Weight of wagon, 1460 lb.; weight of wagon loaded with oats,—1st load, 3640 lb.; 2d load, 3720 lb.; 3d load, 3380 lb.; 4th load, 3540 lb.; 5th load, 3610 lb.; 6th load, 3570 lb. Find the value of the oats at 36 cents a bushel.

*p.* Weight of wagon, 1410 lb.; weight of wagon loaded with rye,—1st load, 4150 lb.; 2d load, 3970 lb.; 3d load, 4220 lb.; 4th load, 4230 lb.; 5th load, 4040 lb.; 6th load, 4160 lb. Find the value of the rye at 52 cents a bushel.

Troy weight is used in weighing gold, silver, and jewels.

### Troy Weight

24 grains (gr.) = 1 pennyweight (pwt.).
20 pennyweights = 1 ounce (oz.).
12 ounces = 1 pound (lb.).

Apothecaries' weight is used in mixing medicines and in selling them at retail.

### Apothecaries' Weight

20 grains (gr.) = 1 scruple (℈).
3 scruples = 1 dram (ʒ).
8 drams = 1 ounce (℥).
12 ounces = 1 pound (℔).

NOTE 1. The pound Troy and the pound Apothecary are equal, each weighing 5760 grains. The pound Avoirdupois weighs 7000 Troy or Apothecary grains.

NOTE 2. The ounce Troy and the ounce Apothecary are each 480 grains; the ounce Avoirdupois is 437½ grains.

QUERY. Which is heavier, a pound of feathers or a pound of gold? An ounce of feathers or an ounce of gold?

*a*. Change 5 lb. Avoirdupois weight to pounds, ounces, etc., Apothecaries' weight.

*b*. How many 5-gr. powders can be made from one Avoirdupois ounce of quinine?

*c*. One Avoirdupois ton of gold is how many Troy pounds?

*d*. Twenty-four Troy pounds equal how many Avoirdupois pounds?

*e*. Reduce 7000 grains to pounds, ounces, pennyweights, and grains.

The standard units in the measurement of time are the **day** and the **year**.

NOTE 1. The **solar day** is the interval between the time when the sun is on a given meridian and the time when it appears on that meridian again. These intervals (solar days) are not perfectly uniform. The average of these intervals is the **mean solar day** — the day noted by our watches and our clocks — the day one twenty-fourth of which is called an hour.

NOTE 2. The **solar year** is the time of one revolution of the earth around the sun, or nearly 365¼ mean solar days. The calendar year of 365 days is nearly 6 hours less than the solar year. Four years of 365 days each, would lack nearly 24 hours (4 times ¼ da.) of being equal to 4 solar years. Hence 1 day is added to 365 every fourth year, with the exception noted below.

NOTE 3. The exact length of a solar year is 365 da. 5 hr. 48 min. 46 sec. Since this lacks 11 min. 14 sec. of being 365¼ days, it follows that if *every* fourth year should contain 366 days, in 400 years the calendar years would amount to 3 days (400 times 11 min. 14 sec.) more than the solar years. Hence three of the years that would otherwise contain 366 days are made to contain 365 days. The years thus changed to 365 days are those ending the centuries (1800, 1900, 2100, etc.) unless the number denoting the year is exactly divisible by 400; as 2000, which will be a leap year.

### Measure of Time

60 seconds (sec.) = 1 minute (min.).
60 minutes = 1 hour (hr.).
24 hours = 1 day (da.).
7 days = 1 week (wk.).
365 days = 1 common year.
52 wk. 1 da. = 1 common year.
366 days = 1 leap year.
52 wk. 2 da. = 1 leap year.
12 months = 1 year.
10 years = 1 decade.
100 years = 1 century.

1. If Jan. 1 of a common year is Monday, Feb. 1 is —— ; March 1 is —— ; April 1 is —— ; May 1 is ——; June 1 is ——; July 1 is —— ; Aug. 1 is —— ; Sept. 1 is —— ; Oct. 1 is —— ; Nov. 1 is ——; Dec. 1 is —— .

2. Jan. 1, 1899, was Sunday. Tell the day of the week for each of the following dates:

1900, Jan. 1; Feb. 1; Feb. 8.
1901, Jan. 1; Feb. 1; Feb. 9.
1902, Jan. 1; Feb. 1; March 1.
1903, Jan. 1; Feb. 1; March 10.
1904, Jan. 1; Feb. 1; April 1.
1906, Jan. 1; March 1; May 1.
1910, Jan. 1; April 1; July 1.

*a.* How many days from Jan. 1, 1900 to Aug. 17, 1900?

*b.* Jan. 1, 1900, falls on Monday. Upon what day of the week does Aug. 17, 1900, fall?

*c.* What month begins on the same day of the week as January in every common year?

*d.* What months begin on the same day of the week as February in every common year?

*e.* If January begins on Sunday, how many Sundays in the month? *f.* How many Mondays? *g.* How many Tuesdays? *h.* How many Wednesdays?

*i.* If a common year begins on Sunday, how many Sundays in the year? *j.* How many Mondays?

*k.* How many days old are you? *l.* How many weeks old? *m.* Upon what day of the week were you born?

For the purpose of measurement, every circumference is supposed to be divided into 360 equal parts. Each of these parts is called an **arc** of 1 degree.

An **angle** is measured by regarding its vertex as the center of a circle, its sides being extended until they cut the circumference. The angle is measured by the arc lying between its sides. If the intercepted arc is an arc of 45 degrees, the angle is an angle of 45 degrees; if the arc is 20 degrees, the angle is 20 degrees, etc.

### Circular and Angular Measure

60 seconds (″) = 1 minute (′).
60 minutes = 1 degree (°).
360 degrees = 1 circumference.

In geography, **a meridian** is a north and south line on the surface of the earth, extending from pole to pole.

**Longitude** is distance east or west in degrees (or parts of degrees) from a given meridian. In this country longitude is usually measured from the meridian that passes through Greenwich, England, or from the one that passes through Washington, D.C.

Find difference in longitude between :

*a.* Rome, 12° 27′ east, and Washington, 77° 2′ 48″ west.

*b.* Washington and Chicago, 87° 37′ 30″ west.

*c.* Chicago and Denver, 104° 59′ 23″ west.

*d.* Denver and San Francisco, 122° 25′ 42″ west.

*e.* San Francisco and Berlin, 13° 23′ 53″ east.

*f.* Washington and Honolulu, 157° 50′ 36″ west.

One degree of longitude corresponds to 4 minutes of time.

EXPLANATORY. The sun seems to move over 360 degrees of longitude in 24 hours; over 1 degree in $\frac{1}{360}$ of 24 hours = 4 minutes.

*a.* When it is noon at Greenwich, what is the time at Washington, 77° 2′ 48″?*

*b.* When it is noon at Washington, what is the time at Greenwich?

*c.* When it is noon at Portland, Maine, 70° 15′ 40″, what is the time at San Francisco, 122° 25′ 42″?

*d.* When it is noon at San Francisco, what is the time at Portland, Maine?

**Standard Railroad Time.** Every railroad train in North America is run on the time of some one of the following meridians:

60th meridian — passes through Labrador — "Atlantic."
75th meridian — passes near Philadelphia — "Eastern."
90th meridian — passes near St. Louis — "Central."
105th meridian — passes near Denver — "Mountain."
120th meridian — passes near Carson City — "Pacific."

Observe that each of the above numbers is 15 greater than the one preceding it; and that 15 degrees of longitude correspond to 1 hour of time.

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*At first the pupil should give an approximate answer to these problems, *considering integral degrees only.* Later, if thought advisable, he may take into the account the parts of degrees. One arc minute corresponds to $\frac{1}{60}$ of 4 minutes of time, or 4 seconds of time. One arc second corresponds to $\frac{1}{3600}$ of 4 minutes of time, or $\frac{4}{60}$ of one second of time. That is, each arc degree corresponds to 4 minutes of time; each arc minute, to 4 seconds of time; each arc second, to $\frac{4}{60}$ of a second of time. Hence, multiplying by 4 the figures standing for degrees, minutes, and seconds of longitude will give the figures standing for minutes, seconds, and 60ths of seconds of time.

*a.* When it is noon by "Atlantic" time, what is the hour of the day by "Eastern" time? *b.* By "Central" time? *c.* By "Mountain" time? *d.* By "Pacific" time?

*e.* When it is noon by "Pacific" time, what is the hour of the day by "Mountain" time? *f.* By "Central" time? *g.* By "Eastern" time? *h.* By "Atlantic" time?

*i.* A man traveling from Philadelphia to Chicago would begin his journey on "Eastern" time. He would arrive at Pittsburg on "Eastern" time. His journey from Pittsburg to Chicago would be made by "Central" time. That his watch may agree with the watches of the train-men, how should he change it on leaving Pittsburg? *j.* If he left Philadelphia at 8.40 A.M. "Eastern" time, and arrived in Chicago the next morning at 8.40 "Central" time, how many hours were required to make the entire journey? *k.* If he arrived in Pittsburg at 6.55 P.M. "Eastern" time and left that city at 6.30 P.M. "Central" time, how long did his train stop at Pittsburg? *l.* If on his journey back from Chicago to Philadelphia he left Chicago at 3.00 P.M., "Central" time, and arrived in Philadelphia next afternoon at 6.00 "Eastern" time, how many hours were required to make his journey? *m.* If the train arrived in Pittsburg at 5.50 A.M. "Central" time, and left at 7.30 A.M. "Eastern" time, how long did it stay at Pittsburg?*

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*** Require the pupil with his geography before him, open at a map of the United States, to think this problem through again and again, until he has a clear perception of all the facts involved. He will then be able to solve any similar problem that may be presented.**

Note. The following problems are to be thought of not in connection with "Standard Railroad Time" but rather with *mean solar time*, or as it is sometimes called "sun-time." It will promote a thoughtful consideration of this subject, if *fractions of degrees* are for the most part eliminated.

See page 183.

*a.* What is the difference of time between two places that are one longitude degree apart? *b.* 10 degrees? *c.* 15 degrees? *d.* 8½ degrees?

*e.* Learn from your geography the longitude (nearest whole degree) of your schoolhouse. *f.* Find the longitude of Denver, Colo. *g.* How many degrees from your schoolhouse to Denver? *h.* What, then, is the difference in time between your home and Denver? *i.* When it is noon at your home, what time is it at Denver? *j.* When it is noon at Denver, what time is it at your home?

*k.* In a similar manner compare the time for your own home and Boston, Mass.

*l.* What is the longitude of Boston? *m.* Of Greenwich, near London? *n.* How many longitude degrees from Greenwich to Boston? *o.* What is the difference of time between Greenwich and Boston? *p.* When it is noon in Greenwich, what time is it in Boston? *q.* When it is noon in Boston, what time is it in Greenwich?

*r.* Learn from your geography the longitude of St. Petersburg in Russia. *s.* How many longitude degrees from St. Petersburg to Boston? *t.* What is the difference of time between St. Petersburg and Boston? *u.* When it is noon at Boston, what time is it in St. Petersburg?

The standard unit of value in the United States is the **dollar.**

### United States Money

| | |
|---|---|
| 10 mills (m.) | = 1 cent (ct. or ¢). |
| 10 cents | = 1 dime (d). |
| 10 dimes | = 1 dollar ($). |

The standard unit of value in Great Britain and Ireland is the **pound.** Its value, reckoned in United States money, is $4.866½.

### English, or Sterling, Money

| | |
|---|---|
| 4 farthings (far.) | = 1 penny (*d*). |
| 12 pence | = 1 shilling (*s*). |
| 20 shillings | = 1 pound (£). |
| 5 shillings | = 1 crown. |
| 21 shillings | = 1 guinea. |

The standard unit of value in France is the **franc.** Its value, reckoned in United States money, is 19.3¢.

The standard unit of value in Germany is the **mark.** Its value, reckoned in United States money, is 23.8¢.

The standard unit of value in Russia is the **ruble.** Its value, reckoned in United States money, is $.515.

*a.* Find the value of a guinea in United States money.

*b.* Find the value of $1000 in English money.

*c.* Find the value of £5000 in United States money.

*d.* Find the value of £1000 in Russian money.

*e.* Find the value of 4000 marks in English money.

*f.* Find the value of 8000 francs in United States money.

# METRIC SYSTEM

All units in the metric system of measures and weights are derived from the primary unit known as the **meter**.

When the length of the primary unit of this system was determined it was supposed to be one ten-millionth of the distance from the equator to the pole. A pendulum that vibrates seconds is nearly one meter long.*

In the names of the derived units of this system the prefix *deka* means 10; *hekto* means 100; *kilo* means 1000; *myria* means 10000; *deci* means tenth; *centi* means hundredth; *milli* means thousandth.

### Linear Measure

| | |
|---|---|
| 10 millimeters (mm.) | = 1 centimeter (cm.). |
| 10 centimeters | = 1 decimeter (dm.). |
| 10 decimeters | = 1 meter (m.). |
| 10 meters | = 1 dekameter (Dm.). |
| 10 dekameters | = 1 hektometer (Hm.). |
| 10 hektometers | = 1 kilometer (Km.). |
| 10 kilometers | = 1 myriameter (Mm.). |

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* *To the Teacher.* If pupils have not before become somewhat familiar with the metric units, the author suggests that at this point it will be best to "make haste slowly." Give a little time each day for at least one week to page 187 before attempting page 188. It may not be necessary to devote all of the arithmetic period for the full week to this page, but it is absolutely necessary that the pupil should become quite familiar with the new terms before proceeding further in this topic. The teacher must provide a meter stick and pupils must use it in actual measuring until they can easily think the different metric units without reference to the more common units of measurement.

The names of the units of **surface measurement** are the same as those used for linear measurement, combined with the word *square;* thus a surface equivalent to a square whose side is a meter is 1 **square meter.**

The pupil, if properly taught to this point, will be able, without difficulty, to fill the blanks in the table of —

### Square Measure

100 square millimeters (sq. mm.) = 1 square centimeter (sq. cm.).[*]
—— square centimeters = 1 square decimeter (sq. dm.).
—— square decimeters = 1 square meter (sq. m.).
—— square meters = 1 square dekameter (sq. Dm.).
—— square dekameters = 1 square hektometer (sq. Hm.).
—— square hektometers = 1 square kilometer (sq. Km.).
—— square kilometers = 1 square myriameter (sq. Mm.).

NOTE. The special unit of surface measure for measuring land is equivalent to a square whose side is ten meters. This unit is called an **are.**

100 centares (ca.) = 1 are (a.).
100 ares = 1 hektare (Ha.).

*a.* In a square decimeter there are —— sq. cm.

*b.* In 2 square decimeters there are —— sq. cm.

*c.* In a 2-decimeter square there are —— sq. cm.

*d.* In a square meter there are —— sq. dm.

*e.* In 2 square meters there are —— sq. dm.

*f.* In a 2-meter square there are —— sq. dm.

*g.* In a square meter there are —— sq. cm.

*h.* In 2 square meters there are —— sq. cm.

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[*] The abbreviation $cm^2$ or qcm. is sometimes used for square centimeter, $m^2$ or qm. for square meter, etc.

The names of the units of **volume measurement** are the same as those used for linear measurement, combined with the word *cubic;* thus, a volume equivalent to a cube whose edge is a meter is **1 cubic meter.**

### Volume Measure

1000 cubic millimeters (cu. mm.) = 1 cubic centimeter (cu. cm.).*
—— cubic centimeters = 1 cubic decimeter (cu. dm.).
—— cubic decimeters = 1 cubic meter (cu. m.).
—— cubic meters = 1 cubic dekameter (cu. Dm.).
—— cubic dekameters = 1 cubic hektometer (cu. Hm.).

NOTE 1. The special unit of capacity, for measuring liquids, grain, small fruits, etc., is the **liter.** It is equal to 1 cubic decimeter. 10 liters (l.) = 1 dekaliter (Dl.), and 1 tenth of a liter = 1 deciliter (dl.), etc. The special unit for measuring wood is the **stere.** It is equal to 1 cubic meter.

*a.* A cubic meter equals —— liters.

*b.* A cubic meter equals —— cubic decimeters.

*c.* A cubic meter equals —— cubic centimeters.

*d.* A cubic decimeter equals —— cubic centimeters.

*e.* Two cubic decimeters equal —— cubic centimeters.

*f.* A 2-decimeter cube equals —— cubic centimeters.

*g.* A 5-centimeter square equals —— square centimeters.

*h.* A 5-centimeter cube equals —— cubic centimeters.

*i.* One tenth of a liter equals —— cubic centimeters.

*j.* One deciliter equals —— cubic centimeters.

*k.* One dekaliter equals —— cubic decimeters.

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* The abbreviation $cm^3$ is often used for cubic centimeter, $m^3$ for cubic meter, etc.

The primary unit of weight is the **gram**. *This equals the weight of one cubic centimeter of pure water.*

### Weight

10 milligrams (mg.) = 1 centigram (cg.).
10 centigrams = 1 decigram (dg.).
10 decigrams = 1 gram (g.).
10 grams = 1 dekagram (Dg.).
10 dekagrams = 1 hektogram (Hg.).
10 hektograms = 1 kilogram (Kg.).

NOTE. The special unit for the weight of very heavy articles is the **tonneau**. It equals the weight of a cubic meter of pure water, or 1000 kilograms.

*a.* The weight of 1 liter of water is —— grams.

*b.* The weight of 6 cubic centimeters of water is ——.

*c.* The weight of a cubic decimeter of water is ——.

*d.* One kilogram of water equals —— cubic centimeters.

*e.* One hektogram of water equals —— cu. cm.

*f.* One dekagram of water equals —— cubic centimeters.

*g.* What is the weight of 20 cu. cm. of iron, if iron is 7½ times as heavy as water?

*h.* What is the weight of 20 cu. cm. of cork, if cork is ¼ as heavy as water?

*i.* What is the weight of a liter of oil, if oil weighs 9 tenths as much as water?

*j.* What is the weight of a cubic meter of limestone, if limestone weighs 2½ times as much as water?

*a.* Find the area of a rectangular surface that is 1 meter long and 6 decimeters wide. Make a diagram of this surface upon the blackboard.*

*b.* Find the area of a rectangular surface that is 2 decimeters long and 5 centimeters wide. Make a diagram of this surface on your slate or paper.

*c.* Find the solid content of a 5-centimeter cube. *d.* A 5-centimeter cube is what part of a cubic decimeter?

*e.* Find the solid content of a 4-decimeter cube. *f.* A 4-decimeter cube is what part of a cubic meter?

*g.* Find the entire surface of a 4-centimeter cube. *h.* The surface of a 4-centimeter cube is what part of a square decimeter?

*i.* Find the area of a rectangular surface that is 2.4 yards by 5 yards. *j.* Find the area of a rectangular surface that is 2.4 meters by 5 meters.

*k.* Which is the larger of the two surfaces described in problems *i* and *j*?

*l.* Find the area of a rectangular surface that is 3.5 yards by 2.5 yards. *m.* Find the area of a rectangular surface that is 3.5 meters by 2.5 meters.

*n.* Find the volume of a rectangular solid that is 3.4 feet by 3 feet by 2 feet. *o.* Find the volume of a rectangular solid that is 3.4 meters by 3 meters by 2 meters.

*p.* Find the volume of a rectangular solid that is 3.5 meters by 2.3 meters by 4.6 meters.

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* Pupils should draw or image every magnitude whose dimensions are given on this page.

1. Estimate in meters the width of the lot upon which the school building stands. Measure it.

2. Estimate in centimeters the width of your desk. Measure it.

3. Estimate in square centimeters the area of a sheet of paper. Measure and compute.

4. Estimate in square meters the area of the blackboard. Measure and compute.

5. Estimate the number of cubic meters of air in the schoolroom. Measure and compute.

6. Estimate in grams the weight of a teaspoonful of water. Weigh it.*

7. Estimate in kilograms your own weight.

8. Estimate in liters the capacity of a water pail.

9. Estimate in kilograms the weight of a gallon of water.

**Table of Equivalents**

| | | |
|---|---|---|
| **Meter** | a little more than 1 yard | 39.37 inches. |
| **Kilometer** | nearly ⅝ of a mile | 3280.8 + feet. |
| **Decimeter** | nearly 4 inches | 3.937 inches. |
| **Are** | nearly $\frac{1}{40}$ of an acre | 3.954 sq. rd. |
| **Stere** | a little more than ¼ cord | 35.3 + cu. ft. |
| **Liter** | a little more than 1 liquid quart | 1.056 + qt. |
| **Gram** | nearly 15½ grains | 15.4 + grains (av.). |
| **Kilogram** | nearly 2¼ pounds | 2.204 + lb. |

NOTE 1. A "nickel" is 2 centimeters in diameter, 2 millimeters in thickness, and weighs 5 grams.

NOTE 2. A silver dollar weighs about 25 grams.

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* Every school should be provided with scales, weights, and measures.

*a.* Is a distance of 50 meters more or less than 50 yards? *b.* Than 54 yards?

*c.* Express approximately a distance of 100 yards in meters giving the nearest integral number.

*d.* Is a distance of 40 kilometers more or less than 25 miles?

*e.* Express approximately a distance of 60 kilometers in miles, giving the nearest integral number. *f.* Express 60 miles in kilometers.

*g.* Is a distance of 8 decimeters more or less than 32 inches?

*h.* Express approximately a distance of 100 inches in decimeters, giving the nearest integral number. *i.* Express 100 decimeters in inches. *j.* Express 4.43 meters in feet and inches. *k.* Express 25 feet 6 inches in meters, true to hundredths.

*l.* Is a piece of land containing 120 ares more or less than three acres?

*m.* Express 120 acres in ares, true to tenths.

*n.* Is a pile of wood containing 60 steres more or less than 15 cords?

*o.* Express 60 cords in steres, true to tenths.

*p.* Does a 28-liter can contain more or less than 28 liquid quarts?

*q.* Express 28 liters in quarts, true to tenths.

*r.* Express 28 quarts in liters, true to tenths.

*s.* Express 10 grams in grains, giving nearest integral number.

*t.* Express 100 grains in grams, giving nearest integral number.

*a.* Find the area of a rectangular surface that is 2 meters long and 16 decimeters wide.

*b.* Find the area of a rectangular surface that is 2 decimeters long and 16 centimeters wide.

*c.* The area described in problem *a* is how many times the area described in problem *b*?

*d.* Find the solid content of a 1.8-meter cube. *e.* Find the area of one of its faces. *f.* Find the perimeter of one of its faces. *g.* Find the area of its entire surface.

*h.* Find the solid content of a 1.8-decimeter cube. *i.* Find the area of one of its faces. *j.* Find the perimeter of one of its faces. *k.* Find the area of its entire surface.

*l.* The solid content of the cube described in problem *d* is how many times the solid content of the cube described in problem *h*?

*m.* The entire surface of the cube described in problem *d* is how many times the entire surface of the cube described in problem *h*?

*n.* What is the weight of a cubic decimeter of metal, if the metal is 8½ times as heavy as water?

*o.* What is the weight of a cubic centimeter of gold, that metal being 19.3 times as heavy as water?

*p.* What is the weight of 500 cubic centimeters of marble, the marble being 2.7 times as heavy as water?

*q.* A rectangular tank is 2.5 meters long and .8 of a meter wide. Put 1500 kilograms of water into this tank. What is the depth of the water?

# RATIO

**Ratio** is relation by quotient. The two numbers (magnitudes) of which the ratio is to be found are called the **terms of the ratio.** The first term is called the **antecedent** and the second term the **consequent.** The ratio is the quotient of the antecedent divided by the consequent.

The usual sign of ratio is the colon. It indicates that the ratio of the two numbers between which it stands is to be found, the number preceding the colon being the antecedent and the number following it, the consequent. The expression, 12 : 4 = 3, is read, the ratio of 12 to 4 is 3.

### EXERCISE

Read and complete the following:

1. 12 : 4 =     4 : 12 =     12 : 2 =
2. 18 : 9 =     9 : 18 =     18 : 6 =
3. 15 : 5 =     5 : 15 =     15 : 10 =
4. 40 : 10 =     10 : 40 =     40 : 20 =

NOTE. It will be observed that the sign of ratio is the sign of division (÷) with the line omitted.

*Every integral number is a ratio.* The number 4 is the ratio of a magnitude 4 (inches, ounces, bushels) to the measuring unit 1 (inch, ounce, bushel). The number 7 is the ratio of 7 yards to 1 yard; of 7 dollars to 1 dollar, or of 7 seconds to 1 second, etc.

*Every fractional number is a ratio.* The fraction $\frac{3}{4}$ is the ratio of the magnitude 3 to the magnitude 4.

So $\frac{12}{4}$ (3) is the ratio of 12 to 4. Observe that in every case the terms of the ratio may be written as the terms of a fraction; the antecedent becoming the numerator and the consequent the denominator of the fraction. *The fraction itself is the ratio.*

### EXERCISE I

Make the terms of the ratio the terms of the fraction; then reduce the fraction to its simplest form.

1. The ratio of 20 to 6 is $\frac{20}{6}=\frac{10}{3}=3\frac{1}{3}$.
2. The ratio of 6 to 20 is $\frac{6}{20}=\frac{3}{10}$.
3. The ratio of 7 to 5 is —; of 5 to 7, —.
4. The ratio of 12 to 1 is —; of 1 to 12, —.

### EXERCISE II

5. $\frac{5}{7}$ is the ratio of 5 to 7; of 10 to 14; of 15 to 21; etc.
6. $\frac{3}{4}$ is the ratio of — to —; of — to —; of — to —; etc.
7. $\frac{6}{5}$ is the ratio of — to —; of — to —; of — to —; etc.
8. 8 is the ratio of 8 to 1; of — to —; of — to —; etc.

### EXERCISE III

Make the necessary reduction and find the ratio: *

*a.* Of 2 feet to 8 inches.
*b.* Of 3 yards to 6 inches.
*c.* Of 6 rods to 3 yards.
*d.* Of 2 rods 5 yards to 1 yard 1 foot.
*e.* Of 3 miles 40 rods to 1 mile 80 rods.

---

* The comparison of two magnitudes involves their measurement by the same standard. To compare feet with inches, the inches may be changed to feet or the feet to inches, or both may be changed to yards.

Not only is number itself ratio, but a large part of the work in arithmetic is merely the changing of the form of the expression of ratios.

### EXERCISE IV

**(Reducing fractions to their lowest terms)**

*a.* Express the ratio of 30 to 40 in its simplest form.

*b.* Express the ratio of 500 to 720 in its simplest form.

*c.* Express the ratio of 425 to 875 in its simplest form.

*d.* Express the ratio of 5 min. to 2 hours in its simplest form.

*e.* Express the ratio of 1 lb. 4 oz. to 5 lb. 8 oz. in its simplest form.

### EXERCISE V

**(Reducing improper fractions to integers)**

*f.* Express the ratio of 400 to 50 in its simplest form.

*g.* Express the ratio of 375 to 25 in its simplest form.

*h.* Express the ratio of 256 to 16 in its simplest form.

*i.* Express the ratio of 3 hours 20 minutes to 50 minutes in its simplest form.

### EXERCISE VI

**(Reducing complex fractions to simple fractions)**

*j.* Express the ratio of $\frac{1}{2}$ to $\frac{3}{4}$ in its simplest form.

*k.* Express the ratio of $\frac{3}{5}$ to $\frac{3}{8}$ in its simplest form.

*l.* Express the ratio of $2\frac{1}{2}$ to $8\frac{1}{2}$ in its simplest form.

## Exercise VII

(Changing common fractions to decimals)

*a.* Express the ratio of 3 to 4 ($\frac{3}{4}$), in hundredths.

*b.* Express the ratio of 20 to 50, in tenths.

*c.* Express the ratio of 30 to 80, in thousandths.

*d.* Express the ratio of 50 sq. rd. to 1 acre 40 rd., in hundredths.

*e.* Express the ratio of 4 ft. 6 in. to 5 yd., in tenths.

## Exercise VIII

(Finding what per cent one number is of another)

*f.* Express the ratio of 16 to 20, in hundredths.

*g.* Express the ratio of 14 to 200, in hundredths.

*h.* Express the ratio of 17 to 25, in hundredths.

*i.* Express the ratio of 16 to $33\frac{1}{3}$, in hundredths.

*j.* Express the ratio of 27 to 500, in hundredths.

## Exercise IX

(Changing "per cent" to a common fraction in its lowest terms, or to a whole or a mixed number)

*k.* A's money equals 40 % of B's money. Express the ratio of A's money to B's money in the form of a fraction in its lowest terms. *l.* Express the ratio of B's money to A's money in its simplest form.

*m.* One number is 50 % more than another number. Express the ratio of the smaller to the larger number in the form of a fraction in its lowest terms. *n.* Express the ratio of the larger to the smaller number in its simplest form.

Note. Observe that the base in percentage corresponds to the consequent in ratio.

The **specific gravity** of a liquid or solid is the ratio of its weight to the weight of the same bulk of water.

1. **A cubic foot of water weighs 62½ lb.** A cubic foot of cork weighs 15 lb. *a.* What is the ratio of the weight of the cork to the weight of the water? *b.* Express the ratio in hundredths. *c.* What is the specific gravity of cork?

2. A certain piece of limestone weighs 37 ounces. Water equal in bulk to the piece of limestone weighs 15 ounces. *d.* What is the ratio of the weight of the limestone to that of the water? *e.* What is the specific gravity of the limestone?

3. A certain bottle holds 10 ounces of water or 9½ ounces of oil. *f.* What is the ratio of the weight of the oil to that of the water? *g.* Express the ratio in hundredths. *h.* What is the specific gravity of the oil?

NOTE. Observe that the weight of water in specific gravity problems corresponds to the consequent in ratio problems.

*i.* What is the ratio of a unit of the first integral order to a unit of the first decimal order?

*j.* What is the ratio of a unit of any order to a unit of the next lower order?

*k.* What ratio corresponds to 6 per cent?

*l.* What is the ratio of a dollar to a dime? *m.* Of a dime to a cent? *n.* Of a cent to a mill?

*o.* What is the ratio of a meter to a decimeter?

*p.* Of a decimeter to a centimeter?

*q.* What is the ratio of a rod to a yard? *r.* Of a yard to a foot? *s.* Of a foot to an inch?

## SOME OLD PROBLEMS IN NEW FORMS*

*a.* What is the ratio of the area of a 2-inch square to the area of a 6-in. square?* *b.* Of the area of a 6-in. square to that of a 2-in. square?

*c.* What is the ratio of the perimeter of a 2-in. square to the perimeter of a 6-in. square? *d.* Of the perimeter of a 6-in. square to the perimeter of a 2-in. square?

*e.* What is the ratio of the area of a 3-cm. square to the area of a 6-cm. square? *f.* Of the area of a 6-cm. square to that of a 3-cm. square?

*g.* What is the ratio of the perimeter of a 3-cm. square to the perimeter of a 6-cm. square? *h.* Of the perimeter of a 6-cm. square to the perimeter of a 3-cm. square?

*i.* What is the ratio of the solid content of a 2-in. cube to the solid content of a 6-in. cube? *j.* Of the solid content of a 6-in. cube to that of a 2-in. cube?

*k.* What is the ratio of the surface of a 2-in. cube to the surface of a 6-in. cube? *l.* Of the surface of a 6-in. cube to the surface of a 3-in. cube?

*m.* What is the ratio of the solid content of a 3-cm. cube to the solid content of a 6-cm. cube? *n.* Of the solid content of a 6-cm. cube to that of a 3-cm. cube?

*o.* What is the ratio of the surface of a 3-cm. cube to the surface of a 6-cm. cube? *p.* Of the surface of a 6-cm. cube to the surface of a 3-cm. cube?

*q.* What is the ratio of a square inch to a square foot? *r.* Of a cubic inch to a cubic foot?

---

* If pupils image the magnitudes compared, they will find no difficulty in the solution of these problems.

*a.* The specific gravity of granite is 2.7.* How much does a cubic foot of granite weigh?

*b.* A certain vessel is exactly large enough to contain 1000 grains of water. It will contain only 700 grains of petroleum. What is the specific gravity of the petroleum? †

*c.* The specific gravity of gold is 19.3. How much does a cubic foot of gold weigh? ‡

*d.* A cubic foot of sulphur weighs 125 lb. What is the specific gravity of sulphur?

*e.* A cubic foot of steel weighs 487.5 lb. What is the specific gravity of steel? §

*f.* What is the ratio of .01 to .001? *g.* Of .1 to .01? *h.* Of 1 to .1?

*i.* What is the ratio of 1 bu. to 1 pk.? *j.* Of 1 pk. to 1 qt.?

*k.* What is the ratio of $\$37\frac{1}{2}$ to \$15? *l.* Of \$15 to $\$37\frac{1}{2}$?

*m.* The ratio of the perimeter of one square to the perimeter of another square is 4. What is the ratio of the areas of the two squares?

*n.* The specific gravity of dry white pine is .55. How much does a cubic foot of it weigh?

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* See page 199. This means that a cubic foot of granite weighs 2.7 times as much as a cubic foot of water.

† This means, — What is the ratio of the weight of petroleum to the weight of the same bulk of water?

‡ This means that gold is 19.3 times as heavy as water, bulk for bulk.

§ How many times as much does it weigh as the same bulk of water?

*a.* The specific gravity of silver is 10.4. How much does a cubic inch of silver weigh?

*b.* The specific gravity of gold is 19.3. How much does a cubic inch of gold weigh?

*c.* The specific gravity of lead is 11.3. How much does a cubic inch of lead weigh?

*d.* The specific gravity of copper is 8.8. How much does a cubic inch of copper weigh?

*e.* The specific gravity of platinum is 21.4. How much does a cubic inch of platinum weigh?

*f.* A cubic foot of rock salt weighs 137½ lb. What is the specific gravity of rock salt?

*g.* A certain piece of marble weighs 40.5 ounces. Water equal in bulk to the piece of marble weighs 15 ounces. What is the ratio of the weight of the marble to the weight of the water? *h.* What is the specific gravity of the marble?

*i.* A certain bottle holds 17 ounces of water or 25½ ounces of nitric acid. What is the ratio of the weight of the nitric acid to the weight of the water? *j.* What is the specific gravity of the nitric acid?

*k.* The specific gravity of seasoned white oak is about .8. Find the approximate weight of 1000 feet of 2-in. white oak plank.

*l.* The ratio of the perimeter of one square to the perimeter of another square is 2½. What is the ratio of the areas of the two squares?

*m.* The ratio of the edge of one cube to the edge of another cube is as 2 to 3. What is the ratio of the solid contents of the two cubes?

# PROPORTION

The terms of a ratio are together called a **couplet**. Two couplets whose ratios are equal are called a **proportion**.

The two couplets of a proportion are often written thus, $6:18 = 10:30$, and should be read, the ratio of 6 to 18 equals the ratio of 10 to 30.

Couplets are sometimes written thus, $20:4::50:10$, and read, 20 is to 4 as 50 is to 10.*

**To find a missing term in a proportion:**

### EXAMPLE I

$$36:12 = x:25$$

The ratio of the first couplet is 3; that is, the antecedent is 3 times the consequent. Since the ratios of the couplets are equal, the ratio of the second couplet must be 3, and its antecedent must be 3 times its consequent. Three times 25 = 75, the missing term.

Find the missing term:

*a.* $90:45::x:180$ *b.* $20:60=x:225$
*c.* $48:12::x:150$ *d.* $30:50=x:175$
*e.* $75:30::x:140$ *f.* $90:20=x:140$

---

* The ratio sign (:) may be regarded as the sign of division (÷) with the horizontal line omitted, and the proportion sign (: :) the sign of equality (=) with an erasure through its center, thus: (==).

### EXAMPLE II

$$36 : 12 = 48 : x$$

Since the ratio of the first couplet is 3, the ratio of the second couplet must be 3, and $x$ must equal 1 third of 48. 1 third of 48 is 16.

Find the missing term:

*a.* $84 : 21 = 172 : x$

*b.* $20 : 60 :: 120 : x$

*c.* $96 : 16 :: 45 : x$

*d.* $25 : 35 = 45 : x$

*e.* $75 : 30 = 125 : x$

*f.* $50 : 25 :: 14 : x$

### EXAMPLE III

$$36 : x = 45 : 15$$

The ratio of each couplet is 3; so each consequent must be 1 third its antecedent, and $x$ must equal 1 third of 36, or 12.

Find the missing term:

*g.* $54 : x = 90 : 30$

*h.* $18 : x :: 65 : 195$

*i.* $75 : x :: 125 : 25$

*j.* $50 : x = 12 : 18$

*k.* $50 : x = 40 : 16$

*l.* $35 : x :: 21 : 3$

### EXAMPLE IV

$$x : 12 = 100 : 25$$

The ratio of each couplet is 4; so each antecedent must be 4 times its consequent, and $x$ must equal 4 times 12, or 48.

Find the missing term:

*m.* $x : 16 :: 51 : 17$

*n.* $x : 96 = 23 : 92$

*o.* $x : 22 = 76 : 19$

*p.* $x : 40 :: 36 : 48$

*q.* $x : 11 :: 24 : 3$

*r.* $x : 27 = 42 : 14$

**To find the missing term of a proportion without finding the ratio:**

The first and fourth terms of a proportion are called the **extremes**, and the second and third terms, the **means**; thus, in the proportion $12 : 6 = 8 : 4$, 12 and 4 are the extremes, and 6 and 8 are the means.

Observe that in the following proportions the product of the means equals the product of the extremes:

$6 : 3 = 8 : 4$; then $6 \times 4 = 3 \times 8$.
$2 : 5 = 4 : 10$; then $2 \times 10 = 5 \times 4$.
$\frac{1}{2} : \frac{1}{4} = 4 : 2$; then $\frac{1}{2} \times 2 = \frac{1}{4} \times 4$.

By algebraic methods it is easy to prove that in every numerical proportion the product of the means is equal to the product of the extremes. Hence the rules:

I. *If the extremes and one mean are given, divide the product of the extremes by the given mean and the quotient will be the required mean.*

II. *If the means and one extreme are given, divide the product of the means by the given extreme and the quotient will be the required extreme.*

### EXAMPLE I

$$30 : 20 = 18 : x$$

$30x =$ the product of the extremes
$20 \times 18 =$ the product of the means
Then $30x = 20 \times 18$, or 360
and $x = 12$

### EXAMPLE II

$$10 : 25 = x : 50$$

Then $25x = 10 \times 50$, or 500
and $x = 20$

### EXAMPLE III

$$40 : x = 25 : 5$$

Then $25x = 40 \times 5$, or $200$

and $x = 8$

### EXAMPLE IV

$$x : 35 = 4 : 28$$

Then $28x = 35 \times 4$, or $140$

and $x = 5$

Find the missing term:

*a.* $24 : 72 = x : 69$

*b.* $\frac{2}{3} : 4 = x : 30$

*c.* $45 : 12 = 75 : x$

*d.* $x : \frac{3}{4} = 40 : 6$

*e.* $35 : x = 14 : 40$

*f.* $\frac{1}{2} : \frac{2}{5} = x : 8$

*g.* $x : 70 = 3 : 21$

*h.* $.6 : .8 = 15 : x$

*i.* If 8 acres of land cost $360, how much will 15 acres cost at the same rate?

$$8 : 15 = 360 : x\,^{*}$$

*j.* If 12 horses consume 3500 lb. of hay per month, how many pounds will 15 horses consume?

*k.* If 11 cows cost $280.50, how many cows can be bought for $433.50 at the same rate?

---

* Observe that in the solution of concrete problems by the method here given the numbers must be regarded as abstract. It would be absurd to talk or think of finding the product of 15 acres and 360 dollars and dividing this by 8 acres. It is true, however, that the ratio of 8 acres to 15 acres equals the ratio of 360 dollars to $x$ dollars. It is also true that in the proportion $8 : 15 = 360 : x$, the product of the means is equal to the product of the extremes.

*a.* If 75 yd. of cloth cost $115.25, how much will 15 yd. cost at the same rate?

$$75 \text{ yd.} : 15 \text{ yd.} = \$115.25 : x$$

*b.* If $2\frac{1}{2}$ acres of land cost $76.20, how much will 15 acres cost at the same rate?

*c.* If 7 tons of coal can be bought for $26, how many tons can be bought for $39?

$$7 \text{ tons} : x \text{ tons} = \$26 : \$39$$

*d.* If 36 lb. of coffee can be bought for $7, how many pounds can be bought for $$17\frac{1}{2}$?

*e.* If sugar sells at the rate of 18 lb. for $1, how much should 63 lb. cost?

$$18 \text{ lb.} : 63 \text{ lb.} = \$1 : x \text{ dollars}$$

*f.* If a post 6 feet high casts a shadow 4 feet long, how high is that telegraph pole which at the same time and place casts a shadow 20 feet long?

*g.* If a post 5 feet high casts a shadow 8 feet long, how high is that steeple which casts a shadow 152 feet long?

*h.* If a train moves 50 miles in 1 hr. 20 min., at the same rate how far would it move in 2 hours?

*i.* If 15 chickens are worth $7.75, how much are 25 chickens worth at the same rate?

*To the Teacher.* The pupil may solve each of the above problems in three ways: (1) He may make use of the fact of the equality of the ratios in a proportion. (2) He may follow the Rules as given on page 205. (3) He may solve "by analysis," somewhat as follows: Prob. *a.* Since 75 yd. cost $115.25, 1 yd. costs $\frac{1}{75}$ of 115.25; but 15 yd. cost 15 times as much as 1 yd.; so 15 yd. cost $15 \times \frac{1}{75}$ of $115.25. That pupil will deserve commendation who can solve each problem by one method and prove his work by solving again in another way.

*a.* If 48 yd. of cloth cost $62.40, how much will 72 yd. cost at the same rate?

*b.* If 3¼ acres of land cost $146.90, how much will 27½ acres cost at the same rate?

*c.* If 5½ tons of coal cost $42.90, how much will 3¾ tons cost at the same rate?

*d.* If 4.3 tons of hay cost 53.75, how much will 17.2 tons cost at the same rate?

*e.* If the profit on .4 of an acre of land is $5.20, what is the profit on 4.4 acres at the same rate?

*f.* If a man's income is $220 for 2½ months, how much is it for one year at the same rate?

*g.* If a train moves 130 miles in 3 hr. 20 min., at the same rate how far will it move in 7 hr. 40 min.?

*h.* If 27 acres of land yield 1512 bushels of oats, how many bushels may be expected from 63 acres?

*i.* If sugar sells at the rate of 21 lb. for $1, how much should be paid for 125 pounds?

*j.* If coffee sells at the rate of 4½ lb. for one dollar, how much should be paid for 63 lb.?

*k.* If 15 dozen eggs will pay for 48 lb. of sugar, how much sugar should be given for 35 dozen eggs?

*l.* If 25½ lb. butter will pay for 36 lb. coffee, how much coffee should be given for 17 lb. butter?

*m.* Thirty-five bushels potatoes at 35¢ a bushel will pay for how many tons of coal at $7 a ton?

*n.* Five cords of wood at $5.50 a cord, will pay for how many bricks at $7 a thousand?

# POWERS AND ROOTS

## THE SQUARE AND SQUARE ROOT

A product obtained by using a number twice as a factor is called **the second power** or **the square** of the number; thus, 25 ($5 \times 5$) is the second power, or the square, of 5.

NOTE. Twenty-five is called the *second* power of 5 because it may be obtained by using 5 *twice* as a factor. It is called the *square* of 5 because it is the number of square units in a square whose side is 5 linear units.

**1.** What is the second power of 2? 8? 3? 5?

**2.** What is the square of 4? 7? 1? 6? 9? 10?

$11^2 = ?$ $12^2 = ?$ $13^2 = ?$ $14^2 = ?$

$15^2 = ?$ $16^2 = ?$ $17^2 = ?$ $18^2 = ?$

*a.* Find the sum of the eighteen squares in **1** and **2**.

The **square root** of a number is one of the two equal factors of the number.

The radical sign, $\sqrt{\ }$ (without a figure above it), indicates that the square root of the number following it is to be taken; thus $\sqrt{64}$, means the square root of 64.

**3.** What is the square root of 144? 81? 49?

**4.** What is the square root of 36? 25? 16?

$\sqrt{9} = ?$ $\sqrt{64} = ?$ $\sqrt{121} = ?$ $\sqrt{100} = ?$

$\sqrt{4} = ?$ $\sqrt{1} = ?$ $\sqrt{400} = ?$ $\sqrt{169} = ?$

*b.* Find the sum of the fourteen results in **3** and **4**.

Any number that can be resolved into two equal factors is a **perfect square.**

1. Tell which of the following are perfect squares and which are not:

9, 10, 12, 16, 18, 25, 32, 36.

NOTE. It is a curious fact that no number, either integral or mixed, can be found which, when multiplied by itself, will give as a product 10, or 12, or 14, or any number that is not a *perfect square.*

Any integral number that is a perfect square is composed of an even number of like prime factors; that is, its prime factors are an even number of 2's, 3's, 5's, 7's, etc.

2. Tell which of the following are perfect squares:

144, $(2 \times 2 \times 2 \times 2 \times 3 \times 3)$; 250, $(2 \times 5 \times 5 \times 5)$; 225, $(5 \times 5 \times 3 \times 3)$.

**To find the square root of an integral number that is a perfect square:** *Resolve the number into its prime factors and take half of them as factors of the root; that is, one half as many 2's, 3's, or 5's, etc., as there are 2's, 3's, or 5's, etc., in the factors of the number.*

3. Find the square root of 1225.

$1225 = 5 \times 5 \times 7 \times 7. \qquad \sqrt{1225} = 5 \times 7 = 35.$

*a.* Find the square root of 441. *b.* Of 400.
*c.* Find the square root of 576. *d.* Of 324.
*e.* Find the square root of 784. *f.* Of 2025.
*g.* Find the square root of 625. *h.* Of 3025.
*i.* Find the square root of 484. *j.* Of 1089.
*k.* Find the square root of 729. *l.* Of 1024.

**1.** The square of $\frac{1}{2}$, $(\frac{1}{2} \times \frac{1}{2})$, is ——.

NOTE 1. A square whose side is $\frac{1}{2}$ (of a linear unit) has an area of $\frac{1}{4}$ (of a square unit). Show this by diagram.

**2.** Answer the following and illustrate by diagram if necessary:

$(\frac{1}{3})^2 = ?$ $(\frac{1}{4})^2 = ?$ $(\frac{1}{5})^2 = ?$ $(\frac{1}{6})^2 = ?$

$(\frac{2}{3})^2 = ?$ $(\frac{3}{4})^2 = ?$ $(\frac{2}{5})^2 = ?$ $(\frac{5}{6})^2 = ?$

*a.* Find the sum of the eight results in **2**.

*b.* A square of sheet brass whose edge is $\frac{5}{12}$ of a foot is what part of a square foot?

**3.** The square root of $\frac{9}{16}$ is ——.

NOTE 2. The side of a square whose area is $\frac{9}{16}$ (of a square unit) is $\frac{3}{4}$ (of a linear unit) in length. Show this by diagram.

NOTE 3. Only those fractions are perfect squares which, when in their lowest terms, have perfect squares for numerators and perfect squares for denominators.

**4.** What is the square root of $\frac{25}{36}$? Of $\frac{16}{25}$? Of $\frac{1}{4}$?

$\sqrt{\frac{144}{25}} = ?$ $\sqrt{\frac{169}{36}} = ?$ $\sqrt{\frac{81}{16}} = ?$ $\sqrt{\frac{121}{144}} = ?$

*c.* Find the sum of the seven results in **4**.

*d.* The area of a square piece of sheet brass is $\frac{81}{144}$ of a square foot. What is the length of the side of the square?

*e.* How long is the side of a square of zinc, the area of which is $\frac{25}{36}$ of a square yard?

*f.* What is the perimeter of a square piece of land whose area is $\frac{16}{36}$ of a square mile?

*g.* What number multiplied by itself will give as a product $\frac{25}{81}$?

**1.** The square of .5 is ——.

NOTE 1. A square whose side is .5 (of a linear unit) has an area of .25 (of a square unit). Show this by diagram.

**2.** Answer the following and illustrate by diagram if necessary:

$.1^2 = ?$ $.2^2 = ?$ $.3^2 = ?$ $.4^2 = ?$

$.5^2 = ?$ $.6^2 = ?$ $.7^2 = ?$ $.8^2 = ?$

$1.2^2 = ?$ $1.5^2 = ?$ $1.6^2 = ?$ $1.8^2 = ?$

*a.* Find the sum of the twelve results in **2**.

*b.* A square of sheet brass whose edge is .9 of a foot is what part of a square foot?

**3.** The square root of .25 is ——.

NOTE 2. A square whose area is .25 (of a square unit) is .5 (of a linear unit) in length. Show this by diagram.

NOTE 3. Only those decimals are perfect squares which, when in their lowest decimal terms, have numerators that are perfect squares and denominators that are perfect squares. The decimal denominators that are perfect squares are 100, 10000, 1000000, etc.

**4.** What is the square root of $\frac{49}{100}$? Of .36? Of .64?

$\sqrt{\frac{144}{100}} = ?$ $\sqrt{1.44} = ?$ $\sqrt{2.25} = ?$ $\sqrt{6.25} = ?$

*c.* Find the sum of the seven results in **4**.

*d.* How long is the edge of a square of zinc whose area is 4.84 square feet? *

*e.* What is the perimeter of a piece of land whose area is .49 of a square mile?

*f.* What number multiplied by itself will give as a product 2.56?

*g.* Is .9 a perfect square?

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* 4.84 square feet is $\frac{484}{100}$ square feet.

A product obtained by using a number three times as a factor is called **the third power**, or **the cube** of the number; thus, 125 ($5 \times 5 \times 5$) is the third power, or the cube of 5.

NOTE. One hundred twenty-five is called the *third* power of 5, because it may be obtained by using 5 *three* times as a factor. It is called the *cube* of 5, because it is the number of *cubic units* in a cube whose edge is 5 linear units.

$1^3 = 1$  $2^3 = 8$  $3^3 = 27$  $4^3 = 64$  $5^3 = 125$

$6^3 = 216$  $7^3 = 343$  $8^3 = 512$  $9^3 = 729$  $10^3 = 1000$

**1.** Find the cube of 12; of 13; of 14; of 15; of 16.

*a.* Find the sum of the five results in **1**.

The **cube root** of a number is one of its three equal factors.

The radical sign with a figure 3 over it indicates that the cube root of the number following it is to be taken; thus, $\sqrt[3]{512}$, means the cube root of 512.

**To find the cube root of an integral number that is a perfect cube:** *Resolve the number into its prime factors and take one third of them as factors of the root; that is, one third as many* 2's, 3's, *or* 5's, *etc., as there are* 2's, 3's, *or* 5's *in the factors of the number.*

**2.** Find the cube root of 216.

$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3.$  $\sqrt[3]{216} = 2 \times 3.$

**3.** Find the cube root of 1728; of 3375; of 2744; of 10648; of 5832.

*b.* Find the sum of the five results in **3**.

*c.* What number used three times as a factor gives a product of 27000?

**1.** Square 42. Then resolve the square of 42 into its prime factors and compare them with the prime factors of 42.

**2.** Cube 42. Then resolve the cube of 42 into its prime factors and compare them with the prime factors of 42.

**3.** Square 45. Then resolve the square of 45 into its prime factors and compare them with the prime factors of 45.

**4.** Cube 45. Then resolve the cube of 45 into its prime factors and compare them with the prime factors of 45.

**5.** Square 27. Then resolve the square of 27 into its prime factors and compare them with the prime factors of 27.

**6.** Cube 27. Then resolve the cube of 27 into its prime factors and compare them with the prime factors of 27.

*a.* Divide the cube of 15 by the square of 15.
*b.* Divide the cube of $\frac{3}{4}$ by the square of $\frac{3}{4}$.
*c.* Divide the cube of .7 by the square of .7.
*d.* Divide the cube of 2.5 by the square of 2.5.

Find the square root of each of the following perfect squares:

*e.* 3025 *f.* 4225 *g.* 5625 *h.* 7225
*i.* $\frac{144}{625}$ *j.* $\frac{25}{576}$ *k.* $\frac{49}{324}$ *l.* $\frac{16}{529}$

Find the cube root of each of the following perfect cubes:

*m.* 1728 *n.* 15625 *o.* 3375 *p.* 9261
*q.* $\frac{8}{125}$ * *r.* $\frac{27}{512}$ *s.* $\frac{64}{343}$ *t.* $\frac{1}{729}$

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* *Find the cube root of the numerator for the new numerator, and the cube root of the denominator for the new denominator.*

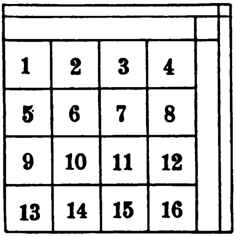
# SQUARE ROOT

## Find the square root of 1795 true to tenths.

Regard the number as representing 1795 1-inch squares. These are to be arranged in the form of a square, and the length of its side noted.

100 1-inch squares = 1 10-inch square.
1700 1-inch squares = 17 10-inch squares.

But 16 of the 17 10-inch squares can be arranged in a square that is 4 by 4; that is, 40 inches by 40 inches. See diagram.



After making this square (40 inches by 40 inches) there are (1700 – 1600 + 95) 195 1-inch squares remaining. From these, additions are to be made to two sides of the square already formed. Each side is 40 inches; hence the additions must be made upon a base line of 80 inches. These additions can be as many inches wide as 80 is contained times in 195.* 195 ÷ 80 = 2 +. The additions are 2 inches wide. These will require 2 times 80, + 2 times 2, = 164 square inches.

After making this square (42 in. by 42 in.) there are (195 – 164) 31 square inches remaining. If further additions are to be made to the square, the 31 square inches must be changed to tenth-inch squares. In each 1-inch square there are 100 tenth-inch squares; in 31 square inches there are 3100 tenth-inch squares. From these, additions are to be made upon two sides of the 42-inch square. 42 inches equal 420 tenth-inches. The additions must be made upon a base line (420 × 2) 840 tenth-inches long. These additions can be as many tenth-inches wide as 840 is contained times in 3100. 3100 ÷ 840 = 3+. The additions are 3 tenth-inches wide. These will require 3 times 840, + 3 times 3, = 2529 tenth-inch squares.

After making this square (42.3 by 42.3) there are (3100 – 2529) 571 tenth-inch squares remaining. (If further additions are to be made to the square, the 571 tenth-inch squares must be changed to hundredth-inch squares.) The square root of 1795 is 42.3+.

* Allow for the little square in the upper right-hand corner.

**Note.** Pupils who have mastered the work on the preceding page will have no difficulty in discovering that the same result may be obtained by the following process:

Find the square root of 1795 true to hundredths.

OPERATION

$$\begin{array}{rr|r}
 & \multicolumn{2}{l}{\overset{\frown}{17}\overset{\frown}{95}.{}^{\vee}(42.36} \\
 & \multicolumn{2}{l}{16} \\
\hline
40 \times 2 = & 80\,^{*} & 195 \\
 & 2 & 164 \\
\hline
420 \times 2 = & 840 & 3100 \\
 & 3 & 2529 \\
\hline
4230 \times 2 = & 8460 & 57100 \\
 & 6 & 50796 \\
\hline
 & & 6304
\end{array}$$

1. *Beginning with the decimal point, group the figures as far as possible into periods of two figures each.*

2. *Find the largest square in the left-hand period and place its root at the right as the first figure of the complete root.*

3. *Subtract the square from the left-hand period and to the difference annex the next period. Regard this as a dividend.*

4. *Take 2 times* 10 *times the root already found as a trial divisor, and find how many times it is contained in the dividend. Write the quotient as the second figure of the root, and also as a part of the divisor. Multiply the entire divisor by the second figure of the root, subtract the product from the dividend, and proceed as before.*

Note. If, in applying the foregoing rule, a dividend is found that will not contain the divisor, annex a zero to the root, a zero to the trial divisor, a new period to the dividend, and proceed as before.

Find the approximate square root (true to tenths):

*a.* Of 875 *b.* Of 1526 *c.* Of 2754 *d.* Of 4150

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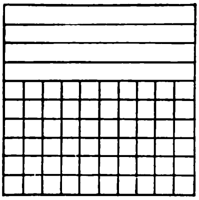
* The entire divisor is 80 and 2; that is, 82.

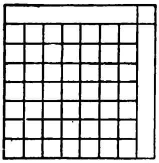
### Find the square root of .6.



**Regard the number as representing .6 of a 1-inch square. .6 of a 1-inch square = 60 tenth-inch squares.**

**But 49 of the 60 tenth-inch squares can be arranged in a square that is 7 by 7; that is, 7 tenths of an inch by 7 tenths of an inch.**

**After making this square there are (60 – 49) 11 tenth-inch squares remaining. If additions are to be made to the square, the 11 tenth-inch squares must be changed to hundredth-inch squares. In each tenth-inch square there are 100 hundredth-inch squares; in 11 tenth-inch squares there are 1100 hundredth-inch squares. From these, additions are to be made upon two sides of the .7-inch square. .7 = 70 hundredths. The additions must be made upon a base line (70 × 2) 140 hundredth-inches long. These additions can be as many hundredth-inches wide as 140 is contained times in 1100. 1100 ÷ 140 = 7+. The additions are 7 hundredth-inches wide. These will require 7 times 140, + 7 times 7, = 1029 hundredth-inch squares.**



**After making this square (.77 by .77) there are (1100 – 1029) 71 hundredth-inch squares remaining. (If further additions are to be made to the square the 77 hundredth-inch squares must be changed to thousandth-inch squares.) The square root of .6, true to hundredths, is .77.**

**NOTE. The work on this page should be first presented orally by the teacher. It must be given very slowly. Great care must be taken that pupils image each magnitude when its word-symbol is spoken by the teacher. Any attempt to move forward more rapidly than this can be done by the slowest pupil, will result in failure so far as that pupil is concerned. The great skill of many pupils in the Illinois School for the Blind in such work as this, is to be attributed mainly to their practice in creating imaginary magnitudes. When the teacher says *tenth-inch square*, they "think a tenth-inch square." Its image comes immediately into consciousness. Teachers of pupils who have sight may obtain invaluable suggestion from the mathematical ability of these pupils who have opportunity for comparatively little sense-perception of magnitudes, but who, from necessity, are constantly trained to "see with the mind's eye."**

**Note.** Pupils who have mastered the work on the preceding page will readily understand the following process. See rule on page 216.

**1.** Find the square root of .6.

OPERATION

$$\begin{array}{r|r}
\multicolumn{2}{r}{.\,\overset{\vee}{}\overparen{60}\,\overparen{00}(.774} \\
\multicolumn{2}{r}{49} \\ \hline
70\times 2=140 & 1100 \\
7 & 1029 \\ \hline
770\times 2=1540 & 7100 \\
4 & 6176 \\ \hline
\multicolumn{2}{r}{924}
\end{array}$$

1. *In grouping decimals for the purpose of extracting the square root it is necessary to begin at the decimal point.*

2. *The square root of any number of hundredths is a number of tenths; the square root of any number of ten thousandths is a number of hundredths, etc.*

**2.** Find the square root of 54264.25.

$$\begin{array}{r|r}
\multicolumn{2}{r}{\overparen{5}\,\overparen{42}\,\overparen{64}.\overset{\vee}{}\overparen{25})232.946} \\
\multicolumn{2}{r}{4} \\ \hline
20\times 2=40 & 142 \\
3 & 129 \\ \hline
230\times 2=460 & 1364 \\
2 & 924 \\ \hline
2320\times 2=4640 & 440.25 \\
9 & 418.41 \\ \hline
23290\times 2=46580 & 21.8400 \\
4 & 18.6336 \\ \hline
232940\times 2=465880 & 3.206400 \\
6 & 2.795316 \\ \hline
\multicolumn{2}{r}{.411084}
\end{array}$$

Observe that the trial divisor is always 2 times 10 times the part of the root already found.

The following numbers are perfect squares. Find their square roots by both the factor method and the method given on the four preceding pages:

*a.* 6889 *b.* 841 *c.* 71824

*d.* 1849 *e.* 729 *f.* 60516

*g.* Find the sum of the six results, *a* to *f*.

*h.* $\frac{81}{625}$ *i.* $\frac{289}{625}$ *j.* $\frac{576}{625}$

*k.* $\frac{4489}{625}$ *l.* $\frac{2916}{625}$ *m.* $\frac{900}{625}$

*n.* Find the sum of the six results, *h* to *m*.

*o.* .81 *p.* .0625 *q.* .04

*r.* 1.21 *s.* .7921 *t.* .0004

*u.* Find the sum of the six results, *o* to *t*.

1. The square of a number represented by one digit gives a number represented by —— or —— digits.

2. The square of a number represented by two digits gives a number represented by —— or —— digits.

3. The square of a number represented by three digits gives a number represented by —— or —— digits.

4. The square root of a perfect square represented by one or two digits is a number represented by —— digit.

5. The square root of a perfect square represented by three or four digits is a number represented by —— digits.

6. The square root of a perfect square represented by five or six digits is a number represented by —— digits.

7. The square root of a perfect square represented by seven or eight digits is a number represented by —— digits.

*a.* What is one of the two equal factors of 9216?

*b.* What is one of the four equal factors of 20736?*

*c.* If a body of 7921 soldiers were arranged in a solid square, how many soldiers would there be on each side?

*d.* How many rods of fence will inclose a square field whose area is 40 acres?

*e.* How many rods long is one side of a square piece of land containing exactly one acre? †

*f.* If the surface of a cubical block is 150 square inches, what is the length of one edge of the cube? ‡

*g.* How many rods of fence will inclose a square piece of land containing 4 acres 144 square rods?

*h.* Find the side of a square equal in area to a rectangle that is 15 ft. by 60 ft.

*i.* Compare the amount of fence required to inclose two fields each containing 10 acres: one field is square, and the other is 50 rods long and —— rods wide.

*j.* Find the area of the largest possible rectangle having a perimeter of 40 feet.

*k.* If a square piece of land is $\frac{1}{4}$ of a square mile, how much fence will be required to inclose it?

*l.* Find the squares of numbers from 10 to 20 inclusive, and memorize them.

*m.* Find the cubes of numbers from 1 to 10 inclusive, and memorize them.

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* To find one of the four equal factors of a number (the 4th root) extract the square root of the square root. Why? What is the fourth root of 81?

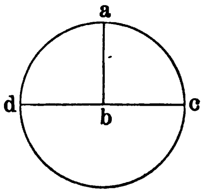
† Find the answer to problem *e* true to hundredths of a rod.

‡ One face of the block is one sixth of the entire surface.

# MENSURATION

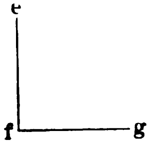
## ANGLES

1. For the purpose of measurement every circumference is considered as divided into 360 equal parts, called **degrees**. Any part of a circumference is an **arc**.

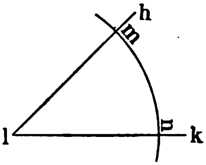


2. Two lines meeting at a point form an **angle**. The point in which the two lines meet is the **vertex** of the angle.

3. Every angle may be regarded as having its vertex at the center of a circle, and the angle is measured by the part of the arc intercepted; thus the angle *hlk* is measured by the arc *mn*.

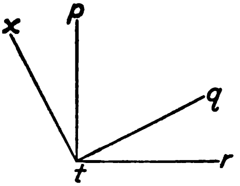


4. The angle *efg* is an angle of 90 degrees. The angle *hlk* is an angle of 45 degrees.



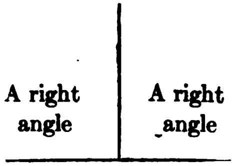
5. An angle of 90° is a **right angle**; an angle of less than 90° is an **acute angle**; an angle of more than 90° is an **obtuse angle**.

*All the angles about a point together equal 4 right angles.*



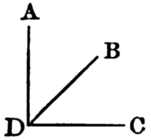
*a*. The angle *xtr* is an angle of about —— degrees.

*b*. The angle *ptq* is an angle of about —— degrees.



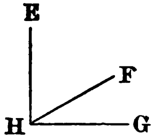
Parallel Lines

1. When one straight line meets another straight line in such a manner that two right angles are formed by the lines, the two lines are said to be **perpendicular** to each other.



2. Two lines side by side extending in the same direction are said to be **parallel**.

3. A line extending in the direction of the horizon is said to be **horizontal**.



4. A line in the direction of a plumb line * is said to be **vertical**.

*a*. The angle formed by a vertical line meeting a horizontal line is an angle of —— degrees.

*b*. An angle that is equal to one half of a right angle is an angle of —— degrees.

*c*. The angle *ADB* is an angle of —— degrees.

*d*. The angle *BDC* is an angle of —— degrees.

*e*. If from a right angle an angle of 30 degrees is taken, the remaining angle is an angle of —— degrees.

*f*. The angle *FHG* is an angle of —— degrees.

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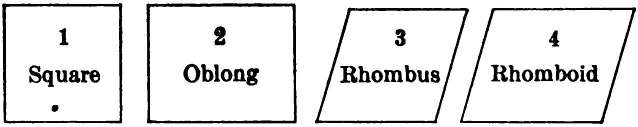
* A plumb line is a cord by which a piece of lead (plumbum), or other heavy material, is suspended.



1. Any side of any one of the above figures is parallel to the opposite side of the same figure. Hence the figures are called **parallelograms.**

2. Each of the above figures has four sides. Hence the figures are called **quadrilaterals.**

3. If all the sides of a figure are equal, the figure is said to be **equilateral.**

4. If all the angles of a parallelogram are right angles (angles of 90°) the figure is said to be **rectangular.**

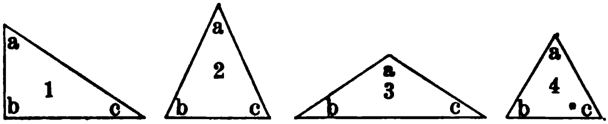
5. In a rhomboid or a rhombus two of the angles are less than right angles and two of them are greater than right angles. Convince yourself by cutting a rhomboid from paper and comparing it with rectangular figures that two of the angles of a rhomboid are as much less than two right angles as the other two are greater than two right angles.

*a.* Which of the above figures are equilateral? *b.* Rectangular? *c.* Not equilateral? *d.* Not rectangular?

*e.* Which of the above figures is an equilateral rectangular parallelogram?

*f.* If one of the angles of a rhombus or a rhomboid is an angle of 80 degrees, what is the number of degrees in each of the other angles?

6. Draw a quadrilateral that is not a parallelogram.



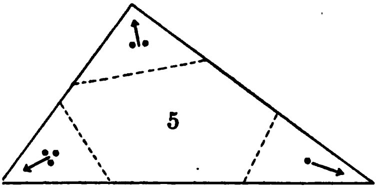
Right Isosceles Isosceles Equilateral

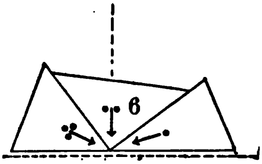
1. A triangle has —— sides and —— angles.

2. A **right triangle** has one right angle; that is, one angle of —— degrees.

3. An **isosceles triangle** has two angles that are equal and two sides that are equal.

4. An **equilateral triangle** has equal sides and equal angles.

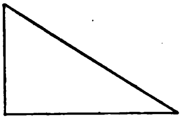




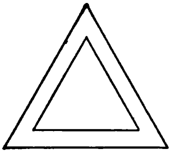
5. Cut from paper a triangle similar to the one shown in Fig. 5. Then cut it into parts as shown by the dotted lines. Rearrange the 3 angles of the triangle as shown in Fig. 6. Compare the sum of the 3 angles with 2 right angles as shown in Fig. 6. Convince yourself that the three angles of this triangle are together equal to two right angles.

6. Cut other triangles and make similar comparisons, until you are convinced that *the sum of the angles of any triangle is equal to two right angles.*

7. Estimate each angle in each of the above figures.



Right



Equilateral and Equiangular

**1.** The sum of the angles of any triangle is equal to —— right angles or —— degrees.

**2.** In a right triangle there is one right angle. The other two angles are together equal to ——.

**3.** In a right triangle one of the angles is an angle of 40°. How many degrees in each of the other two angles? Draw such a triangle.

**4.** Can a right triangle have an obtuse angle?

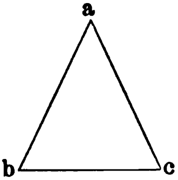
**5.** Convince yourself by drawings and measurements that every equilateral triangle is equiangular.

**6.** Note that in every equiangular triangle each angle is one third of 2 right angles. So each angle is an angle of —— degrees.

**7.** If any one of the angles of a triangle is greater or less than 60°, can the triangle be equiangular? Can it be equilateral?

**8.** If angle *a* of an isosceles triangle measures 50°, how many degrees in angle *b*? In angle *c*?

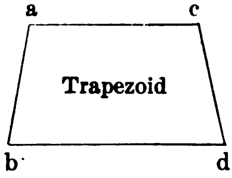
**9.** If the sum of two angles of a triangle is 128°, how many degrees in the third angle of the triangle?

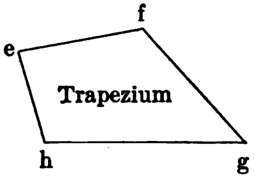


Isosceles

**10.** If one of the angles of a right triangle is an angle of 25°, how many degrees in the other acute angle?

**Quadrilaterals that are not parallelograms:**





**1.** Two of the sides of a trapezoid are parallel and two are not parallel. In the trapezoid represented above the side *ac* is parallel to the side ——.

**2.** No two of the bounding lines of a trapezium are parallel.

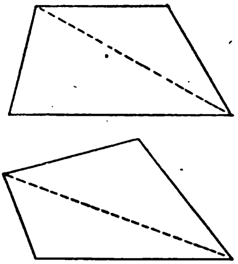
**3.** In the trapezoid represented above no one of the angles is a right angle. Name the angles that are greater than right angles.

**4.** Draw a trapezoid two of whose angles are right angles.

**5.** Can you draw a trapezoid having one and only one right angle?

**6.** Draw a trapezium one of whose angles is a right angle.

**7.** Every quadrilateral (trapezium, trapezoid, or parallelogram) may be divided into two triangles. Remember that the sum of the angles of two triangles is equal to four right angles. Observe that the sum of the angles of the two triangles is equal to the sum of the angles of the quadrilateral. *So the sum of the angles of a quadrilateral is equal to four right angles.*



*a.* If two of the angles of a trapezoid are right angles and the third is an angle of 60°, how many degrees in the fourth angle? Draw such a trapezoid.*

*b.* If the sum of three of the angles of a trapezium is 298°, how many degrees in the fourth angle? Draw such a trapezium.*

*c.* If one of the angles of a triangle is an angle of 80°, and the other two angles are equal, how many degrees in each of the angles? Draw the figure.*

*d.* If one of the angles of a quadrilateral is a right angle, and the other three angles are equal, what kind of a quadrilateral is the figure?

*e.* I am thinking of a right triangle one of whose angles measures 41°. Give the measurements of the other two angles. Draw such a triangle.

*f.* One of the angles of a rhomboid is an angle of 75°. What is the number of degrees in each of the other angles? Draw such a figure.

*g.* An angle that is equal to one fifth of a right angle is an angle of how many degrees? Draw such an angle.

*h.* An angle that is equal to one and one half right angles is an angle of how many degrees? Draw such an angle.

*i.* The smallest angle of a triangle measures 40°. Another of its angles measures 50°. How many degrees in the third angle? Draw the triangle.

*j.* Find the sum of the angles of three triangles.

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* It is not expected that this drawing will be accurate in its angular measurements — simply an approximation to accuracy, to aid the pupil in recognizing the comparative size of angles.

1. **All the geometrical figures on this page are quadrilaterals; that is, *each has four sides.***

2. The first four figures are **parallelograms**; that is, *the opposite sides of each figure are parallel.*

3. The first two figures are **rectangular**; that is, *their angles are right angles.*

4. The first and third are **equilateral**; that is, *the sides are equal.*

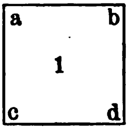
5. There is one equilateral rectangular parallelogram. Which is it?

6. There is one equilateral parallelogram that is not rectangular. Which is it?

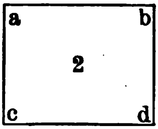
7. There is one rectangular parallelogram that is not equilateral. Which is it?

8. The sum of the angles of each figure on the page is ——— right angles.

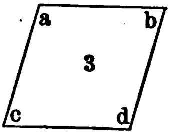
9. Tell as nearly as you can the size of each angle of each figure.



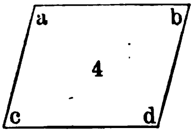
Square



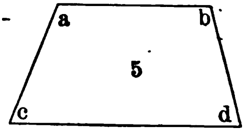
Oblong



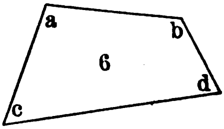
Rhombus



Rhomboid



Trapezoid



Trapezium

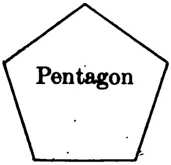


Fig. 1.

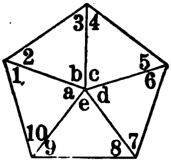


Fig. 2.

1. Every regular pentagon may be divided into —— equal isosceles triangles.

2. The sum of the angles of one triangle is equal to —— right angles; then the sum of the angles of 5 triangles is equal to —— right angles.

3. But the sum of the central angles in figure 2 ($a+b+c+d+e$) is equal to —— right angles; then the sum of all the other angles of the five triangles is equal to 10 right angles less 4 right angles, or 6 right angles = 540°. But the angular space that measures 540°, as shown in Figure 2, is made up of 10 equal angles, so each one of the angles is 1 tenth of 540°, or 54°. Two of these angles, as 1 and 2, make one of the angles of the pentagon; therefore each angle of the pentagon measures 2 times 54°, or 108°.

4. Using the protractor,* construct a regular pentagon as follows:

(*a*) Draw two lines that meet in a point, each line being 2 inches long and the angular space between them being 108°.

(*b*) Regarding the two lines as two sides of a regular pentagon, draw two more sides each 2 inches long and joining those already drawn at an angle of 108°.

(*c*) Complete the figure by drawing the fifth side, and prove your work by measuring the last line drawn and the other two angles of the pentagon.

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* See page 351.

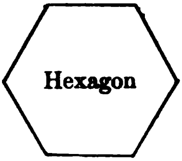

Fig. 1.

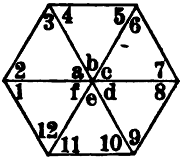

Fig. 2.

1. Every regular hexagon may be divided into —— equal isosceles triangles.

2. The sum of the angles of one triangle is equal to —— right angles, then the sum of the angles of 6 triangles is equal to —— right angles.

3. But the sum of the central angles in Fig. 2 $(a + b + c + d + e + f)$ is equal to —— right angles; then the sum of all the other angles of the six triangles is equal to 12 right angles less 4 right angles, or 8 right angles = 720°. But the angular space that measures 720°, as shown in Fig. 2, is made up of 12 equal angles; so each one of the angles is one 12th of 720°, or 60°. Two of these angles, as 1 and 2, make one of the angles of the hexagon; therefore each angle of the hexagon measures 2 times 60,° or 120°.

4. Using the protractor,* construct a regular hexagon, making each side 2 inches long.

Observe that since all the angular space about a point is equal to 4 right angles, or 360°, and since the space around the central point of the hexagon is divided into 6 equal angles, each of these angles is an angle of (360° + 6) 60°. But each of the other angles of these triangles has been shown to be an angle of 60°; so each triangle is equiangular.

Are the triangles equilateral?

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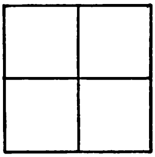
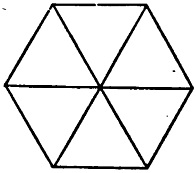
* See page 351.

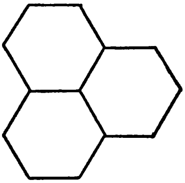


1. Four equal squares may be so joined as to cover all the space about a point. Each angle whose vertex is at the central point of the figure is an angle of 90°. Four times 90° = —— degrees.



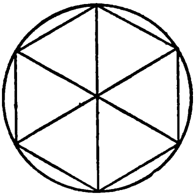
2. Six equal equilateral triangles may be so joined as to cover all the space about a point. Each angle whose vertex is at the central point of the figure is an angle of 60°. Six times 60° = —— degrees.

3. Cut from paper 6 equal equilateral triangles and join them as shown in the figure.



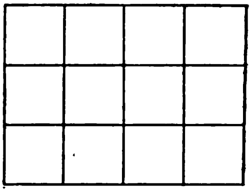
4. Three equal hexagons may be so joined as to cover all the space about a point. Each angle whose vertex is at the central point of the figure is an angle of 120°. Three times 120° = —— degrees.

5. Cut from paper 3 equal hexagons and join them as shown in the figure.



6. Since every regular hexagon may be divided into six equal equilateral triangles, as shown in the figure, it follows that the side of a regular hexagon is exactly equal to the radius of the circle that circumscribes the hexagon.

7. Using compasses, draw a circle and inscribe a hexagon.



**1.** One side of every rectangle may be regarded as its **base.** The side perpendicular to its base is its **altitude.**

**2.** The number of square units in the row of square units next to the base of a rectangle, taken as many times as there are linear units in its altitude, equals the number of square units in its area. In the figure given, we have 4 sq. units × 3 = 12 sq. units.

NOTE 1. In the above, it is assumed that the base and altitude are measured by the same linear unit, and that the square unit takes its name from the linear unit.

NOTE 2. In the actual finding of the area of rectangles for practical purposes, the work is done mainly with abstract numbers and the proper interpretation is given to the result. There can be no serious objection to the rule for finding the area of rectangles, as given in the old books, provided the pupil is able to interpret it.

**To find the area of a rectangle:** "*Multiply its base by its altitude.*"

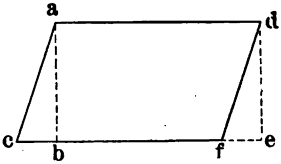
*a.* Find the area of the surface of a cubical block whose edge is 9 inches in length.

*b.* Find the area, in square yards, of a rectangular piece of ground that is 36 feet by 45 feet.

*c.* Find the area, in acres, of a rectangular piece of land that is 92 rods by 16 rods.

*d.* Find the area, in square rods, of a piece of ground that is 99 feet by 66 feet.

*e.* The area of a field 30 rods square is how many times as great as the area of a field 10 rods square? Compare a 30-in. square with a 10-in. square.



**1.** One side of a rhomboid[*] may be regarded as its **base.** The perpendicular distance from the base to the opposite side is its **altitude.**

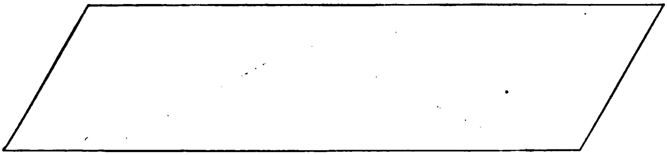
In the figure here given *cf* is the base and *ab* the altitude.

**2.** Convince yourself by measurements and by paper-cutting that from every rhomboid there may be cut a triangle (*abc*), which when placed upon the opposite side (*def*) converts the rhomboid into a rectangle (*adeb*).

Observe that the base of the rectangle is equal to the base of the rhomboid, and the altitude of the rectangle is equal to the altitude of the rhomboid.

**3.** A rhomboid is equivalent to a rectangle having the same base and altitude.

**To find the area of a rhomboid:** *Find the area of a rectangle whose base and altitude are the same as the base and altitude of the rhomboid.* Or, as the rule is given in the older books,—"*Multiply the base by the altitude.*"



*a.* If the above figure represents a piece of land, and is drawn on a scale of $\frac{1}{8}$ inch to the rod, what part of an acre of land does it represent?

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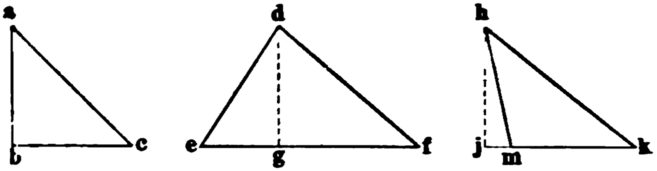
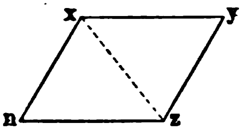
[*] The statements upon this page apply to the rhombus as well as to the rhomboid.



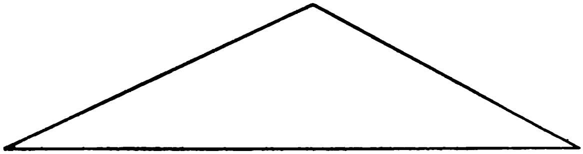
1. One side of a triangle may be regarded as its base The perpendicular distance from its base (or from its base extended) to the opposite angle is its altitude.

2. What is the altitude of the first of the above triangles? Of the second? Of the third?

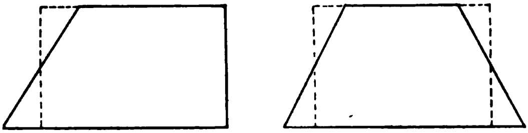


3. Convince yourself by measurement and by paper-cutting that every triangle is one half of a parallelogram having the same base and the same altitude as the triangle.

**To find the area of a triangle:** *Find the area of the parallelogram having the same base and altitude, and take one half of the result.* Or, as the rule is given in the older books, "*Take one half of the product of the base and altitude.*"



*a.* If the above figure represents a piece of land and is drawn on a scale of $\frac{1}{8}$ inch to the rod, how many square rods of land does it represent? *b.* This is what part of an acre of land?



1. Convince yourself by measurement and by paper-cutting that from every trapezoid there may be cut a triangle (or triangles) which, when properly adjusted to another part (or parts) of the trapezoid, will convert the trapezoid into a rectangle.

2. Convince yourself that the rectangle made from a trapezoid is not so long as the longer of the parallel sides of the trapezoid, and not so short as the shorter of the parallel sides of the trapezoid — that *its length is midway between the lengths of the two parallel sides of the trapezoid.*

NOTE. Observe that the length of the rectangle thus formed may be found by dividing the sum of the parallel sides of the trapezoid by 2.

**To find the area of a trapezoid:** *Find the area of the rectangle to which it is equivalent*, or, as the rule is usually given, "*Multiply one half the sum of the parallel sides by the altitude.*"

*a.* Find the area of a trapezoid whose parallel sides are 10 inches and 15 inches respectively, and whose altitude is 8 inches.

*b.* How many acres in a trapezoidal piece of land, the parallel sides being 28 rods and 36 rods respectively, and the breadth (altitude) 25 rods?

*c.* The parallel sides of a trapezoid are 20 inches and 30 inches. Its altitude is 14 inches. Find its area.

**1.** The longest side of a right triangle is the **hypotenuse**. Either of the other sides may be regarded as the **base** and the remaining side as the **perpendicular**.

**2.** In Fig. 1, *AC* is the ——; *BC*, the ——; and *AB*, the ——.

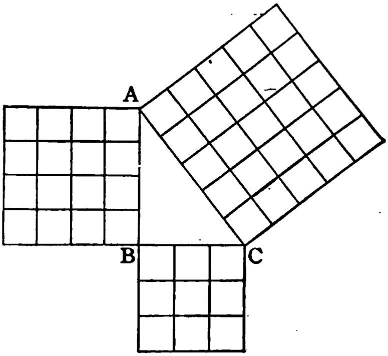


Fig. 1.

**3.** Convince yourself by measurements, etc., that *the square of the hypotenuse of a right triangle is equivalent to the sum of the squares of the other two sides.*

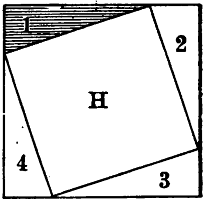


Fig. 2.

Figures 2 and 3 are equal squares. If from Figure 2, the four right triangles, 1, 2, 3, 4, are taken, *H*, the square of the hypotenuse will remain. If from Figure 3, the four right triangles (equal to the four right triangles in Figure 2) are taken, *B*, the square of the base, and *P*, the square of the perpendicular, will remain. When equals are taken from equals the remainders are equal, therefore the square, *H*, equals the sum of the squares *B* and *P*.

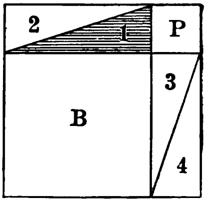


Fig. 3.

**To find the hypotenuse of a right triangle when the base and perpendicular are given:** *Square the base; square the perpendicular; extract the square root of the sum of these squares.*

**4.** Give a rule for finding the perpendicular when the base and hypotenuse are given.

*a.* Find approximately the diagonal of a square whose side is 20 feet.

*b.* Find approximately the distance diagonally across a rectangular floor, the length of the floor being 30 feet and its breadth 20 feet.

*c.* How long a ladder is required to reach a window 25 feet high if the foot of the ladder is 6 feet from the building, and the ground about the building level?

*d.* The base of a right triangle is 40 rods and its perpendicular 60 rods. What is its hypotenuse? *e.* What is its area? *f.* What is its perimeter?

*g.* The area of a certain square piece of land is 2½ acres. Find (in rods) its side. *h.* Find its perimeter. *i.* Find its diagonal, true to tenths of a rod.

*j.* The length of a rectangular piece of land is to its breadth as 4 to 3. Its area is 30 acres. Find its breadth. *k.* Find its perimeter. *l.* Find the distance diagonally across it.

*m.* A ladder 30 feet long is placed against the side of a house. The foot of the ladder is 6 feet from the house. How far up the wall does the ladder reach?

*n.* A ladder with its foot 8 feet from the wall of a house leans against the house and reaches to a point 40 feet above the ground. How long is the ladder?

*o.* How long a rope will be required, allowing 6 feet for sagging and tying, to reach from the top of a 40-foot derrick to a post 30 feet from the derrick?

*p.* How far is it from the center of a 90-foot square to one of the corners of the square?

1. Cut a 3-inch circle from cardboard. By rolling it upon a foot rule, measure its circumference.

2. Measure the diameter of a bicycle wheel; then by rolling it upon the ground or upon the schoolroom floor, measure its circumference.

3. In a similar manner measure the diameters and the circumferences of other wheels until you are convinced that the circumference of a circle is a little more than —— times its diameter.

4. The circumference of a circle is nearly $3\frac{1}{7}$ times the diameter; more accurately, it is $3.141592^+$ times the diameter.

NOTE. It is a curious fact that the diameter of a circle being given in numbers, it is impossible to express in numbers its exact circumference. The circumference being given in numbers, it is impossible to express in numbers its exact diameter. In other words, the exact ratio of the circumference to the diameter is not expressible.

*a.* Find the approximate circumference of a 5-inch circle. *b.* Of a 7-inch circle. *c.* Of a 10-inch circle.*

*d.* Find the approximate diameter of a circle that is 6 ft. in circumference.*

*e.* The circumference of a 6-inch circle is how many times the circumference of a 3-inch circle?

*f.* The diameter of a circle whose circumference is 12 inches is what part of the diameter of a circle whose circumference is 24 inches?

*g.* Find the circumference of a 10-inch pulley. *h.* Of a 15-inch pulley.

---

* In the solution of such problems as these the pupil may use, as the approximate ratio of the circumference to the diameter, 3.14.

*a.* Find the approximate circumference of a circle whose diameter is 3.4 meters.

*b.* Find the approximate diameter of a circle whose circumference is 3.4 decimeters.

*c.* Find the approximate diameter in yards of a circular 1-mile race track; of a half-mile track.

*d.* Find the approximate diameter in meters of a circular 1-kilometer race track; of a half-kilometer track.

*e.* A 28-inch bicycle wheel will make how many revolutions in running one mile?

*f.* A 70-centimeter bicycle wheel will make how many revolutions in running 1 kilometer? *

*g.* The 7-foot drive wheel of a locomotive will make how many revolutions per mile?

*h.* A 10-inch pulley is driven by a belt from a 25-inch pulley. If the 25-inch pulley makes 100 revolutions per minute, how many revolutions per minute will the 10-inch pulley make?

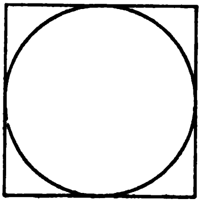
*i.* A 27-inch pulley is driven by a belt from a 6-inch pulley. If the 6-inch pulley makes 100 revolutions per minute, how many revolutions per minute will the 27-inch pulley make?

*j.* Find the inside diameter of a pail whose inside circumference is 24 inches.

*k.* The inside diameter of a certain round silo is 16 feet. The walls of the silo are one foot in thickness. Find the outside circumference of the silo.

---

* The exact answer to such questions as this cannot be found; but the approximation is practically correct.



**The circle and the square compared.**

**The circle is that which is *included within the circumference.***

1. If from a square piece of paper you cut the largest possible circle, what part of the paper will you cut away?

***A circle is a little more than ¾ of the smallest square from which it could have been cut.***

**Observe that the diameter of a circle is equal to the side of the smallest square from which it could have been cut.**

*a.* What is the area of a 1-inch circle?

*b.* What is the area of a 2-inch square? *c.* Of a 2-inch circle?

*d.* What is the area of a 3-inch square? *e.* Of a 3-inch circle?

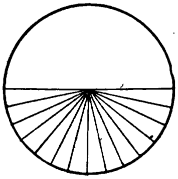
*f.* What is the area of a 4-inch square? *g.* Of a 4-inch circle?

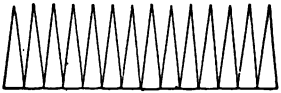
**Regarding ¾ as the ratio of the area of a circle to the area of the smallest square from which it could have been cut, first find the approximate area of each of the following without a pencil; then use .78* as the ratio and solve with a pencil.**

*h.* A 5-inch circle. *i.* A 6-inch circle.
*j.* A 7-inch circle. *k.* An 8-inch circle.
*l.* A 9-inch circle. *m.* A 10-inch circle.
*n.* A 12-inch circle. *o.* A 15-inch circle.
*p.* A 20-foot circle. *q.* A 40-foot circle.

---

* .78 is more nearly accurate than ¾.





**1.** Cut one half of a circular piece of paper as indicated in the diagram.

Observe that if the circle is cut into a very large number of parts and opened as shown in the figure, the circumference of the circle becomes, practically, a straight line.

Observe that a circle may be regarded as made up of an infinite number of triangles whose united bases equal the circumference and whose altitude equals the radius. Hence,

**To find the area of a circle:** I. *Multiply the circumference by* $\frac{1}{4}$ *of the diameter.*

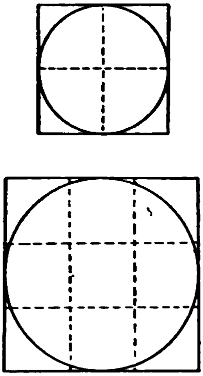
**2.** It has already been stated that if the diameter of a circle is 1, its circumference is $3.141592^+$. Hence the area of a circle whose diameter is 1 is $(3.141592^+ \times \frac{1}{4}) = .785398^+$.

**3.** A circle whose diameter is 2, is 4 times as large as a circle whose diameter is 1; a circle whose diameter is 3, 9 times as large, etc. Hence,

**To find the area of a circle:** II. *Multiply the square of the diameter by* $.785398^+$.

**4.** The approximate area may be found by taking $\frac{3}{4}$ (or .78) of the square of the diameter.

*a.* Using the ratio $\frac{3}{4}$, find the approximate area of a 48-foot circle. *b.* Using the ratio .78, find the approximate area of the same circle.



**The areas of two squares are to each other as the squares of their lengths.**

**The areas of two circles are to each other as the squares of their diameters.**

**Observe that the ratio of the areas of the above squares is $\frac{4}{9}$ (or $\frac{2^2}{3^2}$). But the area of each circle is $.78^+$ of its circumscribed square; so the ratio of the areas of the circles is $\frac{4}{9}$ (or $\frac{2^2}{3^2}$).**

*a.* The diameters of two circles are as 3 to 4. Compare their areas.*

*b.* The area of a 6-inch circle is how many times as great as the area of a 3-inch circle?

*c.* If a 4-inch circle of brass plate weighs 3 ounces, how much will a 6-inch circle weigh, the thickness being the same in each case?

*d.* If a piece of rolled dough 1 foot in diameter is enough for 17 cookies, how many cookies can be made from a piece 2 feet in diameter, the thickness of the dough and the size of the cookies being the same in each case?

*e.* If a piece of wire $\frac{1}{8}$ of an inch in diameter will sustain a weight of 1000 lb., how many pounds will a wire $\frac{1}{4}$ of an inch in diameter sustain?

---

* Their areas are as the square of 3 to the square of 4.

**1.** What is the area of a circle whose diameter is 1 foot? 1 yard? 1 rod? 1 mile?

*a.* What is the area of a circle whose diameter is 2 feet? *b.* 2 rods? *c.* 2 miles?

*d.* Find the approximate area of a circle whose diameter is 60 feet.

*e.* A horse is so fastened with a rope halter that he can feed over a circle 40 feet in diameter. Does he feed over more or less than 5 square rods?

*f.* Find the approximate length (in rods) of the side of a square containing 2 acres.

*g.* Find the approximate diameter (in rods) of a circle whose circumference is one mile.

*h.* Find the approximate circumference of a circle whose diameter is 30 rods.

*i.* The expression "a bicycle geared to 68" means that the machine is so geared that it will move forward at each revolution of the pedal shaft as far as a 68-inch wheel would move forward at one revolution. How far does a bicycle "geared to 68" move forward at each revolution of the pedal shaft? *j.* A bicycle "geared to 70"?

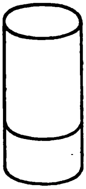
*k.* What is the approximate circumference of the largest circle that can be drawn on the floor of a room 40 ft. by 40 ft. if at its nearest points the circumference is 2 ft. from the edge of the floor?

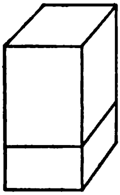
*l.* Which has the greater area, a 10-inch square or a 12-inch circle?

---

*** Require the pupil to make an estimate of the answer to each problem before attempting to solve it with the aid of a pencil.**

Observe that in any cylinder or right prism[*] the number of cubic units in one layer 1 unit high (as indicated in the diagrams) is equal to the number of square units in the area of the base. Thus, if there are 4½ square units in the area of the base there are 4½ cubic units in one layer. The content of the entire solid is as many times the cubic units in one layer, as the solid is linear units in height. Hence the rule as usually given:





**To find the solid content of a cylinder or of a right prism:** "*Multiply the area of the base by the altitude.*"

*To the Teacher:* This rule must be carefully interpreted by the pupil. He must not be allowed the misconception that area multiplied by any number can give solid content, except *through such interpretation as is suggested in the above observation.*

*a.* Find the solid content of a square right prism whose base is 6 in. by 6 in., and whose altitude is 8 inches.

*b.* Find the approximate solid content of a cylinder 6 inches in diameter and 10 inches long.

*c.* Find the approximate capacity in cubic inches of a cylindrical pail 5 inches in diameter and 6 inches deep. *d.* Is the capacity of the pail more or less than 2 liquid quarts?

---

[*] A **right prism** is a solid whose bases, or ends, are similar, equal, and parallel plane polygons, and whose lateral faces are perpendicular to its bases.

*a.* Find the solid content of an octagonal right prism the area of whose base is 24 square inches and whose altitude is 15 inches.

*b.* What is the solid content of a cylinder or of any right prism, the area of whose base is 30 square inches and whose altitude is 12 inches?

*c.* How many cubic feet of earth must be removed to dig a well 6 feet in diameter and 20 feet deep?*

*d.* Find the approximate number of feet of ½-in. lumber required to make the lining of the sides of a cylindrical silo that is 20 feet in diameter and 30 feet deep.

*e.* Find the approximate number of cords of rough stone in a cylindrical pile that is 16 feet in diameter and 6 feet deep.

*f.* Find the approximate number of brick necessary for a solid cylindrical foundation that is 9 feet in diameter and 4 feet high.†

*g.* If the brick and mortar used in the foundation described in problem *f*, weigh 1.9 times as much as the same bulk of water, how much does the entire foundation weigh?‡

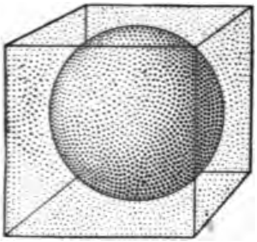
*h.* What is the weight of the water in a cylindrical tank, the diameter of the tank being 8 ft. and the depth of the water in the tank 6 ft. 4 in.? *i.* How many gallons of water in the tank?

---

* The exact number of cubic feet cannot be expressed in figures. An approximation that will answer many practical purposes may be obtained by regarding the circle (base) as ¾ of its circumscribed square. If an answer more nearly accurate is required, use .78 instead of ¾.

† Count 22 bricks to the cubic foot.

‡ Water weighs 62½ lb. to the cubic foot.



1. If from a cube of dried clay you cut the largest possible sphere, what part of the clay will you cut away?*

*A sphere is little more than ½† of the smallest cube from which it could have been cut.*

Observe that the diameter of the sphere equals the edge of the smallest cube from which it could have been cut.

2. What is the solid content of a 1-inch sphere?‡

*a.* What is the solid content of a 2-inch cube? *b.* Of a 2-inch sphere?

*c.* What is the solid content of a 3-inch cube? *d.* Of a 3-inch sphere?

Regarding ½ as the ratio of the solid content of a sphere to the solid content of the smallest cube from which it could have been cut, first find the approximate solid content of each of the following without a pencil; then use .52 as the ratio and solve with a pencil.

*e.* A 5-inch sphere.
*f.* A 6-inch sphere.
*g.* A 7-inch sphere.
*h.* An 8-inch sphere.
*i.* A 9-inch sphere.
*j.* A 10-inch sphere.

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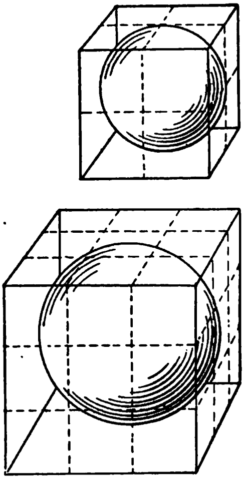
* Nearly one half of it.

† .52+.

‡ A little more than ½ a cubic inch; more accurately, a little more than .52 of a cubic inch.



The solid contents of two cubes are to each other as the cubes of their lengths.

The solid contents of two spheres are to each other as the cubes of their diameters.

Observe that the ratio of the solid contents of the above cubes is $\frac{8}{27}$ (or $\frac{27}{8}$). But the solid content of each sphere is .52$^+$ of its circumscribed cube; so the ratio of the solid contents of the spheres is $\frac{8}{27}$ (or $\frac{27}{8}$).

*a*. The diameters of two spheres are as 3 to 4. Compare their solid contents.*

*b*. The solid content of a 6-inch sphere is how many times as great as the solid content of a 3-inch sphere?

*c*. If a 4-inch sphere of brass weighs 10 lb., how many pounds will a 6-inch sphere of brass weigh?

*d*. If a sphere of dough 1 foot in diameter is enough for 20 loaves of bread, how many loaves can be made from a sphere of dough 2 feet in diameter?

*e*. If the half of a solid 8-inch globe weighs 4 lb., how much will the half of a solid 5-inch globe weigh, the material being of the same kind and quality?

* Their solid contents are as the cube of 3 to the cube of 4.

*a.* The diameters of two circles are as 4 to 5. Compare their areas.

*b.* The diameters of two spheres are as 4 to 5. Compare their solid contents.

*c.* The area of a 6-inch circle is how many times as great as the area of a 4-inch circle?

*d.* The solid content of a 6-inch sphere is how many times as great as the solid content of a 4-inch sphere?

*e.* The circumference of a 6-inch circle is how many times as great as the circumference of a 4-inch circle?

*f.* If it costs $3 to make a circular excavation 6 feet in diameter and 5 feet deep, how much would it cost, at the same rate per cubic foot removed, to make a circular excavation 8 feet in diameter and 5 feet deep?

*g.* The depth of two round cisterns is the same; but the diameter of one of them is 8 feet and of the other 10 feet. Compare them as to capacity.

*h.* I am thinking of two 10-foot iron rods. One of them is $\frac{5}{8}$ of an inch in diameter and the other $\frac{7}{8}$ of an inch. Compare them as to weight.

*i.* I am thinking of two small iron balls. One of them is $\frac{5}{8}$ of an inch in diameter and the other $\frac{7}{8}$ of an inch. Compare them as to weight.

*j.* I am thinking of two grindstones of equal thickness. One is 15 inches in diameter and the other 20 inches. Compare them as to weight.

*k.* A makes bricks that are 10 % thicker, 10 % wider, and 10 % longer than those made by B. Compare the bricks as to amount of material. *l.* Compare them as to surface.

*a.* If a pig whose girth is 2 feet weighs 50 pounds, what is the weight of a similarly proportioned pig whose girth is 4 feet?*

*b.* If a disk of dough 15 inches in diameter is sufficient for 20 doughnuts, how many such doughnuts can be made from a disk 30 inches in diameter?

*c.* The bore of a 10-inch gun is how many times as large as the bore of a 2-inch gun?

*d.* The ball of a 10-inch gun is how many times as large as the ball of a 2-inch gun?

*e.* A square, a pentagon, a hexagon, an octagon, and a circle have equal perimeters. Which has the greatest area? *f.* Which has the least area?

*g.* The capacity of a cistern 6 feet in diameter and 6 feet deep is about 40 barrels. What is the capacity of a cistern 12 feet in diameter and 12 feet deep?

*h.* A 2½-inch pipe is how many times as large as a 1-inch pipe?

*i.* If a man 6 feet tall weighs 190 pounds, how much would a similarly proportioned giant 12 feet tall weigh?*

*j.* In a certain orchard the trees are 15 feet apart each way, and there are 800 trees. How many trees in an orchard of equal size, the trees being 30 feet apart each way?

*k.* A ball of yarn 3 inches in diameter is sufficient for one mitten. How many mittens can be made from a ball 6 inches in diameter?

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* *The solid contents of similar solids are proportional to the cubes of their corresponding dimensions.*

Without the aid of a pencil (except for writing results) give approximate answers, sufficiently accurate for many practical purposes, to the following problems:

*a.* What is the area of a circle 20 inches in diameter?

*b.* What is the volume of a 12-inch globe?

*c.* What is the circumference of a circle that is 40 rods in diameter?

*d.* What is the capacity, in gallons, of a tank 6 ft. by 5 ft. by 2 ft.?

*e.* Is the capacity of a cylindrical pail 6 inches in diameter and 5 inches deep, more or less than ½ a gallon?

*f.* A piece of land 41 rods by 24 rods is how many acres?

*g.* What is the capacity in barrels (31½ gal.) of a tank 4 ft. by 4 ft. by 2 ft.?

*h.* What is the weight of one cord of green hickory wood?

*i.* What is the weight of a stick of seasoned white pine timber 12 in. by 12 in. and 30 feet long?

*j.* What is the weight of the water that will fill a cylindrical tank 8 feet in diameter and 8 feet deep?

*k.* A circular half-mile race track incloses how many acres?

*l.* If the 6-foot drive wheel of a locomotive makes 840 revolutions in moving a certain distance, how many revolutions will a 7-foot wheel make in moving the same distance?

*m.* Twenty-four kilometers is how many miles?

# SHORT METHODS IN MULTIPLICATION AND DIVISION

**1. To multiply a number by 50:** *Multiply* ½ *of the number by* 100. Why?

46 × 50   ½ of 46 × 100 = 2300  
47 × 50   ½ of 47 × 100 = 2350

Multiply:

44 by 50  
36 by 50  
64 by 50  
82 by 50  
32 by 50  
38 by 50  
42 by 50  
76 by 50  
35 by 50  
27 by 50  
55 by 50  
43 by 50

*a.* Find the sum of the twelve products.

**2. To multiply a number by 51:** *Take* 50 *times the number, to which add the number itself.*

48 × 51   50 times 48 = 2400   2400 + 48 = 2448  
37 × 51   50 times 37 = 1850   1850 + 37 = 1887

Multiply:

26 by 51  
46 by 51  
24 by 51  
66 by 51  
34 by 51  
32 by 51  
42 by 51  
84 by 51  
35 by 51  
29 by 51  
43 by 51  
33 by 51

*b.* Find the sum of the twelve products.

**3. To multiply a number by 52:** *Take* 50 *times the number, to which add twice the number.*

34 × 52   50 times 34 = 1700   1700 + 68 = 1768
45 × 52   50 times 45 = 2250   2250 + 90 = 2340

Multiply:

26 by 52
18 by 52
24 by 52
38 by 52
14 by 52
32 by 52
27 by 52
17 by 52
35 by 52

*a.* Find the sum of the nine products.

**4. To multiply a number by 49:** *Take* 50 *times the number, from which subtract the number itself.*

24 × 49   50 times 24 = 1200   1200 − 24 = 1176
33 × 49   50 times 33 = 1650   1650 − 33 = 1617

Multiply:

18 by 49
28 by 49
16 by 49
22 by 49
34 by 49
46 by 49
27 by 49
35 by 49
43 by 49

*b.* Find the sum of the nine products.

**5. To multiply a number by $33\frac{1}{3}$:** *Multiply* $\frac{1}{3}$ *of the number by* 100. Why?

$36 \times 33\frac{1}{3}$   $\frac{1}{3}$ of $36 \times 100 = 1200$
$37 \times 33\frac{1}{3}$   $\frac{1}{3}$ of $37 \times 100 = 1233\frac{1}{3}$

Multiply:

24 by $33\frac{1}{3}$
18 by $33\frac{1}{3}$
30 by $33\frac{1}{3}$
27 by $33\frac{1}{3}$
21 by $33\frac{1}{3}$
33 by $33\frac{1}{3}$
28 by $33\frac{1}{3}$
22 by $33\frac{1}{3}$
35 by $33\frac{1}{3}$

*c.* Find the sum of the nine products.

**6. To multiply a number by $34\frac{1}{3}$:** *Take $33\frac{1}{3}$ times the number, to which add the number itself.* Why?

| | | |
|---|---|---|
| $24 \times 34\frac{1}{3}$ | $33\frac{1}{3}$ times $24 = 800$ | $800 + 24 = 824$ |
| $25 \times 34\frac{1}{3}$ | $33\frac{1}{3}$ times $25 = 833\frac{1}{3}$ | $833\frac{1}{3} + 25 = 858\frac{1}{3}$ |

Multiply:

18 by $34\frac{1}{3}$
21 by $34\frac{1}{3}$
30 by $34\frac{1}{3}$
27 by $34\frac{1}{3}$
33 by $34\frac{1}{3}$
36 by $34\frac{1}{3}$
24 by $34\frac{1}{3}$
39 by $34\frac{1}{3}$
15 by $34\frac{1}{3}$

*a.* Find the sum of the nine products.

**7. To multiply a number by $35\frac{1}{3}$:** *Take $33\frac{1}{3}$ times the number, to which add twice the number.* Why?

| | | |
|---|---|---|
| $24 \times 35\frac{1}{3}$ | $33\frac{1}{3}$ times $24 = 800$ | $800 + 48 = 848$ |
| $25 \times 35\frac{1}{3}$ | $33\frac{1}{3}$ times $25 = 833\frac{1}{3}$ | $833\frac{1}{3} + 50 = 883\frac{1}{3}$ |

Multiply:

21 by $35\frac{1}{3}$
18 by $35\frac{1}{3}$
33 by $35\frac{1}{3}$
30 by $35\frac{1}{3}$
66 by $35\frac{1}{3}$
39 by $35\frac{1}{3}$
27 by $35\frac{1}{3}$
36 by $35\frac{1}{3}$
31 by $35\frac{1}{3}$

*b.* Find the sum of the nine products.

**8. To multiply a number by $32\frac{1}{3}$:** *Take $33\frac{1}{3}$ times the number, from which subtract the number itself.* Why?

| | | |
|---|---|---|
| $21 \times 32\frac{1}{3}$ | $33\frac{1}{3}$ times $21 = 700$ | $700 - 21 = 679$ |
| $22 \times 32\frac{1}{3}$ | $33\frac{1}{3}$ times $22 = 733\frac{1}{3}$ | $733\frac{1}{3} - 22 = 711\frac{1}{3}$ |

Multiply:

15 by $32\frac{1}{3}$
24 by $32\frac{1}{3}$
33 by $32\frac{1}{3}$
21 by $32\frac{1}{3}$
30 by $32\frac{1}{3}$
39 by $32\frac{1}{3}$
18 by $32\frac{1}{3}$
27 by $32\frac{1}{3}$
36 by $32\frac{1}{3}$

*c.* Find the sum of the nine products.

**9. To multiply a number by 25:** *Multiply* $\frac{1}{4}$ *of the number by* 100. Why?

$48 \times 25$ $\quad \frac{1}{4}$ of $48 \times 100 = 1200$
$49 \times 25$ $\quad \frac{1}{4}$ of $49 \times 100 = 1225$

Multiply:

36 by 25
40 by 25
24 by 25
32 by 25
28 by 25
16 by 25
33 by 25
29 by 25
19 by 25

*a.* Find the sum of the nine products.

**10. To multiply a number by 26:** *Take* 25 *times the number, to which add the number itself.* Why?

$36 \times 26$ $\quad$ 25 times $36 = 900$ $\quad 900 + 36 = 936$
$37 \times 26$ $\quad$ 25 times $37 = 925$ $\quad 925 + 37 = 962$

Multiply:

36 by 26
28 by 26
44 by 26
48 by 26
24 by 26
32 by 26
45 by 26
25 by 26
35 by 26

*b.* Find the sum of the nine products.

**11. To multiply a number by 27:** *Take* 25 *times the number, to which add twice the number.* Why?

$36 \times 27$ $\quad$ 25 times $36 = 900$ $\quad 900 + 72 = 972$
$37 \times 27$ $\quad$ 25 times $37 = 925$ $\quad 925 + 74 = 999$

Multiply:

48 by 27
32 by 27
28 by 27
52 by 27
16 by 27
24 by 27
37 by 27
17 by 27
26 by 27

*c.* Find the sum of the nine products.

**12. To multiply a number by 24:** *Take* 25 *times the number, from which subtract the number itself.* Why?

$32 \times 24$ 25 times $32 = 800$ $800 - 32 = 768$

$33 \times 24$ 25 times $33 = 825$ $825 - 33 = 792$

Multiply:

24 by 24
44 by 24
28 by 24
16 by 24
36 by 24
32 by 24
17 by 24
37 by 24
35 by 24

*a.* Find the sum of the nine products.

**13. To multiply a number by 20:** *Multiply* $\frac{1}{5}$ *of the number by* 100. How may a number be multiplied by 21? By 22? By 19?

$35 \times 21$ 20 times $35 = 700$ $700 + 35 = 735$

Multiply:

45 by 21
35 by 21
36 by 21
45 by 22
35 by 22
36 by 22
45 by 19
35 by 19
36 by 19

*b.* Find the sum of the nine products.

**14. To multiply a number by $16\frac{2}{3}$:** *Multiply* $\frac{1}{6}$ *of the number by* 100. How may a number be multiplied by $17\frac{2}{3}$? By $18\frac{2}{3}$? By $15\frac{2}{3}$?

$24 \times 17\frac{2}{3}$ $16\frac{2}{3}$ times $24 = 400$ $400 + 24 = 424$

Multiply:

18 by $17\frac{2}{3}$
30 by $17\frac{2}{3}$
36 by $17\frac{2}{3}$
18 by $18\frac{2}{3}$
30 by $18\frac{2}{3}$
36 by $18\frac{2}{3}$
18 by $15\frac{2}{3}$
30 by $15\frac{2}{3}$
36 by $15\frac{2}{3}$

*c.* Find the sum of the nine products.

**15. To multiply a number by 12½:** *Multiply ⅛ of the number by* 100. How may a number be multiplied by 13½? By 14½? By 11½?

32 × 13½ 12½ times 32 = 400 400 + 32 = 432

Multiply:

24 by 13½ 24 by 14½ 24 by 11½
16 by 13½ 16 by 14½ 16 by 11½
40 by 13½ 40 by 14½ 40 by 11½

*a.* Find the sum of the nine products.

**16. To multiply a number by 125:** *Multiply ⅛ of the number by* 1000. How may a number be multiplied by 126? By 127? By 124?

96 × 125 ⅛ of 96 × 1000 = 12000
96 × 126 125 times 96 = 12000 12000 + 96 = 12096

Multiply:

120 by 126 120 by 127 120 by 124
320 by 126 320 by 127 320 by 124
240 by 126 240 by 127 240 by 124

*b.* Find the sum of the nine products.

**17. To multiply a number by 250:** *Multiply ¼ of the number by* 1000. How may a number be multiplied by 251? By 252? By 249?

48 × 251 250 times 48 = 12000 12000 + 48 = 12048

Multiply:

60 by 251 60 by 252 60 by 249
72 by 251 72 by 252 72 by 249
96 by 251 96 by 252 96 by 249

*c.* Find the sum of the nine products.

**18. To square $2\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$, etc.:** *Multiply the integer by the integer plus* 1, *and add* $\frac{1}{4}$ *to the product.*

$$2\tfrac{1}{2} \times 2\tfrac{1}{2} = \overline{2 \text{ times } 2} + \overline{2 \text{ times } \tfrac{1}{2}} + \overline{\tfrac{1}{2} \text{ of } 2} + \overline{\tfrac{1}{2} \text{ of } \tfrac{1}{2}}$$

But $\overline{2 \text{ times } \frac{1}{2}} + \overline{\frac{1}{2} \text{ of } 2} = 1$ time 2; and $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{4}$

Hence, $2\frac{1}{2} \times 2\frac{1}{2} = \overline{2 \times 3} + \frac{1}{4} = 6\frac{1}{4}$

$3\frac{1}{2} \times 3\frac{1}{2} = \overline{3 \times 4} + \frac{1}{4} = 12\frac{1}{4}$

Multiply:

$4\frac{1}{2}$ by $4\frac{1}{2}$ $5\frac{1}{2}$ by $5\frac{1}{2}$ $6\frac{1}{2}$ by $6\frac{1}{2}$

$7\frac{1}{2}$ by $7\frac{1}{2}$ $8\frac{1}{2}$ by $8\frac{1}{2}$ $9\frac{1}{2}$ by $9\frac{1}{2}$

$1\frac{1}{2}$ by $1\frac{1}{2}$ $2\frac{1}{2}$ by $2\frac{1}{2}$ $3\frac{1}{2}$ by $3\frac{1}{2}$

*a.* Find the sum of the nine products.

**19. To square 25, 35, 45, etc.:** *Multiply the tens' figure* * *by the tens' figure increased by* 1; *regard the product as hundreds, to which add* 25.

To explain this rule, think of 25 as 2 *tens and* $\frac{1}{2}$ *of a ten*, and apply the explanation given under preceding article.

$25 \times 25 = \overline{2 \times 3}$ hundred and $25 = 625$

$35 \times 35 = \overline{3 \times 4}$ hundred and $25 = 1225$

$45 \times 45 = \overline{4 \times 5}$ hundred and $25 = 2025$

Multiply:

55 by 55 65 by 65 75 by 75

85 by 85 95 by 95 15 by 15

*b.* Find the sum of the six products.

---

* The author is aware that the expressions "*Multiply the tens' figure*" and "*the tens' figure increased by 1*" are tabooed by the hypercritical. But it is believed that neither obscurity nor misconception will arise from this use of the word *figure*. The word as here used clearly means the *form value of the figure* — the *number* which the figure by virtue of its shape represents.

**20. To multiply $2\frac{1}{4}$ by $2\frac{3}{4}$, $3\frac{1}{5}$ by $3\frac{4}{5}$, etc.:** *Multiply the integer by the integer plus* 1, *and to the product add the product of the fractions.*

Observe that this rule will apply only when the integer of the multiplicand and the integer of the multiplier are the same, and the sum of the fractions is 1.

$$2\tfrac{1}{4} \text{ by } 2\tfrac{3}{4} = \overline{2 \text{ times } 2} + \overline{2 \text{ times } \tfrac{1}{4}} + \overline{\tfrac{3}{4} \text{ of } 2} + \overline{\tfrac{3}{4} \text{ of } \tfrac{1}{4}}$$

$$\text{But } \overline{2 \text{ times } \tfrac{1}{4}} + \overline{\tfrac{3}{4} \text{ of } 2} = 1 \text{ time } 2, \text{ and } \tfrac{3}{4} \text{ of } \tfrac{1}{4} = \tfrac{3}{16}$$

$$\text{Hence } 2\tfrac{1}{4} \times 2\tfrac{3}{4} = \overline{2 \times 3} + \overline{\tfrac{3}{4} \text{ of } \tfrac{1}{4}} = 6\tfrac{3}{16}$$

$$3\tfrac{1}{5} \times 3\tfrac{4}{5} = \overline{3 \times 4} + \tfrac{4}{5} \text{ of } \tfrac{1}{5} = 12\tfrac{4}{25}$$

$$2\tfrac{1}{3} \times 2\tfrac{2}{3} = \overline{2 \times 3} + \tfrac{2}{3} \text{ of } \tfrac{1}{3} = 6\tfrac{2}{9}$$

Multiply:

| | | |
|---|---|---|
| $4\frac{1}{3}$ by $4\frac{2}{3}$ | $5\frac{1}{4}$ by $5\frac{3}{4}$ | $6\frac{1}{3}$ by $6\frac{2}{3}$ |
| $7\frac{1}{4}$ by $7\frac{3}{4}$ | $8\frac{2}{3}$ by $8\frac{1}{3}$ | $9\frac{1}{4}$ by $9\frac{3}{4}$ |

*a.* Find the sum of the six products.

**21. To multiply 24 by 26, 33 by 37, etc.:** *Multiply the tens' figure by the tens' figure increased by* 1; *regard the product as hundreds, to which add the product of the units' figures.*

Observe that this rule will apply only when the tens' figure of the multiplicand and the tens' figure of the multiplier are alike, and the sum of the units' figures is 10.

$$22 \times 28 = \overline{2 \times 3} \text{ hundred and } 16 = 616$$

$$33 \times 37 = \overline{3 \times 4} \text{ hundred and } 21 = 1221$$

Multiply:

| | | |
|---|---|---|
| 21 by 29 | 23 by 27 | 24 by 26 |
| 31 by 39 | 32 by 38 | 34 by 36 |

*b.* Find the sum of the six products.

**22. To multiply a number by 15:** *Multiply the number by* 10, *and to the product add* $\frac{1}{2}$ *of the product.*

64 × 15  10 times 64 = 640  640 + 320 = 960
45 × 15  10 times 45 = 450  450 + 225 = 675

Multiply:

24 by 15
46 by 15
82 by 15
32 by 15
34 by 15
66 by 15
35 by 15
43 by 15
75 by 15

*a.* Find the sum of the nine products.

**23. To multiply a number by 99:** *Take* 100 *times the number, from which subtract the number itself.* How may a number be multiplied by 98?

36 × 99  100 times 36 = 3600  3600 − 36 = 3564
42 × 98  100 times 42 = 4200  4200 − 84 = 4116

Multiply:

35 by 99
27 by 99
62 by 99
44 by 99
54 by 99
75 by 99
35 by 98
46 by 98
28 by 98

*b.* Find the sum of the nine products.

**24. To multiply a number by 75:** *Multiply* $\frac{3}{4}$ *of the number by* 100. How may a number be multiplied by $66\frac{2}{3}$? By $62\frac{1}{2}$? By $87\frac{1}{2}$?

Multiply:

64 by 75
48 by 75
52 by 75
24 by $66\frac{2}{3}$
36 by $66\frac{2}{3}$
63 by $66\frac{2}{3}$
64 by $87\frac{1}{2}$
48 by $87\frac{1}{2}$
56 by $37\frac{1}{2}$

*c.* Find the sum of the nine products.

**25. To divide a number by 25; by $33\frac{1}{3}$; by $12\frac{1}{2}$; by $16\frac{2}{3}$; by 20; by 50.** (See pp. 44 and 45, of this book.)

$$850 \div 25 = \overline{8 \text{ times } 4} + 2 = 34$$
$$933\tfrac{1}{3} \div 33\tfrac{1}{3} = \overline{9 \text{ times } 3} + 1 = 28$$
$$637\tfrac{1}{2} \div 12\tfrac{1}{2} = \overline{6 \text{ times } 8} + 3 = 51$$
$$750 \div 16\tfrac{2}{3} = \overline{7 \text{ times } 6} + 3 = 45$$
$$960 \div 20 = \overline{9 \text{ times } 5} + 3 = 48$$
$$450 \div 50 = \overline{4 \text{ times } 2} + 1 = 9$$

Divide:

1275 by 25
950 by $16\frac{2}{3}$
800 by $16\frac{2}{3}$
$1166\frac{2}{3}$ by $33\frac{1}{3}$
880 by 20
800 by $12\frac{1}{2}$
$762\frac{1}{2}$ by $12\frac{1}{2}$
950 by 50
800 by $33\frac{1}{3}$

*a.* Find the sum of the nine quotients.

Note. Without a pencil, tell the integral quotient and the remainder resulting from the incomplete division of 1584 by 25.

**26. To divide a number by 125; by 250:** *Observe that* 125 *is contained in each thousand of a number*, 8 *times; that* 250 *is contained in each thousand of a number*, 4 *times.*

$$7125 \div 125 = \overline{7 \text{ times } 8} + 1 = 57$$
$$8500 \div 250 = \overline{8 \text{ times } 4} + 2 = 34$$

Divide:

13250 by 250
12000 by 125
15000 by 125
13500 by 250
12500 by 125
15000 by 250
13750 by 250
12625 by 125
15000 by 500

*b.* Find the sum of the nine quotients.

Note. Without a pencil, tell the integral quotient and the remainder resulting from the incomplete division of 15450 by 250.

NOTE. In this discussion and in the solution of problems in this connection, all the numbers are regarded as abstract.

**1.** Divide 180 by 42.

OPERATION NO. 1

42)180(4$\frac{2}{7}$
168
$\frac{12}{42}=\frac{2}{7}$

OPERATION NO. 2

$$\frac{180}{42}=\frac{\cancel{2}\times 2\times \cancel{3}\times 3\times 5}{\cancel{2}\times \cancel{3}\times 7}=\frac{30}{7}=4\frac{2}{7}$$

Observe that the striking out of the factors 2 and 3 from the dividend and its divisor does not change their ratio — the quotient.

**2.** Divide 420 by 35.

OPERATION NO. 1

35)420(12
35
70
70

OPERATION NO. 2

$$\frac{420}{35}=\frac{2\times 2\times 3\times \cancel{5}\times \cancel{7}}{\cancel{5}\times \cancel{7}}=\frac{12}{1}=12$$

Observe that if all the factors of one of the numbers are canceled the number becomes 1 and not 0. The factor 5 is 5 times 1; the factor 7, 7 times 1. Hence in the above problem there really remain in the divisor, after the cancellation, the factors 1 and 1 = 1 × 1 = 1.

**3.** Divide 48 × 8 × 4 = 1536 by 8 × 4 × 4 = 128.

OPERATION NO. 1

128)1536(12
128
256
256

OPERATION NO. 2

$$\frac{\overset{12}{\cancel{48}}\times \cancel{8}\times \cancel{4}}{\cancel{8}\times \cancel{4}\times \cancel{4}}=\frac{12}{1}=12$$

Observe that it is not necessary to obtain the *prime* factors of a dividend and its divisor to employ cancellation in finding the quotient. In the above the composite factor 4 is stricken out of the divisor and out of the 48 of the dividend.

*a.* Divide 36 × 7 × 4 × 9 by 2 × 7 × 3 × 2.

*b.* Divide 12 × 15 × 24 × 21 by 5 × 8 × 7.

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* See page 74.

1. Divide 56 × 35 = 1960 by 15 × 8 = 120.

OPERATION NO. 1

$$\begin{array}{l} 120)1960(16\frac{1}{3} \\ \quad\underline{120} \\ \quad\ \ 760 \\ \quad\ \ \underline{720} \\ \quad\ \ \ \frac{40}{120}=\frac{1}{3} \end{array}$$

OPERATION NO. 2

$$\frac{\overset{7}{\cancel{56}} \times \overset{7}{\cancel{35}}}{\underset{3}{\cancel{15}} \times \cancel{8}} = \frac{49}{3} = 16\frac{1}{3}$$

Observe that in the above the factor 5 is stricken out of 15 and 35, and the factor 8 is stricken out of the divisor and out of the 56 of the dividend.

How many cords of wood :

*a.* In a pile 32 feet by 8 feet by 4 feet ?*

*b.* In a pile 40 feet by 16 feet by 6 feet ?

*c.* In a pile 32 feet by 30 feet by 10 feet ?

*d.* Find the sum of the three results, *a* to *c*.

How many acres of land :

*e.* In a piece 180 rods by 28 rods ? †

*f.* In a piece 64 rods by 96 rods ?

*g.* In a piece 136 rods by 32 rods ?

*h.* Find the sum of the three results, *e* to *g*.

*i.* Multiply 64 by 96 and divide the product by 16 × 24 × 2.

*j.* Multiply 250 by 72 and divide the product by $16\frac{2}{3}$ × 3 × 24.

*k.* Multiply 125 by 48 and divide the product by 25 × 3 × 8.

*l.* Find the sum of the three results, *i* to *k*.

---

* Think of a cord as 8 feet by 4 feet by 4 feet.

† Think of an acre as 40 rods by 4 rods.

Find the cost:

*a*. Of 346 acres of land at $50 per acre.
*b*. Of 346 acres of land at $51 per acre.
*c*. Of 346 acres of land at $52 per acre.
*d*. Of 346 acres of land at $49 per acre.
*e*. Find the sum of the four results, *a* to *d*.

*f*. Of 243 ft. iron pipe at $33\frac{1}{3}$¢ a foot.
*g*. Of 243 ft. iron pipe at $34\frac{1}{3}$¢ a foot.
*h*. Of 243 ft. iron pipe at $35\frac{1}{3}$¢ a foot.
*i*. Of 243 ft. iron pipe at $32\frac{1}{3}$¢ a foot.
*j*. Find the sum of the four results, *f* to *i*.

*k*. Of 260 lb. butter at 25¢ a pound.
*l*. Of 260 lb. butter at 26¢ a pound.
*m*. Of 260 lb. butter at 27¢ a pound.
*n*. Of 260 lb. butter at 24¢ a pound.
*o*. Find the sum of the four results, *k* to *n*.
*p*. Of 350 lb. coffee at $12\frac{1}{2}$¢ a pound.
*q*. Of 350 lb. coffee at $13\frac{1}{2}$¢ a pound.
*r*. Of 350 lb. coffee at $14\frac{1}{2}$¢ a pound.
*s*. Of 350 lb. coffee at $11\frac{1}{2}$¢ a pound.
*t*. Of 330 lb. coffee at $16\frac{2}{3}$¢ a pound.
*u*. Of 330 lb. coffee at $17\frac{2}{3}$¢ a pound.
*v*. Of 330 lb. coffee at $15\frac{2}{3}$¢ a pound.
*w*. Of 240 lb. coffee at 25¢ a pound.
*x*. Of 240 lb. coffee at 26¢ a pound.
*y*. Of 240 lb. coffee at 27¢ a pound.
*z*. Find the sum of the ten results, *p* to *y*.

Find the cost:

*a.* Of 2½ tons coal at $ 2½ per ton.

*b.* Of 3½ tons coal at $ 3½ per ton.

*c.* Of 4½ tons coal at $ 4½ per ton.

*d.* Find the sum of the three results, *a* to *c*.

*e.* Of 25 tons of meal at $ 25 per ton.

*f.* Of 35 acres of land at $ 35 per acre.

*g.* Of 45 M ft. of lumber at $ 45 per M.

*h.* Find the sum of the three results, *e* to *g*.

*i.* Of 23 yd. cloth at 27¢ a yard.

*j.* Of 36 yd. cloth at 34¢ a yard.

*k.* Of 42 yd. cloth at 48¢ a yard.

*l.* Find the sum of the three results, *i* to *k*.

*m.* Of 3240 ft. lumber at $ 15 per M.

*n.* Of 2460 ft. lumber at $ 15 per M.

*o.* Of 1620 ft. lumber at $ 16 per M.

*p.* Find the sum of the three results, *m* to *o*.

*q.* Of 99 lb. butter at 23¢ a pound.

*r.* Of 99 lb. butter at 28¢ a pound.

*s.* Of 98 lb. butter at 24¢ a pound.

*t.* Find the sum of the three results, *q* to *s*.

*u.* Paid $ 15.50 for ribbon at 16⅔¢ a yard. How many yards did I buy?

*v.* Paid $ 24.75 for ribbon at 12½¢ a yard. How many yards did I buy?

*w.* Paid $ 36.50 for ribbon at 25¢ a yard. How many yards did I buy?

*x.* Find the sum of the three results, *u* to *w*.

# BUSINESS PROBLEMS

## THE EXTENSION CLERK

NOTE. In large wholesale houses, clerks are sometimes employed in *extending bills.* These clerks receive the bills upon which the amount, description, and price of the goods sold are given, and *extend* each line; that is, they carry into the proper place the value of each purchase. They also add the columns of figures representing these values.

CHICAGO, June 12, 1907.

*Illinois School for the Blind,*
*Jacksonville, Ill.*

Bought of DURAND & KASPER CO.

| | | | | A | | B |
|---|---|---|---|---|---|---|
| 1 | box Oatmeal Wafers . . . . . . | 26½ | .07½ | * – | – | |
| 1 | Cheddar Cheese . . . . . . . . | 45 | .12½ | – | – | |
| 1 | bag Rio Coffee . . . . . . . . | 150 | .16½ | – | – | |
| 1 | bbl. Granulated Sal Soda . . . . | 325 | .70 | † – | – | |
| 5 | cases Elgin Corn . . . . . . . . | 5/2 | 1.10 | ‡ – | – | |
| 8 | " Columbian Corn . . . . . . | 8/2 | .80 | – | – | |
| 5 | " Wayne Co. Peas gal . . . | 5/1 | 4.60 | – | – | |
| 2 | " 2 lb Salmon . . . . . . . | 2/2 | 2.85 | – | – | |
| 3 | " Tomatoes gal . . . . . . . | 3/1 | 2.50 | – | – | |
| 1 | " Col. Riv. Salmon . . . . . | 1/4 | 1.40 | – | – | |
| 1 | box Oyster Crackers . . . . . . | 22 | .06½ | – | – | |
| ¼ | chest Japan Tea . . . . . . . . | 40 | .35 | – | – | |
| 5 | bags Navy Beans . . . . . . . | 764 | 2.00 | § – | – | |

*a.* Copy, extend, and find the amount of the above bill. The dashes indicate where the extension figures should appear. Put the amount of the bill in column B, opposite the last line in column A.

* 26½ lb. @ 7½ ¢ a pound. † 325 lb. @ 70 ¢ per cwt. ‡ 5 cases; 2 doz. in a case; $1.10 a dozen. § 764 lb. @ $2.00 a bushel.

CHICAGO, June 13, 1907.

*Mr. Ralph T. Dodge,*
*Earlville, Illinois.*

Terms: 10 days less 6%.
30 days less 5%.
60 days less 3%.
After 60 days, net.

Bought of A. C. McCLURG & Co.
Salesman, *Gould.*

| | | | A | | B | |
|---|---|---|---|---|---|---|
| 1 | Ebers . . | In the Desert . . . . . . . . . | 1 | 50 | | |
| 2 | Ruskin . . | Stones of Venice . . . . . 4.50 | * – | – | | |
| 2 | " . . | Seven Lamps of Architecture 1.50 | – | – | | |
| 1 | Coolidge . | What Katy Did . 1.25 15% . . | | | † – | – |
| 1 | Herrick . | Web of Life . . . . . . . . | 1 | 50 | | |
| 2 | Warner .. | Golden House . . . . . . 2.00 | – | – | | |
| 2 | Potter . . | Uncanonized . . 1.5020 % . . | | | – | – |
| 2 | Thompson | Political Economy 1.00 6% . . | | | – | – |
| 3 | Steel . . . | Hosts of the Lord . . . . 1.50 | – | – | | |
| 2 | Besant . . | Fourth Generation . . . . 1.50 | – | – | | |
| 1 | Garland . | Eagle's Heart . . . . . . . . | 1 | 50 | | |
| 2 | Seawell . | House of Egremont . . . . 1.50 | – | – | | |
| 2 | Harris . . | On the Wing of Occasions . 1.50 | – | – | | |
| 1 | Taylor . . | Cobbler of Nimes 1.25 20% . . | | | – | – |
| 2 | Putnam . | Book of Common Worship 1.25 10% | | | – | – |
| 4 | Ade . . . | Fables in Slang . . . . . 1.00 | – | – | | |
| | | | – | – | | |
| | | Less 25 % . . | ‡ – | – | § – | – |
| | | | | | – | – |

*a.* Copy, extend, and find the amount of the above bill. *b.* How much must Mr. Dodge remit if paid in 10 days? *c.* In 30 days? *d.* In 60 days? *e.* In 75 days?

NOTE. In this bill the *list price* of all books upon which the discount is exactly 25% should appear in column A. The *net price* of all other books is carried directly into column B. The sum of column A less 25% of itself is carried into column B. The sum of column B is the *amount* of the bill; but a special discount, as shown at the top of the bill, is allowed for prompt payment. The dashes indicate where the extension figures should appear.

* Two books @ $4.50 each. † $1.25 less 15% of itself. ‡ 25% of the footing of column A. § The footing of column A less 25% of itself.

*Dr. J. C. Gordon,* CHICAGO, June 14, 1901.
*Jacksonville, Ill.*
Bought of JAMES B. CLOW & SONS.
Salesman, *Hamilton.*

| | | Price | A | | B | |
|---|---|---|---|---|---|---|
| 24 | ½″ C. I. Ells . . . . . . . . . | .06 | * – | – | | |
| 36 | ¾″ " " " . . . . . . . . . | .08 | – | – | | |
| 36 | 1″ " " " . . . . . . . . . | .10½ | – | – | | |
| 24 | ¾″ " " Tees . . . . . . . . | .12 | – | – | | |
| 36 | 1″ " " " . . . . . . . . . | .15 | – | – | | |
| | 30, 10, & 5% off | | *a* – | – | *b* – | – |
| 6 | 1½″ Flange Unions . . . . . | .78 | – | – | | |
| 12 | 2″ " " . . . . . . | 1.00 | – | – | | |
| 6 | 2½″ " " . . . . . . | 1.25 | – | – | | |
| 3 | 4″ " " . . . . . . | 2.10 | – | – | | |
| | 35% off | | *a* – | – | *b* – | – |
| † 507′ | 0″ ¾″ Black Pipe . . . . . . | .11½ | – | – | | |
| 519′ | 0″ 1″ " " . . . . . . | .16½ | – | – | | |
| 209′ | 4″ 1½″ " " . . . . . . | .27 | – | – | | |
| 101′ | 1″ 3″ " " . . . . . . | .75½ | – | – | | |
| | 38.2% off | | *a* – | – | *b* – | – |
| 6 | ¾″ Globe Valves . . . . . . . | 2.20 | – | – | | |
| 1 | 3″ " " . . . . . . . . | 16.00 | – | – | | |
| | 50 & 10% off | | *a* – | – | *b* – | – |
| | Boxing & Drayage | | | | | 45 |
| | | | | | – | – |

*a.* Copy, extend, and find the amount of the above bill.

NOTE. In this bill *list prices* are given. In column A, values according to list prices are to appear; in column B, net values. The dashes indicate where the extension figures should appear.

* 24 ½-inch cast-iron ells at 6 ¢ each. † 507 feet, 0 inches.

*a* & *b*. The footing should appear at *a*; the footing, less the discount, at *b*.

*Mr. J. H. Collins,* CHICAGO, June 15, 1907.
*Springfield, Illinois.*

Terms: 10 days less 2%. Bought of MARSHALL FIELD & Co.
30 days less 1%. Salesman, *Brett.*
60 days no discount.

| | | | Amt. | Price | A | | B | |
|---|---|---|---|---|---|---|---|---|
| * | 1 | Muslin . . . Old Glory . . . . . | 52 | .06 | *a* – | – | | |
| | 1 | " Dwight . . . . . | 52 | .07 | *b* – | – | | |
| | 1 | " Aurora . . . . . | 52 | .03¾ | *c* – | – | *d* – | – |
| | 1 | Calico . . . Katrina . . . . . | 44 | .05 | – | – | | |
| | 1 | " Simpson . . . . . | 42 | .04¾ | – | – | – | – |
| | 3 | Gingham 32 30 26 . . . . . | 88 | .08 | – | – | – | – |
| † | 4 | Knitting Cotton . . . . . . .<br>30 & 10 % off | 4 | .65 | *e* –<br>*f* – | –<br>– | *g* – | – |
| | 7 | Coats' Thread . . . . . . . .<br>7 % off | 7<br>. | .55 | *e* –<br>*f* – | –<br>– | *g* – | – |
| ‡ | 2 | Towel . . . No. 726 . . . . . . . | 2 | 1.37½ | – | – | | |
| § | 6 | Crash . . . 6/40 . . . . . . . . | 240 | .12¼ | – | – | – | – |
| | 2 | Brush . . . No. 3642 . . . . . | 2 | 1.15 | – | – | | |
| | 2 | Comb . . . No. 7260 . . . . . | 2 | 1.05 | – | – | – | – |
| | | | | | | | – | – |

*a.* Copy, extend, and find the amount of the above bill.

*b.* How much must Mr. Collins remit if paid in 10 days or less?

*c.* How much if paid in 30 days?

*d.* How much if paid in 60 days?

* 1 piece of muslin containing 52 yd. @ 6¢ a yard.

† 4 dozen balls @ 65¢ a dozen.

‡ 2 dozen towels, No. 726, @ $1.37½ per dozen.

§ 6 pieces crash, 40 yd. in each piece.

*a*, *b*, *c*, *d*, etc. The sum of *a*, *b*, and *c* should appear at *d*; *f* is the discount, *g* is *e* less the discount.

Tax extension clerks compute what each property holder must pay for the several purposes for which taxes are exacted. To aid in doing this work extensive tables are usually provided by which the tax on any assessed value at any given rate may be found without arithmetical process — by mere reference to the tables.

In the tax extension required on page 270, the following substitute for these tables may be employed to advantage. This table shows the tax on any number of hundred dollars from 1 to 9. It is apparent that the tax on any number of thousands of dollars would be ten times as much as the amount shown in the table; on any number of tens of dollars, one tenth as much, etc. "R. & B." stands for *Road and Bridge;* "Lib." for *Library.*

TAXES IN SCHOOL DISTRICT No. 4, AURORA, ILL.

| Assessed value. | State. | County. | Bond. | Town. | R. & B. | Lib. | City. | School. | Total. |
|---|---|---|---|---|---|---|---|---|---|
| $100 | .50 | .75 | .16 | .08 | .35 | .15 | 2.21 | 2.31 | 6.51 |
| 200 | 1.00 | 1.50 | .32 | .16 | .70 | .30 | 4.42 | 4.62 | 13.02 |
| 300 | 1.50 | 2.25 | .48 | .24 | 1.05 | .45 | 6.63 | 6.93 | 19.53 |
| 400 | 2.00 | 3.00 | .64 | .32 | 1.40 | .60 | 8.84 | 9.24 | 26.04 |
| 500 | 2.50 | 3.75 | .80 | .40 | 1.75 | .75 | 11.05 | 11.55 | 32.55 |
| 600 | 3.00 | 4.50 | .96 | .48 | 2.10 | .90 | 13.26 | 13.86 | 39.06 |
| 700 | 3.50 | 5.25 | 1.12 | .56 | 2.45 | 1.05 | 15.47 | 16.17 | 45.57 |
| 800 | 4.00 | 6.00 | 1.28 | .64 | 2.80 | 1.20 | 17.68 | 18.48 | 52.08 |
| 900 | 4.50 | 6.75 | 1.44 | .72 | 3.15 | 1.35 | 19.89 | 20.79 | 58.59 |

1. How much is Mr. Smith's school tax if he lives in School District No. 4, Aurora, Ill., and if his property is assessed at $1450?

| | |
|---|---|
| Tax on $100 (table) $2.31; on $1000, | $23.10 |
| Tax on $400 (table), | 9.24 |
| Tax on $100 (table) $2.31; on $50, | 1.16* |
| Total school tax, | $33.50 |

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* In many states it is provided by law that "*In the extension of taxes, the fraction of a cent shall be extended as one cent.*"

### TAXES IN SCHOOL DISTRICT NO. 4, AURORA, ILLINOIS

| | Value. | | State. | County. | Bond. | Town. | R. & B. | Library. | City. | School. | Total. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| A . . . . . . . | $736 | 00 | | | | | | | | | |
| B . . . . . . . | 845 | 00 | | | | | | | | | |
| C . . . . . . . | 2621 | 00 | | | | | | | | | |
| D . . . . . . . | 524 | 00 | | | | | | | | | |
| E . . . . . . . | 1389 | 00 | | | | | | | | | |
| F . . . . . . . | 621 | 00 | | | | | | | | | |
| G . . . . . . . | 235 | 00 | | | | | | | | | |
| H . . . . . . . | 5175 | 00 | | | | | | | | | |
| I . . . . . . . | 811 | 00 | | | | | | | | | |
| J . . . . . . . | 375 | 00 | | | | | | | | | |

1. Make an enlarged copy of the above, upon which extend the taxes of A, B, C, etc.

NOTE. It will be well for the student to find each tax item (1) by means of the table given on the preceding page, and (2) by finding the product of the assessed value and the rate. The two results should be identical.

Find the total of each person's tax by adding the eight tax items. Prove the work by finding the tax of each person at $6.51 per hundred dollars of assessed valuation. The amount thus found will be a few cents less than the amount found by adding. See foot-note on page 269.

It is sometimes necessary in business to find the compound amount of money at interest. This is done by adding the interest to the principal at the end of a specified period and using the sum thus found as a new principal upon which the interest is computed and the amount obtained as before. This process is repeated as many times as there are interest-bearing periods. To find the compound interest, subtract the original principal from the compound amount.

*a*. What is the compound amount of $ 200 for 2 years, interest at 6 % per annum to be compounded annually?

*b*. What is the compound interest of $200 for 2 years, at 6 % per annum to be compounded annually?

*c*. How much more is the compound interest of $200 for 2 years at 6 %, than the simple interest of the same sum for the same time and rate? Why?

*d*. Without solving the two interest problems in full, tell how much more the compound amount of $ 500 for 2 years at 6 % is, than the simple amount of the same sum for the same time and rate.

*e*. Find the compound amount of $ 350 for 2 yr. 6 mo. at 6 % per annum, interest to be compounded annually.*

*f*. Find the compound amount of $ 350 for 2 yr. 6 mo. at 6 % per annum, interest to be compounded semi-annually.† Compare the answer to this problem with the answer to problem *e*.

*g*. Find the compound amount of $ 525.40 for 2 yr. 4 mo. 12 da. at 6 %, interest to be compounded annually.

---

* Interest must be computed for three periods — the first two, one year each, and the third, six months.

† This calls for the compound amount for five periods at 3 %.

**Showing the compound amount of $1.00 at 2, 3, 4, 5, and 6 per cent, for any number of periods from 1 to 20.**

| | 2 per cent. | 3 per cent. | 4 per cent. | 5 per cent. | 6 per cent. |
|---|---|---|---|---|---|
| 1 | 1.020,000 | 1.030,000 | 1.040,000 | 1.050,000 | 1.060,000 |
| 2 | 1.040,400 | 1.060,900 | 1.081,600 | 1.102,500 | 1.123,600 |
| 3 | 1.061,208 | 1.092,727 | 1.124,864 | 1.157,625 | 1.191,016 |
| 4 | 1.082,432 | 1.125,509 | 1.169,859 | 1.215,506 | 1.262,477 |
| 5 | 1.104,081 | 1.159,274 | 1.216,653 | 1.276,282 | 1.338,226 |
| 6 | 1.126,162 | 2.194,052 | 1.265,319 | 1.340,096 | 1.418,519 |
| 7 | 1.148,686 | 1.229,874 | 1.315,932 | 1.407,100 | 1.503,630 |
| 8 | 1.171,659 | 1.266,770 | 1.368,569 | 1.477,455 | 1.593,848 |
| 9 | 1.195,093 | 1.304,773 | 1.423,312 | 1.551,328 | 1.689,479 |
| 10 | 1.218,994 | 1.343,916 | 1.480,244 | 1.628,895 | 1.790,848 |
| 11 | 1.243,374 | 1.384,234 | 1.539,454 | 1.710,339 | 1.898,299 |
| 12 | 1.268,242 | 1.425,761 | 1.601,032 | 1.795,856 | 2.012,197 |
| 13 | 1.293,607 | 1.468,534 | 1.665,074 | 1.885,649 | 2.132,928 |
| 14 | 1.319,479 | 1.512,590 | 1.731,676 | 1.979,932 | 2.260,904 |
| 15 | 1.345,868 | 1.557,967 | 1.800,944 | 2.078,928 | 2.396,558 |
| 16 | 1.372,786 | 1.604,706 | 1.872,981 | 2.182,875 | 2.540,352 |
| 17 | 1.400,241 | 1.652,848 | 1.947,901 | 2.292,018 | 2.692,773 |
| 18 | 1.428,246 | 1.702,433 | 2.025,817 | 2.406,619 | 2.854,339 |
| 19 | 1.456,811 | 1.753,506 | 2.106,849 | 2.526,950 | 3.025,600 |
| 20 | 1.485,947 | 1.806,111 | 2.191,123 | 2.653,298 | 3.207,136 |

1. Solve again the problems on page 271, using in the work, items from the above table.

NOTE. In savings banks the interest due depositors is compounded usually semiannually; that is, the interest, if not withdrawn, is added to the principal every 6 months.

*a.* Twelve hundred dollars deposited in a savings bank that pays interest at the rate of 4 % per annum and compounds it every 6 mo., will amount to how much in 10 years?

*b.* Find the compound amount of $600 for 8 years at 6 %, interest compounded annually.

It is often important to know the exact present value of a *promise to pay* a sum of money after a specified number of years, months, or days. The rule for solving such problems is, — *Divide the amount to be paid by the amount of* $1.00 *for the given time and at the current rate.* Discuss the reasonableness and accuracy of this rule in its application to the special cases presented on this page.

*a.* What is the present value of a good note for $ 477, drawing no interest, and due one year hence, the current rate being 6 % ?

*b.* Prove your answer to problem *a* to be correct by finding the amount of it for one year at 6 per cent.

*c.* What is the present worth of $ 500 due 2 years hence, the current rate for money being 6 %, interest payable annually ? *

*d.* If a father should give to one son a note for $ 500 payable in 2 years without interest, and to another son an amount of cash equal to your answer to problem *c*, and the second son should loan his money for two years at 6 % with interest payable annually, and at the end of the first year should collect the interest due and loan it for one year at 6 %, how would the amounts received by the two sons compare at the end of two years ?

*e.* What is the present value of a good note for $ 600, that matures two years hence and draws interest from the present time at 8 %, the interest being payable annually, and the current rate for money being 6 %, interest payable annually ? †

---

* Since money is worth 6%, interest payable annually, one dollar in two years would amount, not to $1.12, but to $1.1236. See table on preceding page.

† Observe that this problem calls for the present value of $48 due one year hence, and $648 due 2 years hence.

*a.* The dividend on certain stocks being prospectively for all time 8% per annum, how much shall one pay per share (par value $100) in order to realize 6% on the investment?

*b.* The current rate of interest continuing to be 6% for all time, would an 8% bond running 20 years be worth as much as the 8% stock mentioned in problem *a*?

*c.* The dividend on certain stocks being prospectively for all time 5% per annum, how much shall one pay per share (par value $100) in order to realize 6% per annum on the investment?

*d.* The current rate of interest continuing to be 6% per annum for all time, would a 5% bond running 20 years be worth more or less than the 5% stock mentioned in problem *c*?

*e.* How much, including brokerage,* must one invest in 8% stocks at 38% premium, in order to have an annual income of $1000?

*f.* How much, including brokerage, must one invest in 4% stocks at 28% discount, in order to have an annual income of $1000?

*g.* A man invests, not including brokerage, $27,500 in bank stock at a premium of 10%. If the stock yields a semiannual dividend of 3½%, what is his annual income?

*h.* To secure 5% on the investment, how much must one pay for stock that yields an annual dividend of 6%?

---

* Stocks are often bought or sold through a broker. The commission for transacting the business is called the **brokerage**. The usual brokerage is ⅛ of 1% reckoned on the par value, or 12½¢ per $100 share.

*a.* A house cost $5500. The average annual necessary expenditures are: repairs, $120; taxes, $52; insurance, $12.50. The house rents for $35 a month. What per cent, net, does the owner realize on the investment?

*b.* A house cost $1000. The average annual necessary expenditures are: repairs, $25; taxes, $12.75; insurance, $6.40. The house rents for $10 a month. What per cent, net, does the owner realize on the investment?

*c.* A house rents for $25 per month, and the owner pays $60 for taxes, insurance, and repairs. On what sum does the house pay a net profit of 5% per annum?

*d.* Three flats that cost $8500 rent for $35, $30, and $25 per month. If the flats on the average are vacant $\frac{1}{12}$ of the time, and if the annual cost of repairs, insurance, taxes, water, fuel, and janitor services is $565, what is the net income on the investment?

*e.* A farm cost $10000, including buildings that are valued at $3000. If the farm and buildings rent for $600 per annum, and the owner pays annually for repairs $62, taxes $55, and insurance $13; and if the annual depreciation on the value of the buildings is equal to 2% of their original value, what is the net per cent of income on the investment?

*f.* To make 5% on the investment, how much should be paid for a house that rents for $30 a month, if repairs, taxes, and insurance average $125 per annum?

*g.* Land that cost $70 per acre and rents for $4 per acre pays what per cent on the investment, if the average annual cost of fencing and taxes is 70¢ per acre?

## LATHS

**NOTE.** A lath is 4 feet long and $1\frac{1}{2}$ inches wide. Laths are laid about $\frac{3}{8}$ of an inch apart. Hence 1 bunch, 50 laths, will cover (including the spaces between them) a strip 4 feet wide and 50 times $\overline{1\frac{1}{2}+\frac{3}{8}}$ inches long. Such a strip contains a little more than —— square feet. Making large allowance for defective laths and wasted pieces, builders calculate that *a bunch of laths will cover* 3 *square yards.*

*a*. Estimate the laths necessary for two rooms, walls and ceilings, the rooms being 15 ft. by 16 ft. and 14 ft. by 18 ft. with ceilings 10 ft. above the floors. Make no allowance for windows and doors in the making of this estimate. In practice, if the openings are numerous or large and "close figures" are required, such allowance may be made. In such cases it is best to make the computation as though there were no openings and make the proper deduction from the amount obtained.

*b*. Estimate the laths necessary for the walls of a 5-room house, the entire length of wall, including closets and halls, being 163 feet, and the height of wall being 8 feet, an allowance of 200 square feet to be made for openings.

*c*. Estimate the laths necessary for 6 ceilings, the dimensions being as follows: 12 by $13\frac{1}{2}$, 12 by 14, 10 by $11\frac{1}{2}$, 9 by 13, $4\frac{1}{2}$ by 10, and 4 by 6.

*d*. Estimate the cost of putting the laths on each of the 6 ceilings mentioned in example *c*, at 6 ¢ a square yard.

*e*. Estimate the cost of metal laths for each of the 6 ceilings in example *c*, at 25 ¢ a square yard.

## SHINGLES

NOTE. Shingles are of variable widths; but in buying and selling and in making estimates, each 4 inches in width is counted as 1 shingle. A bunch of shingles is 20 inches wide; so there are 5 shingles in each course. There are 25 courses in each end of the bunch; hence there are —— shingles in one end of the bunch and —— shingles in the entire bunch. Four bunches are 1000 (1 M) shingles. When shingles are laid "4½ inches to the weather," each shingle may be thought of as covering a space 4 inches by 4½ inches, or 18 square inches. So 8 shingles are required for each square foot to be covered. To cover 100 square feet (1 square) 800 shingles will be required. But since there is always some waste and often short measure, builders calculate 900 shingles for each "square" (10 ft. by 10 ft.) to be covered. If the shingles are to be laid only 4 inches to the weather, 1 M will be required for each square to be covered.

*a.* Estimate the shingles (to be laid 4 inches to the weather) for a double roof that is 36 feet long and whose rafters are 20 feet long. In giving the answer to this problem remember that lumber dealers will not divide a bunch of shingles.

*b.* Allowing 10% for waste, 1 thousand shingles laid 5 inches to the weather will cover how many square feet?

*c.* Allowing 10% for waste and laying the shingles 5 inches to the weather, how many bunches must be purchased for a double roof that is 60 feet long and whose rafters are 26 feet long?

*d.* How many bunches of shingles laid 4½ inches to the weather will be needed for the roof of a barn 30 ft. wide and 60 ft. long?

*e.* How many bunches of shingles will be needed for the roof described in *d*, if they are laid 5 inches to the weather?

## SIDING

**NOTE.** **Siding** is about ½ in. thick and 5½ in. wide, but is counted as though 1 in. thick and 6 in. wide. A 16-foot piece of siding contains 8 ft. of lumber. When it is laid "4 in. to the weather," what per cent of itself must be added to the number of sq. ft. to be covered to denote the number of feet of lumber required?

*a.* Estimate the siding for a building 30 ft. by 40 ft. and 18 ft. to the plates,* the roof running lengthwise and its highest point being 9 ft. higher than the plates.

*b.* Allowing 7 % for waste, 1 M ft. siding laid "4 in. to the weather" will cover how many sq. ft. of surface?

*c.* Enough siding was purchased to complete a job, provided the siding was laid "4 in. to the weather." It was laid "4½ in. to the weather." What part of the amount purchased was unused?

## MATCHED FLOORING

**NOTE.** Common matched flooring is, omitting the tongue, about 5 in. in width, but since it is made from 6-inch boards, it is called 6-inch flooring and is counted as 1 foot of lumber for every 2 linear feet of flooring. To find the amount of flooring needed for a given surface *add ⅕ of itself to the number of feet to be covered.*

*d.* Estimate the amount of 6-inch flooring needed for two floors throughout a building, main part 24 by 36, with an ell 18 by 20.

*e.* Find the cost at $27 per M of the flooring for the building described in problem *d.*

*f.* One thousand feet of 6-inch matched flooring will cover how many feet of surface?

---

* The **plates** are the horizontal timbers upon which the rafters rest near their lower ends. The height to the plates is practically the height to the gables, which in this case are 30 ft. by 9 ft.

## SHEATHING

**NOTE. A somewhat low grade of "matched flooring" is often used for the sheathing of buildings; that is, for a first covering of the framework. The siding covers the sheathing.**

*a.* Estimate the sheathing of "matched flooring" required for a building 18 feet by 24 feet and 15 feet to the plates, the roof running lengthwise of the building, and the highest point being 6 feet higher than the plates.

## STOCK BOARDS

**NOTE. Barns are sometimes covered with stock boards 1 foot in width. These boards are put on vertically and the cracks between them are covered with battens.**

*b.* Estimate the number of stock boards required for covering a barn that is 40 feet by 60 feet and 24 feet to the plates, the roof running lengthwise of the barn and its highest point being 10 feet higher than the plates. Consider the openings sufficient to make up for the waste in cutting the lumber and for defective boards.

## SIDEWALKS

*c.* Find the cost of lumber for 240 feet of sidewalk 6 feet wide, the walk to be made of 2-inch plank laid on three 2 by 6 stringers and the lumber to cost $17.50 per M.

*d.* Find the cost of the lumber required for 80 feet of walk, 3 feet wide, the walk to be made of 1-inch boards laid on two 2 by 4 stringers, the lumber to cost $16 per M. *e.* How many boards each 1 foot wide and 12 feet long are required for this walk? *f.* How many 2 by 4, 16's?

## BRICK AND MORTAR

NOTE. For data for estimating brick, see page 168 of this book.

*a.* Estimate the brick for the outer walls of a building, the outside perimeter of which is 240 feet, the height of the walls to the gables being 29 feet and the thickness, 3 bricks, or, including the mortar, about 13 inches. Make no allowance for openings or "double corners," but assume that the material saved at these points will be sufficient to lay the gables.

*b.* To lay 1000 brick, about 1⅛ bbl. lime and 13 bu. sand are required. Find the amount of lime and sand necessary to lay the brick in the outer walls of the building described in problem *a.*

*c.* Estimate the brick for the wall of a well 22 feet deep and 20 feet inside diameter, the brick to be laid without mortar and the wall to be 12 inches in thickness.*

## STONE

NOTE. For data for estimating stone, see page 167 of this book.

*d.* Estimate the stone (cords) for the foundation of the outer walls of the building described in problem *c* above, the foundation to be 5 feet high and 20 in. in thickness. Make no allowance for "double corners" or openings. The material saved at these points may be used in the expansion of the wall at the bottom.

*e.* Which is the cheaper, a 12-inch (practically 13-inch) wall of brick or an 18-inch wall of stone, the brick costing $6 per M, and the stone $8 per cord?

---

* Use 3⅐ as the ratio of diameter to circumference, regard the length of the wall as equal to the circumference of a circle 21 feet in diameter, and compute 25 bricks to the cubic foot.

## PLASTERING

*a.* Find the cost, at 28¢ per square yard, of plastering three rooms, walls and ceilings, the dimensions of the rooms being as follows: Room 1, 18 ft. by 20 ft.; Room 2, 14 ft. by 16 ft.; Room 3, 13 ft. by 19 ft.; height of each room, 10 ft. Make no allowance for openings.

## WATER

NOTE. In computing capacity for water, it is customary to regard 7½ gallons as equal to 1 cubic foot. Water weighs 62½ lb. to the cubic foot.

*b.* When the water in the well described in problem *c*, page 280, is 12 feet deep, how many gallons of water in the well?

*c.* A tank 10 feet deep and 12 feet in diameter will contain how many gallons of water?

*d.* What is the weight of the water that will exactly fill the tank described in problem *c*?

*e.* Estimate the weight of water sufficient to fill a tank as large as your schoolroom to the depth of 3 feet.

*f.* Estimate the number of gallons of water that will cover the floor of your schoolroom to the depth of 1 in.

*g.* What is the weight of water to the acre when there is a rainfall of 1 inch?

*h.* A heavy shower raised the water in a cistern 8 feet long and 6 feet wide, 2 feet 6 inches. The water came from a roof 30 feet long and 22 feet wide. The rainfall was how many inches?

*i.* How many wells or cisterns 4 feet in diameter are equal in capacity to a well or cistern 20 feet in diameter, the depth being the same in each case?

## WEIGHT OF IRON

**NOTE.** Iron is about 7¼ times as heavy as water, the same bulk being considered.

*a.* How much does a cubic foot of iron weigh? *b.* A cubic inch?

*c.* Estimate the weight of a bar of iron, 1 inch by 6 inches by 12 feet.

*d.* Estimate the weight of a 4-inch iron ball. *e.* Of a 2-inch iron ball.

*f.* Estimate the weight of an iron rod 1 inch in diameter and 12 feet long. *g.* Of an iron rod 2 inches in diameter and 12 feet long.

*h.* Estimate the weight of a sheet of boiler iron, 8 feet square and ⅜ of an inch thick.

*i.* Compare the weight of an iron ball 6 inches in diameter with the weight of one 2 inches in diameter.

## WEIGHT OF ICE

NOTE. The specific gravity of a body that floats on water is less than 1; its weight is less than 62½ lb. to the cubic foot. The fraction that indicates the part of a floating body that is below the surface of the water, also indicates its specific gravity. Thus, if three fourths of a floating body is immersed, its specific gravity is ¾, and its weight is ¾ of 62½ lb. to the cubic foot.

*j.* Observe a piece of ice floating in water. Estimate its specific gravity and its weight per cubic foot.

*k.* Estimate the weight of a cube of ice 2 feet square and 18 inches thick.

*l.* Estimate the number of tons of ice that can be stored in an icehouse 20 ft. square inside measurement, if the depth of the ice is 12 ft. and 1 ft. of space next to the walls is filled with sawdust.

# AGRICULTURAL PROBLEMS

NOTE. The efforts that are being made to introduce the elements of agricultural science into rural schools, the fact that many of the pupils will soon be, directly or indirectly, interested in such problems, and that many teachers for the rural schools are educated in the city schools, are the reasons for introducing this topic in the pages of a common school arithmetic.

*a.* If a certain piece of land, with an average annual expenditure of $17 per acre, gives an average annual return of $20 per acre, and is therefore considered worth $40 an acre as an investment, what is that piece of land worth which, with the same average expenditure, will give an average annual return of $29 per acre?

*b.* One piece of land produces 40 bushels of corn to the acre; another piece produces 60 bushels. If the cost of producing and marketing is $8 per acre in each case, and if the average price of corn is 25 cents a bushel, and if the difference in yield is due to the nature of the soils and not to differences in the methods of cultivation, what is the relative value of the two pieces of land as an investment?

*c.* If the cost of producing, threshing, and marketing an acre of wheat, including labor, "wear and tear" of machinery, taxes, and fertilizer enough fully to maintain the fertility of the soil, is $9.50, and if the yield is 25 bushels, worth 70¢ per bushel, how much is the land worth per acre if money is worth 6% per annum?

**Shrinkage in dressing.** Beeves shrink from 43% to 55% in dressing. This means that a carcass of dressed beef weighs from 43% to 55% less than the live weight of the animal. Hogs shrink from 15% to 22%.

### BEEVES

| No. | Live Weight. | Cost per Cwt. | Dressed Weight. | Weight of Hide. |
|---|---|---|---|---|
| 1 . . . . | 1740 | $5.20 | 990 | 105 |
| 2 . . . . | 1230 | 4.30 | 660 | 65 |
| 3 . . . . | 1450 | 4.75 | 750 | 80 |
| 4 . . . . | 1570 | 5.00 | 810 | 85 |
| 5 . . . . | 1080 | 4.10 | 590 | 48 |
| 6 . . . . | 1190 | 4.60 | 610 | 52 |
| 7 . . . . | 1240 | 4.40 | 690 | 60 |
| 8 . . . . | 1630 | 5.00 | 830 | 87 |
| 9 . . . . | 1470 | 4.80 | 820 | 63 |
| 10 . . . . | 1360 | 4.90 | 710 | 56 |

*a.* Find the per cent of shrinkage in each of the above cases. *b.* On the entire lot of ten head.

*c.* If the tongue, liver, heart, and offal are worth enough to pay for slaughtering, and the hide is worth 7 cents per pound, what is the cost per cwt. of each carcass of beef?

*d.* Find the average cost of the dressed beef per cwt.

### HOGS

11. Live weight, 450 lb.; dressed weight, 370 lb.
12. Live weight, 650 lb.; dressed weight, 540 lb.
13. Live weight, 260 lb.; dressed weight, 220 lb.
14. Live weight, 350 lb.; dressed weight, 290 lb.

*e.* Find the per cent of shrinkage in each of the above cases.

The chemist divides the dry part of food constituents into three groups, namely: **proteids** (or **protein**), **carbohydrates**, and **fat.** Protein contains nitrogen and is a "muscle former." The proteids include gluten, albumin, casein, etc. The carbohydrates contain no nitrogen and are "fat formers." The carbohydrates consist of carbon, hydrogen, and oxygen, and include sugar, starch, and cellulose. Fat contains no nitrogen and is itself a "fat former."

The **nutritive ratio** is the ratio of the muscle formers to the fat formers in any article of food. Since fat is a more valuable food constituent than the carbohydrates, it is customary to multiply the digestible fat by 2¼ before making the comparison.

The theoretical nutritive ratio adapted to the needs of a horse doing light work is about 1 : 7.

A ratio like that of timothy hay, 1 : 16.1, is said to be a *wide ratio.* A ratio like that of wheat bran, 1 : 3.9, is said to be a *narrow ratio.*

Rule for finding the nutritive ratio when the weight of digestible food ingredients in a specified amount of food is given:

*Add* 2¼ *times the fat to the carbohydrates and divide the sum by the protein. The quotient is the second term of the ratio, the first term being* 1.

ALFALFA HAY

| PROTEIN | CARBOHYDRATES | FAT |
|---|---|---|
| 10.6 * | 37.3 | 1.4 |
| | 3.1 | 2¼ |
| | 10.6)40.4(3.8 | 2.8 |
| | 31 8 | 3 |
| | 8 60 | 3.1 |
| | 8 48 | |

First, multiply the fat (1.4 lb.) by 2¼.

Second, add the product (3.1) to the pounds of carbohydrates.

Third, divide the sum (40.4) by the number of pounds of protein (10.6); this gives the second term of the ratio, 3.8.

The first term of the ratio is 1, and the ratio is 1 : 3.8.

* See table on next page.

FROM FARMERS' BULLETIN NO. 22

| | PROTEIN. | CARBOHYDRATES. | FAT. |
|---|---|---|---|
| Corn fodder, green | 1.1 | 12.1 | .4 |
| Rye fodder, green | 2.0 | 14.1 | .4 |
| Oat fodder, green | 2.4 | 18.0 | 1.0 |
| Red clover, green | 3.1 | 14.8 | .7 |
| Alfalfa, green | 3.9 | 11.2 | .4 |
| Cowpea, green | 1.7 | 8.1 | .2 |
| Soy bean, green | 2.8 | 11.8 | .6 |
| Rape, green | 2.2 | 8.6 | .3 |
| Corn silage | 1.2 | 14.6 | .9 |
| Corn fodder, dry | 2.3 | 32.3 | 1.1 |
| Corn stover, dry | 2.0 | 33.2 | .6 |
| Timothy hay | 2.9 | 43.7 | 1.4 |
| Red clover hay | 7.4 | 38.1 | 1.8 |
| Alsike clover hay | 8.1 | 41.7 | 1.4 |
| Alfalfa hay | 10.6 | 37.3 | 1.4 |
| Cowpea hay | 10.8 | 38.4 | 1.5 |
| Soy bean hay | 10.8 | 38.7 | 1.5 |
| Wheat straw | .4 | 36.3 | .4 |
| Rye straw | .6 | 40.6 | .4 |
| Oat straw | 1.2 | 38.6 | .8 |
| Soy bean straw | 2.3 | 40.0 | 1.0 |
| Corn, grain | 7.1 | 66.1 | 4.8 |
| Oats, grain | 9.2 | 48.3 | 4.2 |
| Wheat bran | 12.0 | 41.2 | 2.9 |
| Oil meal | 30.6 | 38.7 | 2.9 |
| Cotton seed meal | 37.0 | 16.5 | 12.6 |
| Whole milk | 3.4 | 4.8 | 3.7 |
| Skim milk | 3.0 | 5.0 | .3 |

It is estimated that on the average digestible protein is worth about 4 cents a pound; digestible carbohydrates, about four tenths of 1 cent a pound, and digestible fat about 1 cent a pound.

*a.* Find the nutritive ratio of corn silage.

*b.* Regarding 1 : 7 as a medium ratio, is the ratio of corn silage narrow or wide?

*c.* Find the nutritive ratio of wheat bran. *d.* Is this ratio narrow or wide?

*e.* The widest ratio* of any of the feeds named on page 286 is 1 : 93. Which feed is it? *

*f.* The narrowest ratio of any of the feeds named on page 286 is 1 : 1.2. Which feed is it? *

*g.* Is the nutritive ratio of corn (grain) wider or narrower than 1: 7?

*h.* Which has the narrower nutritive ratio, whole milk or skimmed milk?

*i.* Which has the wider nutritive ratio, timothy hay or red clover hay?

*j.* Which of the green feeds named on page 286 has the narrowest ratio? *k.* Which has the widest ratio?

*l.* Which of the different kinds of hay named on page 286 has the narrowest ratio? *m.* Which has the widest ratio?

*n.* Which has the narrower ratio, corn (grain) or oats (grain)?

*o.* Compare the nutritive ratio of alfalfa hay and wheat bran.

*p.* Compare the nutritive ratio of cowpea hay and soy bean hay.

*q.* Compare the nutritive ratio of green corn fodder and corn silage.

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*** Observe that a small amount of protein and a large amount of carbohydrates make a wide ratio, while a large amount of protein and a small amount of carbohydrates make a narrow ratio.**

**A ration is said to be balanced** when it contains the muscle formers and the fat formers in proper proportion to meet the requirements of the animal to be fed. Theoretically this varies from 1:4 to 1:8 or 9 according to the age and condition of the animal and the kind of service required. A growing animal requires food having a narrower ratio than a mature animal. The same may be said of a horse engaged in heavy work as compared with one that is idle; and of a cow giving a large amount of milk as compared with one that gives a small amount of milk. At many of the agricultural experiment stations, experiments are being made to determine more accurately the nutritive ratio of food adapted to the needs of animals under varying conditions. The following problems will help the pupil to understand what he may hear or read about *the balanced ration.*

*a.* What is the nutritive ratio of a feed consisting of corn (grain) and skimmed milk mixed in the proportion of 100 lb. of corn to 200 lb. of skimmed milk? *

*b.* What is the nutritive ratio of a feed consisting of corn (grain) and green alfalfa mixed in the proportion of 100 lb. of corn to 200 lb. of alfalfa?

*c.* What is the nutritive ratio of a feed consisting of equal parts of corn (grain) and oats (grain)? †

*d.* Which of the three feeds described in problems *a*, *b*, and *c* is the best balanced ration for swine, provided the desirable nutritive ratio is 1:5.9?

*e.* Would the addition of oil meal to the ration described in problem *a* make the ration narrower or wider?

---

* Using the table on page 286, find the number of pounds of protein, carbohydrates, and fat in the 300 lb. of feed. Then find the nutritive ratio as directed on page 285.

† The "parts" are to be taken by weight in this problem and in similar problems on the following page.

The theoretical nutritive ratio of food adapted to the needs of a milch cow weighing 1000 lb. and giving daily 22 lb. (about 10¼ quarts) of milk is 1 : 5.7.* Find the nutritive ratio of each of the following combinations of feeds and note whether it is narrower or wider than 1 : 5.7:

*a.* Three parts timothy hay and one part corn (grain).

*b.* Three parts red clover hay and one part corn (grain).

*c.* Three parts alfalfa hay and one part corn (grain).

*d.* Two parts corn silage and one part alfalfa hay.

*e.* Four parts corn silage, one part wheat bran, and one part alfalfa hay.

The theoretical nutritive ratio of food adapted to the needs of a horse engaged in light work is 1 : 7. Find the nutritive ratio of each of the following combinations of feeds and note whether it is narrower or wider than 1 : 7:

*f.* Two parts timothy hay and one part corn (grain).

*g.* Two parts timothy hay and one part oats (grain).

*h.* One part timothy hay, one part red clover hay, and one part corn (grain).

*i.* One part timothy hay, one part red clover hay, and one part oats (grain).

*j.* Two parts alsike clover hay and one part corn (grain).

*k.* Two parts alfalfa hay and one part corn (grain).

---

* These figures were taken from the Wolf-Lehmann feeding standards. Later investigations may result in a theoretical nutritive ratio for the animal described in No. 1 slightly wider than 1 : 5.7.

1. According to the Wolf-Lehmann feeding standards, a cow weighing 1000 lb. and giving 22 lb. of milk per day should receive for food, 2.5 lb. protein, 13 lb. carbohydrates, and .5 lb. fat.*

*a.* Would a ration made up of 40 lb. silage and 20 lb. alfalfa hay furnish the required amount of protein?† *b.* Would it furnish the required amount of carbohydrates? *c.* Would it furnish the required amount of fat?

*d.* Would a ration consisting of 20 lb. timothy hay and 10 lb. corn (grain) provide the necessary amount of protein? *e.* Would it provide the necessary amount of carbohydrates? *f.* Would it provide the necessary amount of fat? *g.* Is it a balanced ration? *h.* In what food ingredient is it deficient? *i.* Would the substitution of 2 lb. of oil meal for 2 lb. of the corn in this ration improve it? *j.* Would the required amount of protein thus be provided?

*k.* Would a ration consisting of 20 lb. alsike clover hay, 5 lb. corn (grain), and 5 lb. bran provide the necessary amount of protein?‡ *l.* Would it provide the necessary amount of carbohydrates? *m.* Would it provide the necessary amount of fat? *n.* Is it a balanced ration?

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* See footnote on page 289.

† See table on page 286. The figures in the table give the number of pounds in 100 lb. of feed. 10 lb. contain 1 tenth as much as 100 lb.; 40 lb. contain 4 times as much as 10 lb.; 10 lb. corn silage contain .12 lb. protein; 40 lb. contain .48 lb.; 10 lb. alfalfa hay contain 1.06 lb. protein; 20 lb. contain 2.12 lb.

‡ 5 lb. contain 1 half as much as 10 lb.; 10 lb. corn (grain) contain .71 lb. protein; 5 lb. contain .35+ lb.

At 4¢ per pound for digestible protein, .4¢ a pound for digestible carbohydrates, and 1¢ a pound for digestible fat, find the value of each of the following:

*a.* One ton of timothy hay.

*b.* One ton of corn silage.

*c.* One ton of alsike clover hay.

*d.* One ton of alfalfa hay.

*e.* One hundred pounds of skimmed milk.

*f.* One bushel (56 lb.) of corn.

*g.* One bushel (32 lb.) of oats.

*h.* One hundred pounds of whole milk.

*i.* One ton of oil meal.

*j.* One ton of corn stover,* dry.

*k.* One ton of wheat bran.

*l.* One ton of cotton seed meal.

*m.* One ton of green corn fodder.

*n.* One ton of green rape.

*o.* One ton of green alfalfa.

*p.* One ton of green rye fodder.

*q.* One ton of soy bean straw.

*r.* One ton of wheat straw.

*s.* One ton of rye straw.

*t.* One ton of oat straw.

*u.* One ton of oats (grain).

*v.* One ton of corn (grain).

*w.* One ton of cowpea hay.

*x.* One ton red clover hay.

---

* The dry stalks of corn from which the ears have been removed.

**Note.** The most expensive food constituent is protein. Oil meal and cotton seed meal usually cost more per ton than corn or oats because of the greater amount of protein in these feeds. Hence it becomes important to know not only how much digestible protein there is in a ton of any given feed, but how much may be produced per acre of this high-priced food ingredient.

*a.* One and one half tons of timothy hay is regarded as a good yield per acre. How many pounds of digestible protein in 1½ tons of timothy hay? *

*b.* Two and one half tons of red clover hay is regarded as a good yield per acre. How many pounds of digestible protein in 2½ tons of red clover hay?

*c.* Five tons of alfalfa hay is regarded as a good yield per acre. How many pounds of digestible protein in 5 tons of alfalfa hay?

*d.* Comparing the answers to problems *a* and *b*, how many times as much digestible protein may be expected from an acre of red clover as from an acre of timothy?

*e.* Comparing the answers to problems *b* and *c*, how many times as much digestible protein may be expected from an acre of alfalfa as from an acre of red clover?

*f.* Comparing the answers to problems *a* and *c*, how many times as much digestible protein may be expected from an acre of alfalfa as from an acre of timothy?

*g.* Fifty bushels of shelled corn and 1½ tons of stover are regarded as a good yield per acre. How many pounds of digestible protein in 50 bushels of shelled corn and 1½ tons of stover?

*h.* How many pounds of digestible protein in 50 bushels of oats and 1 ton of oat straw?

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* See table on page 286.

NOTE. There are ten essential plant-food elements. Three of these are gases, *oxygen*, *hydrogen*, and *nitrogen*; three are non-metallic solids, *carbon*, *sulphur*, and *phosphorus*; four are metals, *iron*, *potassium*, *calcium*, and *magnesium*. (Some writers regard *chlorine*, *silicon*, and *sodium* as essential plant-food elements.) Only three of these elements have a money value as plant-food ingredients: these are **nitrogen**, **phosphorus**, and **potassium**. All the others are naturally provided in abundance for all soils in which the water supply is adequate.* All crops remove from the soil some of the plant-food supply.

### POUNDS OF FERTILIZING ELEMENTS IN ONE TON†

| | N = NITROGEN. N | P = PHOSPHORUS. P | K = POTASSIUM. K |
|---|---|---|---|
| Timothy hay | 24 | 3 | 24 |
| Clover hay | 40 | 5 | 30 |
| Alfalfa hay | 45 | 4 | 25 |
| Corn, grain | 35 | 6 | 7 |
| Corn, stover | 16 | 2 | 17 |
| Bran | 53 | 25 | 25 |
| Oat straw | 12 | 2 | 20 |
| Milk | 11 | 2 | 3 |
| Butter | 2 | ¼ | ½ |
| Farm animals (average) | 45 | 18 | 3 |
| Farm manure (fresh) | 10 | 2 | 10 |

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* Lime is sometimes purchased and spread upon the land, not as necessary plant-food, but for the purpose of neutralizing the acidity of the soil.

† Many writers give the fertilizing constituents of feeds and fertilizers as *ammonia*, *phosphoric acid*, and *potash*. In this table the number of pounds of fertilizing **elements** is given. To change from one of these methods of expression to the other, it is only necessary to remember that 82 % of ammonia is nitrogen, 44 % of phosphoric acid is phosphorus, and 83 % of potash is potassium.

*a.* One and one half tons of timothy hay per acre remove how much phosphorus per acre from the soil?

*b.* Two and one half tons of clover hay per acre remove how much phosphorus per acre from the soil?

*c.* Five tons of alfalfa hay per acre remove how much phosphorus per acre from the soil?

*d.* Fifty bushels of corn (2800 lb.) and 1½ tons of stover (stalks and leaves) per acre take how much phosphorus per acre from the soil?

*e.* A farmer sold 8½ tons of oat straw. How much phosphorus did he sell?

*f.* A dairyman sold 5 cans (each 68 lb.) of milk per day for a full year of 365 days. How much phosphorus did he sell?

*g.* A dairyman sold 15 lb. of butter per day for a full year. How many pounds of phosphorus did he sell?

*h.* A farmer sold 1½ tons of corn (grain) and with the money received for it bought 1 ton of bran. Did he thereby increase or diminish the amount of nitrogen on his farm? *i.* The amount of phosphorus? *j.* The amount of potassium?

*k.* A farmer sold in one year 24000 lb. of farm animals. How many pounds of phosphorus did he sell?

*l.* A farmer sold 10 tons of oat straw and bought 1 ton of bran. Did he thereby increase or diminish the amount of nitrogen on his farm? *m.* The amount of phosphorus? *n.* The amount of potassium?

*o.* Compare the amount of phosphorus in 50 bushels of corn with that in 1½ tons of stover.

*a.* A "commercial fertilizer" is valued mainly for the amount of nitrogen, phosphorus, and potassium it contains. Sometimes the nitrogen is reckoned at 15¢ a pound, the phosphorus at 12¢, and the potassium at 6¢. At these rates find the fertilizing value of one ton of average fresh farm manure.

*b.* A certain so-called Special Potato Fertilizer was found to contain 4.21% of ammonia, 7.35% of available phosphoric acid, and 9.87% of potash. Find the number of pounds of nitrogen (82% of the ammonia) per ton. *c.* Find the number of pounds of available phosphorus (44% of the phosphoric acid) per ton. *d.* Find the number of pounds of potassium (83% of the potash) per ton. *e.* At the prices given in problem *a*, find the value of this fertilizer per ton.*

*f.* A certain grade of steamed bone meal offered for sale in Illinois was found to contain 1.15% of nitrogen and 13.4% of phosphorus. At the prices given in problem *a*, find the value per ton.

*g.* A certain grade of potassium chloride (so-called "muriate of potash") offered for sale in Michigan was found to contain 48% of potash. At the price given in problem *a*, find the value per ton.

*h.* A certain kind of fertilizer known as Alkaline Bone, offered for sale in Maryland, was found to contain 5% of potash and 12% of available phosphoric acid. At the prices given in problem *a*, find its value per ton.

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* It is perhaps true that only those engaged in market gardening or other intensive agriculture, or in experimental work, can afford to buy nitrogen at such a price, since through the growing of the legume plants (clover, alfalfa, beans, peas, etc.) it may be obtained from the air at comparatively small expense.

*a.* How many tons of farm manure required to replace the phosphorus removed from an acre of land by a 50-bushel (2800 lb.) crop of corn and 1½ tons of stover?

*b.* How many tons of farm manure required to replace the phosphorus removed from one acre of land by a 3-ton crop of clover?

*c.* How many pounds of the fertilizer described in problem *f*, page 295, required to replace the phosphorus removed by a 100-bushel crop of corn, taking into the account the grain only?

*d.* If a farmer sells timothy hay, and for each ton sold brings back to the farm a ton of manure, will his farm thereby gain or lose in its fertility?

*e.* If a farmer sells oat straw, and for each ton sold brings back to the farm 1½ tons of manure, will his farm thereby gain or lose in its fertility?

*f.* Counting nitrogen at 2¢ a pound (see footnote, page 295), phosphorus at 12¢, and potassium at 6¢, how many dollars worth of fertility in a ton of timothy hay? *g.* In a ton of clover hay? *h.* In a ton of alfalfa hay? *i.* In a ton of corn (grain)? *j.* In a ton of corn stover? *k.* In a ton of bran? *l.* In a ton of oat straw? *m.* In a ton of milk? *n.* In a ton of butter? *o.* In a ton of farm animals? *p.* In a ton of farm manure? *

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* The real agricultural value of a ton of fresh farm manure is usually more than the answer to this question (see the answer to problem *a*, on page 295). In the production of ordinary farm crops at the Experiment Stations, it has given a return in five years of about $2.25 per ton, counting average market prices for the crops grown, although the effects produced by farm manure vary with the soil and season and are not always attributable solely to its plant-food content.

# PRACTICAL APPROXIMATIONS

So far as practicable, solve the following problems without the aid of a pencil. At least, *exercise the judgment* on every problem *before making any figures.*

*a.* The specific gravity * of iron being about 7¼, how much does a cubic foot of it weigh? *b.* How much does a cubic inch of iron weigh?

*c.* A 4-inch iron ball weighs about —— pounds. *d.* A 2-inch iron ball weighs about —— pounds.

*e.* An iron rod, 1 inch in diameter and 12 feet long, weighs about —— pounds. *f.* An iron rod 2 inches in diameter and 12 feet long weighs about —— pounds.

*g.* A sheet of boiler iron, 8 feet square and ⅜ of an inch thick, weighs about —— pounds.

*h.* What is the weight of the water that will fill a tank 2 feet wide, 2 feet deep, and 10 feet long?

*i.* The specific gravity of limestone is about 2½. What is the weight of a piece of limestone that is 4 feet square and 3 inches thick?

*j.* What is the weight of a stick of timber 12 inches by 12 inches and 20 feet long, if its specific gravity is .7?

*k.* What is the weight of 1000 feet of green 1-inch lumber if its specific gravity is .9?

---

* When we say that the specific gravity of iron is about 7¼, we mean that it weighs about 7¼ times as much as water, the same bulk being considered.

*a.* The specific gravity of sand is about 2. How much does a load (27 cu. ft.) of it weigh?

*b.* If the specific gravity of granite is 2.7, how much does a cubic yard of it weigh?

*c.* The specific gravity of brick and mortar is nearly 2. What is the weight of a cubic foot of brick wall?

*d.* If a brick 8 in. by 4 in. by 2 in. weighs 4½ pounds, is its specific gravity more or less than 2? That is, does 1 cubic foot of bricks weigh more or less than exactly twice as much as 1 cubic foot of water (62½ lb.)?

*e.* If the specific gravity of Athens (Illinois) limestone is 2.4, and if a cord of it is equal to 100 solid feet, how much does a cord of this stone weigh?

*f.* If bricks 8 in. long, 4 in. wide, and 2 in. thick are laid on their largest face, how many bricks will be required for a walk 6 feet wide and 100 feet long, making some allowance for imperfect bricks and breakage in handling?

*g.* What is the capacity in gallons of a tank 5 feet long, 2 feet wide, and 2 feet deep?

*h.* Give the dimensions of a tank the capacity of which is 225 gallons.

*i.* A certain cylindrical tank is 8 feet in diameter. Each foot in depth will contain how many gallons?

*j.* One inch of rainfall will give how many pounds of water on a horizontal surface 30 feet by 40 feet?

*k.* Two inches of rainfall will give how many barrels (31½ gal.) of water on a horizontal surface 40 feet by 60 feet?

*a.* A one-inch rainfall will give how many barrels (31½ gal.) of water on the schoolhouse lot?

*b.* Your schoolroom floor is what part of an acre?

*c.* The distance around your schoolroom is what part of a mile?

*d.* How many bushels of oats would be required to fill your schoolroom to the depth of 3 feet?

*e.* A shed as large as your schoolroom would hold how many cords of wood?

*f.* Give the dimensions of a crib that will hold 1000 bushels of corn.

*g.* Give the dimensions of a pile of wood that contains 40 cords.

*h.* A piece of land 100 feet long and 5 rods wide is what part of an acre?

*i.* How many cubic inches in a cylinder 8 inches in diameter and 10 inches long?

*j.* How many cubic inches in an 8-inch sphere?

*k.* If 468.25 is multiplied by .5106, will the product be more or less than 234?[*]

*l.* If 484.079 is multiplied by .251, will the product be more or less than 121?

*m.* If 2480 is multiplied by .2479 will the product be more or less than 620?

*n.* If 6400 is multiplied by .74, the product will be how many less than ¾ of 6400?

*o.* If 4800 is multiplied by 1.6, the product will be how many more than 1½ times 4800?

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[*] Observe that 234 is ½ of 468.

*a.* If 25.2314 is divided by $\frac{1}{2}$, will the quotient be more or less than 51?

*b.* If 250 is divided by .26, will the quotient be more or less than four times 250?

*c.* If cheese is worth 17 cents a pound, how much should be paid for 2 lb. 3 oz.? *d.* For 3 lb. 5 oz.? *e.* 1 lb. 7 oz.? *f.* 4 lb. 9 oz.? *g.* 2 lb. 11 oz.?

*h.* If meat is worth 15 cents a pound, how much should be paid for 1 lb. 4 oz.? *i.* For 2 lb. 6 oz.? *j.* 1 lb. 7 oz.? *k.* 2 lb. 9 oz.? *l.* 3 lb. 13 oz.?

*m.* If cheese is worth $12\frac{1}{2}$ cents a pound, how much should be paid for 2 lb. 8 oz.? *n.* For 2 lb. 4 oz.? *o.* 3 lb. 1 oz.? *p.* 4 lb. 9 oz.? *q.* 4 lb. 15 oz.?

*r.* A wagon box 10 feet by 3 feet by 16 inches will contain how many bushels of shelled corn?

*s.* The surface of a sphere is equal to 4 times the area of a circle having the same diameter as the sphere. How many square inches in the surface of a 10-inch sphere?

*t.* How many square inches in the surface of a 20-inch sphere?

*u.* If from a cylinder of wood the largest possible cone is cut, exactly $\frac{2}{3}$ of the wood will be cut away. The solid content of a cone is therefore exactly $\frac{1}{3}$ of a cylinder having the same base and the same altitude (length). How many cubic inches in a cone the diameter of whose base is 8 inches and whose altitude is 12 inches?

*v.* How many bushels of grain in a conical pile whose diameter is 6 feet and whose altitude is 4 feet?

*a*. At 10 cents a square foot, what is the cost of lining a cylindrical tank (curved surface and bottom) whose diameter is 10 feet and whose depth is 12 feet?

A sphere is exactly ⅔ and a cone exactly ⅓ of a cylinder of the same dimensions. Find the solid content and compare the following:

*b*. A 6-inch cube.

*c*. A cylinder 6 in. in diameter and 6 in. long.

*d*. A 6-inch sphere.

*e*. A cone, base 6 in. in diameter, altitude 6 in.

*f*. A cubic foot of steel weighs 490 lb. What is the weight of a cylinder of steel 1 foot in diameter and 1 foot long? *g*. Of a sphere of steel 1 foot in diameter? *h*. Of a cone of steel, base 1 foot in diameter, altitude 1 foot?

*i*. Does a circular piece of land 20 rods in diameter contain more or less than 2 acres?

At sight, give approximate answers to the following:

*j*. Interest of $450.25 for 1 yr. 3 da. at 6%?

*k*. Interest of $4000 for 29 da. at 6%?

*l*. Interest of $250 for 1 yr. 8 mo. 5 da. at 6%?

*m*. Interest of $500 for 1 yr. 11 mo. 16 da. at 6%?

*n*. Interest of $751.27 for 2 yr. 6 mo. 1 da. at 4%?

*o*. Interest of $149.75 for 1 yr. 7 mo. 29 da. at 6%?

*p*. Interest of $298.97 for 1 yr. 6 mo. 4 da. at 8%?

*q*. Interest of $495 for 15 da. at 6%?

*r*. Interest of $1200 for 6 da. at 8%?

*s*. Interest of $600 for 20 da. at 4%?

*t*. Interest of $397.28 for 2 yr. 6 mo. at 6%?

At sight, give answers to the following, that are *true to dollars;* then, with the aid of a pencil, if necessary, obtain answers that are *true to cents:*

*a.* Cost of 2970 lb. coal at $4.50 per ton? *
*b.* Cost of 3520 lb. hay at $11.50 per ton? †
*c.* Cost of 1490 lb. straw at $4.25 per ton?
*d.* Cost of 2460 lb. bran at $10 per ton? ‡
*e.* Cost of 2310 lb. oil meal at $21 per ton?
*f.* Cost of 2240 lb. beef at $6.10 per cwt.?
*g.* Cost of 1560 lb. pork at $4.50 per cwt.?
*h.* Cost of 2150 lb. flour at $3.05 per cwt.?
*i.* Cost of 1200 lb. lard at $5.90 per cwt.?
*j.* Cost of 1400 lb. tallow at $3.55 per cwt.?
*k.* Cost of 1000 lb. nails at $3.05 per cwt.?
*l.* Cost of 2240 ft. lumber at $15 per M? §
*m.* Cost of 4500 lath at $2.50 per M?
*n.* Cost of 6740 brick at $6 per M? ||
*o.* Cost of 1997 ft. lumber at $27.50 per M?
*p.* Cost of 198 lb. butter at 27½ cents per lb.? ¶
*q.* Cost of 203½ lb. cheese at 16 cents per lb.?
*r.* Cost of 2440 lb. oats at 24¼ cents per bu.? **
*s.* Cost of 1680 lb. oats at 23½ cents per bu.?

* 2970 lb. is nearly 1½ tons.

† What is the cost of 3500 at $12 per ton?

‡ At $10 a ton, how much does 1 lb. cost?

§ At $15 per M, how much is 1 foot worth?

|| 6740 brick are nearly 6¾ M.

¶ 200 lb. butter at 27½ cents is worth how much?

** At 24¢ a bushel, 1 lb. of oats is worth how much?

# MISCELLANEOUS PROBLEMS

*a.* Each edge of a cube is diminished by $\frac{1}{10}$ of its length. By what fraction of itself is the volume diminished? *b.* By what fraction of itself is the surface diminished?

*c.* How many cubical blocks, each edge of which is $\frac{1}{3}$ ft., are equivalent to a block 8 ft. long, 4 ft. wide, and 2 ft. thick?

*d.* The circumference of one of my carriage wheels is 12 feet. The circumference of another wheel on the same carriage is 14 feet. How far has the carriage run when the smaller wheel has made exactly 300 more revolutions than the larger wheel?

*e.* Is the capacity of a cylindrical pail 6 inches in diameter and 5 inches deep more or less than $\frac{1}{2}$ of a gallon?

*f.* At what rate per cent must I invest $800 that in 1 year 6 months it will amount to $854?

*g.* If exactly $\frac{5}{6}$ of a stick of timber floating in the water is submerged, and if the timber is 12 inches by 12 inches and 30 feet long, how many pounds does it weigh?

NOTE. If $\frac{5}{6}$ of the timber is submerged, it weighs $\frac{5}{6}$ as much as its own bulk of water.

*h.* How many pickets are required to inclose a square $2\frac{1}{2}$-acre lot if the pickets are 3 inches wide and 3 inches apart?

*a.* If a person traveling as expeditiously as possible from Boston to San Francisco should mail a letter to his friend in Boston every day at noon, how often should the letters be received in Boston?

*b.* If a man and a boy, the boy doing exactly one half as much work as the man, can hoe one and one half acres of corn in one and one half days, how many acres can 6 men hoe in 6 days?

*c.* John and James sold apples together. The first day they sold 60 apples at the rate of 5 apples for 2 cents, and received 24 cents. The second day they divided the apples. John took 30 of the larger apples and sold them at the rate of 2 for 1 cent. James took the remaining 30 apples and sold them at the rate of 3 for one cent. They received 25 cents. Why did they receive one cent more the second day than the first?

*d.* A pile of four-foot wood stands upon a hillside. The pile is 8 feet long and 4 feet high (measured vertically). Does the pile contain one cord?

*e.* A man had shingles enough to cover his house if he laid them 4 inches to the weather. He laid them $4\frac{1}{2}$ inches to the weather. What part of the shingles provided remained? Explain.

*f.* If on a line of railroad connecting Chicago and San Francisco one passenger train leaves Chicago daily at 6 o'clock A.M., and makes the journey to San Francisco in exactly five days, and one train leaves San Francisco daily at 6 o'clock P.M., and makes the journey to Chicago in exactly five days, a person taking the train at Chicago will meet how many passenger trains while going to San Francisco?

# ALGEBRA

In algebra, letters as well as figures are used to represent numbers. Known numbers, or those which may be known without solving a problem, when not expressed by figures, are usually represented by the first letters of the alphabet, as, $a$, $b$, $c$, $d$.

**1.** Find the perimeter of an oblong when its length and breadth are given.

Let $a =$ the length[*]

Let $b =$ the breadth

Then $2a + 2b$, or $(a + b) \times 2 =$ the perimeter

**To find the perimeter of an oblong:** *Multiply the sum of the numbers denoting its length and breadth by* 2.

Unknown numbers, or those which are to be found by the solution of a problem, are usually represented by the last letters of the alphabet, as, $x$, $y$, $z$.

**2.** There are two numbers whose sum is 48, and the second is three times the first. What are the numbers?

Let $x =$ the first number

Then $3x =$ the second number

and $x + 3x = 48$

$4x = 48$

$x = 12$

$3x = 36$ The numbers are 12 and 36.

---

[*] That is, the *number* of units in the length.

The sign of multiplication is usually omitted between two letters representing numbers, and between figures and letters; thus, $a \times b$ is usually written $ab$; $b \times 4$ is written $4b$. $6ab$ means 6 times $a$ times $b$, or $6 \times a \times b$.

Find the numerical value of each of the following expressions, if $a = 8$, $b = 5$, and $c = 2$:

**1.** $a + b + c$

**2.** $2ab$

**3.** $a + b - c$

**4.** $3abc$

**5.** $2a + b + c$

**6.** $2ab + 5c$

**7.** $a + b - 2c$

**8.** $ab + bc$

**9.** Find the sum of the eight results in **1** to **8**.

Find the numerical value of each of the following expressions, if $a = 20$, $b = 4$, $c = 2$:

**10.** $3(a + b)$[*]

**11.** $a \div b$

**12.** $2(a - b)$

**13.** $(a + b) \div c$[†]

**14.** $4(a + b + c)$

**15.** $(a + b) \div 3c$

**16.** $2(a + b - c)$

**17.** $(a + 2b) \div 2c$

**18.** Find the sum of the eight results in **10** to **17**.

**19.** If $2x = 20$, $x = ?$

**20.** If $5x = 40$, $x = ?$

**21.** If $6x = 72$, $x = ?$

**22.** If $2x + 3x = 60$, $x = ?$

**23.** If $3x + 4x = 56$, $x = ?$

**24.** If $7x - 3x = 40$, $x = ?$

**25.** If $5x - x = 36$, $x = ?$

**26.** If $5x + 2x = 84$, $x = ?$

**27.** If $7x - 3x = 160$, $x = ?$

---

[*] This means 3 times the sum of $a$ and $b$.

[†] This means the sum of $a$ and $b$ divided by $c$.

A **coefficient** is a number that indicates how many times a literal quantity* is to be taken; thus, in the expression $4\,ab$, 4 is the coefficient of $ab$.†

When no coefficient is expressed, it is understood that 1 is the coefficient; thus, in the expression $4\,a + b$, the coefficient of $b$ is 1.

The **terms** of an algebraic expression are the parts that are separated by the sign + or −. There are three terms in the following: $ab + 3\,c + 4\,abc$.

**Positive terms** are preceded by the plus sign.
**Negative terms** are preceded by the minus sign.

If no sign is expressed, the term is understood to be positive.

When the literal part of two or more terms is the same, the terms are said to be **similar**.

Unite the terms in each of the following algebraic expressions into one equivalent term:

1. $5\,x + 3\,x + 2\,x$
2. $4\,ab + 2\,ab + 3\,ab$
3. $4\,x + 5\,x - 3\,x$
4. $2\,ab + 5\,ab - 4\,ab$
5. $6\,a - 2\,a + 4\,a$
6. $3\,bc - 5\,bc + 6\,bc$
7. $3\,b + 4\,b - 2\,b$
8. $bx + 2\,bx + 3\,bx$

Referring to the problems given above, use the following words in complete sentences:

coefficient terms positive
negative similar literal

---

* The word *quantity* in algebra means number. The expression *literal quantity* means number expressed by letters.

† The term *coefficient* is sometimes applied to the literal part of an expression; thus, in the expression $abc$, $ab$ is the coefficient of $c$. Usually, however, the term *coefficient* has reference to the *numerical coefficient.*

Regarding the following *positive* numbers as representing *gains* and the *negative* numbers as representing *losses*, find the total gain (or loss) in each case; that is, find the algebraic sum of the numbers in each group:

$$\begin{array}{rrrrrr}
\textbf{1.} & \textbf{2.} & \textbf{3.} & \textbf{4.} & \textbf{5.} & \textbf{6.}\\
70 & 85 & 45 & 8\,a & 9\,b & 2\,ab\\
\underline{25} & \underline{-35} & \underline{-65} & \underline{3\,a} & \underline{-3\,b} & \underline{-6\,ab}\\
95 & 50 & -20 & & &
\end{array}$$

NOTE. The positive sums of examples **1**, **2**, **4**, and **5** indicate actual gain; the negative sums of examples **3** and **6** indicate actual loss.

Regarding each of the following *positive* numbers as representing a *rise* and each of the *negative* numbers as representing a *fall* of the mercury in a thermometer, find the total rise (or fall) in each case; that is, find the algebraic sum of the numbers in each group:

$$\begin{array}{rrrrrr}
\textbf{7.} & \textbf{8.} & \textbf{9.} & \textbf{10.} & \textbf{11.} & \textbf{12.}\\
16 & 12 & 8 & 8\,a & 7\,b & 5\,c\\
10 & -4 & 4 & 2\,a & -3\,b & 3\,c\\
\underline{5} & \underline{6} & \underline{-16} & \underline{3\,a} & \underline{4\,b} & \underline{-12\,c}\\
31 & 14 & -4 & & &
\end{array}$$

NOTE. The positive sums of examples **7**, **8**, **10**, and **11** indicate actual rise; the negative sums of examples **9** and **12** indicate actual fall.

Find the sums:

$$\begin{array}{ccc}
\textbf{13.} & \textbf{14.} & \textbf{15.}\\
12+4-6 & 3\,a+2\,b+c & 2\,ab+6\,bc\\
3+5+2 & 2\,a-b+3\,c & 5\,ab-2\,bc\\
4+6-3 & a+5\,b-6\,c & 3\,ab+bc\\
\underline{5-2+8} & \underline{7\,a-2\,b+3\,c} & \underline{-ab+3\,bc}
\end{array}$$

1. Find the sum of $+8a$ and $+6a$.
2. Find the sum of $+9a$ and $-5a$.[*]
3. Find the sum of $-7a$ and $-4a$.
4. Find the sum of $-12a$ and $+3a$.
5. Find the sum of $+15a$ and $-6a$.
6. Find the sum of $-17a$ and $+8a$.

**7.** A boat sailed 16 miles north, then 9 miles south. How far was it from the starting point?

| OPERATION | INTERPRETATION |
|---|---|
| $\begin{array}{r} +16 \text{ miles} \\ -\;9 \text{ miles} \\ \hline +\;7 \text{ miles} \end{array}$ | The "+7 miles" indicates that the boat was 7 miles north of the starting point. |

**8.** A boat sailed 14 miles north, then 19 miles south. How far was it from the starting point?

| OPERATION | INTERPRETATION |
|---|---|
| $\begin{array}{r} +14 \text{ miles} \\ -19 \text{ miles} \\ \hline -\;5 \text{ miles} \end{array}$ | The "$-5$ miles" indicates that the boat was 5 miles south of the starting point. |

Add the following:

9. $x + y + z$, $3x - 2y + 3z$, and $4x - y + 2z$.[†]
10. $5x + 2y - 3z$, $2x - 3y + 4z$, and $2x - 2y - 2z$.
11. $6x - 3y - z$, $x - y - z$, and $-2x + 3y - 3z$.

---

[*] Think of one person, D, as stepping north or south. Think of $+a$ as standing for a step north and $-a$ as standing for a step south. The question in each case (first six problems) is, How far and in what direction is D from the starting point? The answer to problem **1**, $+14a$, indicates that he is 14 steps north of the starting point. Interpret the answers to problems **2**, **3**, **4**, **5**, and **6**.

[†] Remember that when no sign precedes an algebraic quantity it must be regarded as positive; that when no coefficient is expressed 1 is understood; thus, $x$ means $+1x$.

Regarding the following minuends as representing A's gain (or loss), and the subtrahends as representing B's gain (or loss), subtract B's from A's: *

| | MON. | TUES. | WED. | THURS. | FRI. | SAT. |
|---|---|---|---|---|---|---|
| A, | 80 | 60 | 20 | $7a$ | $-4b$ | $-12c$ |
| B, | 30 | $-20$ | 50 | $2a$ | $-6b$ | $-6c$ |
| | 50 | 80 | $-30$ | $5a$ | $2b$ | $-6c$ |

NOTE. The positive differences for Monday, Tuesday, Thursday, and Friday indicate that A's gain was greater (or his loss less) than B's. The negative differences for Wednesday and Saturday indicate that A's gain was less (or his loss greater) than B's.

Regard the following minuends as representing distances one boat sails from a given point, and the subtrahends as representing distances another boat sails from the same point. Distances sailed north are here represented by positive numbers, and distances sailed south by negative numbers. Find how far the first boat is from the second:

| | **1.** | **2.** | **3.** | **4.** | **5.** | **6.** |
|---|---|---|---|---|---|---|
| 1st B., | 75 | 85 | 15 | $8a$ | $-4b$ | $-15c$ |
| 2d B., | 25 | $-35$ | 35 | $2a$ | $-9b$ | $-5c$ |
| | 50 | 120 | $-20$ | | | |

NOTE. The positive differences in examples **1**, **2**, **4**, and **5** indicate that the first boat is north of the second boat. The negative differences in examples **3** and **6** indicate that the first boat is south of the second boat.

From the foregoing learn that the subtraction of a positive number gives the same result as the addition of an equal negative number; and the subtraction of a negative number the same result as the addition of an equal positive number.

---

* Remember that positive numbers are to represent gains and negative numbers losses.

Hence the rule for algebraic subtraction:

*Conceive the sign (or signs) of the subtrahend to be changed* (– to + and + to –), *then proceed as in addition.*

1. A gained $1200 and lost $250; B gained $500 and lost $350. How much more was A's wealth increased by the two transactions than B's?

$$\begin{array}{ccc}
\textbf{1.} & \textbf{2.} & \textbf{3.} \\
\$1200-\$250 & 12a-5b & 14a-3b \\
\underline{\$500-\$350} & \underline{5a-7b} & \underline{6a-5b} \\
\$700+\$100 & & 
\end{array}$$

4. C gained $1500 and $650; D gained $600 and lost $250. How much more was C's wealth increased by the two transactions than D's?

$$\begin{array}{ccc}
\textbf{4.} & \textbf{5.} & \textbf{6.} \\
\$1500+\$650 & 15a+13b & 13a+5b \\
\underline{\$600-\$250} & \underline{6a-5b} & \underline{4a-3b} \\
\$900+\$900 & & 
\end{array}$$

7. E gained $1300 and lost $450; F gained $400 and $250. How much more was E's wealth increased by the two transactions than F's?

$$\begin{array}{ccc}
\textbf{7.} & \textbf{8.} & \textbf{9.} \\
\$1300-\$450 & 13a-9b & 17a-8b \\
\underline{\$400+\$250} & \underline{4a+5b} & \underline{4a+6b} \\
\$900-\$700 & & 
\end{array}$$

Review the foregoing and observe that in every instance *subtracting a positive number is equivalent to adding an equal negative number*, and *subtracting a negative number is equivalent to adding an equal positive number.*

1. From $+\ 6a$ subtract $+2a$.*
2. From $+\ 9a$ subtract $-5a$.
3. From $-\ 8a$ subtract $-3a$.
4. From $-11a$ subtract $+5a$.
5. From $+12a$ subtract $-9a$.
6. From $-15a$ subtract $+4a$.

7. Two boats, F and G, start from the same point. F sails 15 miles north and G sails 6 miles south. F is how far and in what direction from G?

**OPERATION**

$$\begin{array}{r} +15 \text{ miles} \\ -\ \ 6 \text{ miles} \\ \hline +21 \text{ miles} \end{array}$$

**INTERPRETATION**

The "+ 21 miles" indicates that F is 21 miles north of G.

8. Two boats, F and G, start from the same point. F sails 12 miles south and G sails 18 miles south. F is how far and in what direction from G?

**OPERATION**

$$\begin{array}{r} -12 \text{ miles} \\ -18 \text{ miles} \\ \hline +\ \ 6 \text{ miles} \end{array}$$

**INTERPRETATION**

The "+ 6 miles" indicates that F is 6 miles north of G.

9. From $6x + 3y - 2z$ subtract $2x - y + z$.
10. From $4x - 5y + 3z$ subtract $x + 3y + 2z$.
11. From $5x - y + z$ subtract $2x + 2y + 2z$.
12. From $2x - 6y + 3z$ subtract $5x + 2y + z$.

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* Think of two persons, D and E, starting from the same point and stepping north or south. Think of $+a$ as standing for a step north and $-a$ as standing for a step south. D takes six steps ($6a$) north and E takes two steps ($2a$) north. The question is, How far and in what direction is D from A? The answer, $+4a$ indicates that D is 4 steps north of A. Interpret the answers to problems **2**, **3**, **4**, **5**, and **6**.

**1.**

$$\begin{array}{l} 8+4-3 \\ 4 \\ \hline 32+16-12 \end{array}$$

**2.**

$$\begin{array}{l} a+3b-4c \\ 4 \\ \hline 4a+12b-16c \end{array}$$

**3.**

$$\begin{array}{l} 3a-2b+c \\ d \\ \hline 3ad-2bd+cd \end{array}$$

**4.**

$$\begin{array}{l} 2a+3b-5c \\ 2d \\ \hline 4ad+6bd-10cd \end{array}$$

**Observe that in the above examples we multiply each term of the multiplicand by the multiplier. As a matter of convenience it is best to begin with the term of the multiplicand on the left side and proceed from left to right.**

**5.** Prove example **1** by uniting the terms of the multiplicand, and comparing 4 times the number thus obtained with the number obtained by uniting the terms of the product.

**6.** Verify example **2** by letting $a=5$, $b=3$, and $c=2$.

**7.** Verify example **3** by giving the following values to the letters: $a=7$, $b=4$, $c=3$, $d=5$.

**8.** Verify example **4** by giving any values you may choose to each letter.

**9.** Multiply $3ab-2bc+5c$ by $2d$.

**10.** Multiply $2ax+4bx-y$ by 5.

**11.** Multiply $3bc+ab-bc$ by $3d$.

**12.** Multiply $x-y+z$ by $3ab$.

**13.** Multiply $ax+bx-cx$ by $2y$.

**14.** Verify each of the above problems by giving the following values to the letters: $a=3$, $b=2$, $c=4$, $d=5$, $x=7$, $y=6$, $z=8$.

## THE EXPONENT

$a \times a$, or $aa$, which means $a$ multiplied by $a$, is usually written $a^2$. This is read $a$ square or $a$ second power.

$b^3$ (to be read $b$ cube or $b$ third power) means $b$ taken three times as a factor. It is $b \times b \times b$.

$a^4$ (to be read $a$ fourth power, or simply $a$ fourth) means that $a$ is taken four times as a factor. It is $a \times a \times a \times a$.

The small figure at the right of a letter tells the number of times the letter is to be used as a factor. The figure so used is called an **exponent**. When the exponent is 1, it is not usually expressed; thus $a$ means $a^1$.

On the supposition that $a = 2$, $b = 3$, and $c = 4$, find the numerical value of each of the following expressions:

1. $a^2 + 2\,ab + b^2$
2. $5\,a^2b - 2\,bc$
3. $3\,ab^2 + 5\,bc^2$
4. $a^3b^3 - c^3$
5. $4\,a^2b^2 + 3\,b^2c^2$
6. $a^2b^2 + c^2$
7. $2\,a^2b + 2\,ab^2$
8. $a^2b^2c^2 - ab^3$
9. $3\,b^2c^2 + 5\,ab$
10. $2\,a^2b^2c^2$

## MULTIPLICATION

**1.**

$$\begin{array}{l} 4\,ax + 2\,by + c \\ a^2 \\ \hline 4\,a^3x + 2\,a^2by + a^2c \end{array}$$

**2.**

$$\begin{array}{l} 3\,b^2x + 2\,by - c \\ 2\,b^2 \\ \hline 6\,b^4x + 4\,b^3y - 2\,b^2c \end{array}$$

Observe that in the above examples the *product of the coefficients* and the *sum of the exponents* is taken.

**3.** Verify each of the above examples by letting $a = 2$, $b = 3$, $c = 4$, $x = 5$, $y = 6$.

**1.** Multiply $2a + 3b + 4c$ by $x$.

**2.** Multiply $x^2 + xy + y^2$ by $2x$.

**3.** Multiply $a^2 - ab + b^2$ by $ab$.

**4.** Multiply $x^2 + 2xy + y^2$ by $2y^2$.

**5.** Multiply $a^2 + 2ab + b^2$ by $ab$.

Multiply:

**6.** $x + 4$ by $x + 3$

OPERATION

$$\begin{array}{r}
x + 4 \phantom{{}+12} \\
x + 3 \phantom{{}+12} \\
\hline
x^2 + 4x \phantom{{}+12} \\
3x + 12 \\
\hline
x^2 + 7x + 12
\end{array}$$

**7.** $x + y$ by $x + y$

OPERATION

$$\begin{array}{r}
x + y \phantom{{}+y^2} \\
x + y \phantom{{}+y^2} \\
\hline
x^2 + xy \phantom{{}+y^2} \\
xy + y^2 \\
\hline
x^2 + 2xy + y^2
\end{array}$$

Multiply:

**8.** $y + 5$ by $y + 3$

**9.** $y + 2z$ by $y + 2z$

**10.** $x - 4$ by $x - 3$

**11.** $x - y$ by $x - y$

**12.** $a - b$ by $a - b$

**13.** $a + b$ by $a + b$

**14.** $a + b$ by $a - b$

**15.** $a^2 + b^2$ by $a + b$

**16.** $a^2 - b^2$ by $a - b$

**17.** $a + b$ by $a^2 + b^2$

**18.** $a^2b + b^2$ by $a + b$

**19.** $ab^2 + b$ by $a + b$

**20.** $ax + bx$ by $ax + bx$

**21.** $2ax + 4$ by $3ax + 3$

**22.** $3ax - 5$ by $x - 3$

**23.** $4ax - y$ by $x - y$

**24.** $ax - bx$ by $a + b$

**25.** $2a + 3b$ by $5b - 2b$

**26.** $m + 5n$ by $m - 3n$

**27.** $6m + 9$ by $2m + 5$

**28.** Multiply $a + b$ by $a + b$ and the product thus obtained by $a + b$; that is, use $a + b$ three times as a factor.

**29.** Verify problem **28** by giving the following values to the letters: $a = 2$, $b = 3$.

**1.**

$$\begin{array}{r@{}l} 4) & \underline{12 + 5 \times 4 - 8} \\ & \;3 \;+\; 5 \;-\; 2 \end{array}$$

**2.**

$$\begin{array}{r@{}l} a) & \underline{12\,a + 2\,ab + 4\,a} \\ & 12 \;+ 2\,b \;+ 4 \end{array}$$

**3.**

$$\begin{array}{r@{}l} 2) & \underline{2^3 + 3 \times 2^2 - 2} \\ & 2^2 + 3 \times 2 \;-1 \end{array}$$

**4.**

$$\begin{array}{r@{}l} b) & \underline{ab^3 + cb^2 + 3\,b} \\ & ab^2 + cb \;+ 3 \end{array}$$

Observe that in examples **1** to **4** we divide each term of the dividend by the divisor.

**5.** Prove examples **1** and **3**, by (1) reducing each dividend to its simplest form, (2) dividing it, so reduced, by the divisor, and (3) comparing the result with the quotient reduced to its simplest form.

**6.** Verify example **2**, by letting $a = 3$, and $b = 5$.

**7.** Verify example **4**, by letting $a = 3$, $b = 5$, and $c = 7$.

**8.** $(6 \times a \times a \times a \times a \times a) \div (2 \times a \times a) = 6\,a^5 \div 2\,a^2 = 3\,a^3$.

Observe that to divide one algebraic term by another we must find the *quotient of the coefficients* and the *difference of the exponents.*

**9.** $6\,a^5b \div 2\,a$ **10.** $8\,x^3b^3 \div 2\,a$

**11.** $4\,a^4b^2 \div 2\,a$ **12.** $10\,a^2b^4 \div 2\,a$

**13.**

$$2\,a)\underline{6\,a^5b + 4\,a^4b^2 - 8\,a^3b^3 + 10\,a^2b^4}$$

**14.** Verify problem **13**, by letting $a = 3$, and $b = 5$.

**15.**

$$ab)\underline{a^5b + a^4b^2 + a^3b^3 + a^2b^4 + ab^5}$$

**16.** Verify problem **15**, by letting $a = 2$, and $b = 3$.

**17.** Verify problem **15**, by letting $a = 3$, and $b = 2$.

**1.** Divide $4a^3x + 8a^2x^2 + 6ax^3$ by $2ax$.

**2.** Multiply the quotient of problem **1** by $2ax$.

**3.** Verify problems **1** and **2**, by letting $a = 2$, and $x = 3$.

**4.** Divide $3ab^3 + 6a^2b^2 + 9a^3b$ by $3ab$.

**5.** Multiply the quotient of problem **4** by $3ab$.

**6.** Verify problems **4** and **5**, by letting $a = 3$, and $b = 5$.

**7.** Divide $2x^3y + x^2y^2 - xy^3$ by $xy$.

**8.** Multiply the quotient of problem **7** by $xy$.

**9.** Verify problems **7** and **8**, by letting $x = 2$, and $y = 3$.

**10.** Divide $5a^3y^2 - 2a^2y^3 + a^2y^4$ by $a^2y$.

**11.** Multiply the quotient of problem **10** by $a^2y$.

**12.** Verify problems **10** and **11**, by letting $a = 1$, and $y = 2$.

**13.** Divide $3b^4x + b^3x^2 - 3b^2x^3$ by $bx$.

**14.** Multiply the quotient of problem **13** by $bx$.

**15.** Verify problems **13** and **14**, by letting $b = 3$, and $x = 4$.

Observe that when the divisor is a positive number, each term of the quotient has the same sign as the term in the dividend from which it was derived.

$$\begin{array}{r} 2\overline{)8 - 6} \\ 4 - 3 \end{array}$$ One half of $+8$ is $+4$; one half of $-6$ is $-3$.

**16.**

$$2x\overline{)4x^5 - 6x^4 + 8x^3 - 2x^2 + 6x}$$

**17.** Verify problem **16**, by letting $x = 2$.

When an expression consisting of two or more terms is to be treated as a whole, it may be inclosed in a parenthesis.

$$\left\{\begin{array}{l} 12 + (5 + 3) = ? \\ 12 + 5 + 3 = ? \end{array}\right. \qquad \left\{\begin{array}{l} 7a + (3a + 2a) = ? \\ 7a + 3a + 2a = ? \end{array}\right.$$

Observe that removing the parenthesis makes no change in the results.

$$\left\{\begin{array}{l} 12 - (5 + 3) = ? \\ 12 - 5 - 3 = ? \end{array}\right. \qquad \left\{\begin{array}{l} 7a - (3a + 2a) = ? \\ 7a - 3a - 2a = ? \end{array}\right.$$

Observe the change in signs made necessary by the removal of the parenthesis.

A careful study and comparison of the foregoing problems will make the reasons for the following apparent:

I. *If an expression within a parenthesis is preceded by the plus sign, the parenthesis may be removed without making any changes in the signs of the terms.*

II. *If an expression within a parenthesis is preceded by a minus sign, the parenthesis may be removed; but the sign of each term in the parenthesis must be changed — the sign + to − and the sign − to +.*

(1) Remove the parenthesis, (2) change the signs if necessary, (3) combine the terms:

1. $15 - (6 + 4)$
2. $15b - (12b - 4b)$
3. $18 + (4 - 3)$
4. $18c + (9c - 3c)$
5. $27 - (8 + 3)$
6. $24d - (5d + 3d)$
7. $45 + (12 - 3)$
8. $36x - (5x + 4x)$
9. $75 - (18 + 27)$
10. $45y + (8y + 7y)$

**To multiply an expression inclosed in a parenthesis:**

$6(7+4)=?$* $\qquad 6(7a+4b)=?$

$6(a+b)=?$ *Ans.* $6a+6b$.

$a(b+c)=?$ *Ans.* $ab+ac$.

Observe that in multiplying the sum of two numbers by a third number, the sum may be found and multiplied; or each number may be multiplied and the sum of the products found.

1. In the last three examples given above, let $a=5$, $b=3$, and $c=2$; then perform again the operations indicated, and compare the results with those obtained when the letters were employed.

$6(7-4)=?$† $\qquad 6(7a-4b)=?$

$6(a-b)=?$ *Ans.* $6a-6b$.

$a(b-c)=?$ *Ans.* $ab-ac$.

Observe that in multiplying the difference of two numbers by a third number, the difference may be found and multiplied; or each number may be multiplied and the difference of the products found.

2. In the last three examples given above, let $a=5$, $b=3$, and $c=2$; then perform again the operations indicated and compare the results with those obtained when the letters were employed.

If $a=5$, $b=3$, and $c=2$, find the value of the following:

3. $3(a+b)-2(b+c)$
4. $4(a+2b)-3(b-c)$
5. $2(2a-b)+2(2b-c)$

---

* This means that the sum of 7 and 4 is to be multiplied by six; or that the sum of six 7's and six 4's is to be found.

† This means that the difference of 7 and 4 is to be multiplied by six; or that the difference of six 7's and six 4's is to be found.

A statement of numerical equality is called an **equation**.*

EQUATIONS

1. $2 + 4 + 6 = 3 + 5 + 4$
2. $a + b + c = 40 - 12$
3. $2x + x + 4 = 10 + 9$

Every equation is made up of two members. The part of the equation which is on the left of the sign of equality is called the **first member**; the part on the right of the sign of equality, the **second member**.

I. *If the same number is added to each member of an equation, the equality will not be destroyed.*

If $x = 8$, then $x + 4 = 8 + 4$.
If $a + b = 16$, then $a + b + c = 16 + c$.

II. *If the same number is subtracted from each member of an equation, the equality will not be destroyed.*

If $x = 8$, then $x - 3 = 8 - 3$.
If $a + b = 16$, then $a + b - c = 16 - c$.

III. *If each member of an equation is multiplied by the same number, the equality will not be destroyed.*

If $x = 8$, then $4x = 4$ times 8, or 32.
If $a + b = 16$, then $4a + 4b = 4$ times 16, or 64.

IV. *If each member of an equation is divided by the same number, the equality will not be destroyed.*

If $x = 8$, then $\frac{x}{4} = \frac{8}{4}$, or 2.

If $a + b = 16$, then $\frac{a}{4} + \frac{b}{4} = \frac{16}{4}$, or 4.

---

* Some authors apply the term *equation* to numerical equalities only when they contain an unknown quantity.

Any term in an equation may be transposed from one member of the equation to the other; but its *sign must be changed when the transposition is made.*

If $x + 5 = 15$, then $x = 15 - 5$, or 10.*
If $y - 6 = 27$, then $y = 27 + 6$, or 33.†
If $a + b + c = 18$, then $a + b = 18 - c$.
If $x + y - z = 25$, then $x + y = 25 + z$.

**To find the number for which $x$ stands, in an equation in which there is no other unknown number:**

**1.**

Equation, $x + 2x + 3x - 5 = 13$
Transposing, $x + 2x + 3x = 13 + 5$
Uniting, $6x = 18$
Dividing, $x = 3$

**2.**

Equation, $2x + 3x + 6 = 5x - 2x + 18$
Transposing, $2x + 3x - 5x + 2x = 18 - 6$
Uniting, $2x = 12$
Dividing, $x = 6$

Find the value of $x$:

**3.** $x + 4 = 12$
**4.** $3x + 2x - 4 = x + 16$
**5.** $x + 3x = 8$
**6.** $5x - 7 = 3x + 5$
**7.** $5x - 2 = 23$
**8.** $7x + 2x - x = 3x + 35$
**9.** $3x - x = 44$
**10.** $5x - 4x - 3x + 6x = 44$
**11.** $7x + x = 144$
**12.** $6x - 8 - 2x = 3x + 5$
**13.** $5x + 6 = 131$
**14.** $4x - 2x + 10 = x + 15$

---

*** Observe that 5 is subtracted from each member of the equation.**

**† Observe that 6 is added to each member of the equation.**

**1.**

$$\frac{x}{2}+2x=30$$

Multiplying both members of the equation by 2, the denominator of the fraction in the equation, we have

$$x+4x=60$$

Uniting terms, $5x=60$

Dividing by 5, $x=12$

**2.**

$$\frac{x}{2}+\frac{x}{3}+5x=70.$$

Multiplying by 2, $x+\frac{2x}{3}+10x=140$

Multiplying by 3, $3x+2x+30x=420$ *

Uniting terms, $35x=420$

Dividing by 35, $x=12$

Find the value of $x$:

**3.** $\frac{x}{5}+\frac{x}{3}=8$

**4.** $\frac{x}{3}+5x=64$

**5.** $\frac{x}{2}-\frac{x}{3}=6$

**6.** $2x-\frac{x}{3}=50$

**7.** $\frac{x}{3}+\frac{x}{2}=15$

**8.** $\frac{x}{3}+3x=40$

**9.** $\frac{x}{4}+\frac{x}{5}=9$

**10.** $\frac{x}{5}+2x=33$

**11.** $\frac{x}{4}=x-12$

**12.** $25+\frac{x}{3}=30$

---

* Observe that the equation might have been cleared of fractions by multiplying both its members by 6, the l.c.m. of 2 and 3.

## PROBLEMS LEADING TO EQUATIONS CONTAINING ONE UNKNOWN QUANTITY WITHOUT FRACTIONS

**1.** John and Henry together have 60 oranges, and Henry has three times as many as John. How many has each?

Let $x =$ the number John has
Then $3x =$ the number Henry has
and $x + 3x =$ the number they together have
But they together have 60.
Therefore, $x + 3x = 60$
Uniting, $4x = 60$
Dividing, $x = 15$
Multiplying, $3x = 45$

Therefore, John has 15 oranges and Henry has 45 oranges.

**2.** The sum of two numbers is 275, and the greater is four times the less. What are the numbers?

**3.** Robert has a certain sum of money, and Harry has five times as much; together they have $216. How many dollars has each?

**4.** One number is four times another, and their difference is 270. What are the numbers?

**5.** Peter has a certain number of marbles and William has 8 more than Peter; together they have 96 marbles. How many has each?

**6.** Sarah has a certain number of pennies and her sister has nine more than twice as many; together they have 93. How many has each?

**7.** Two times Reuben's money plus three times his money equals 175 dollars. How many dollars has he?

1. Mr. Rice bought a horse and carriage for \$450. He paid twice as much for the carriage as for the horse. Find the cost of each.

2. Find two numbers whose difference is 65, and one of them 6 times the other.

3. Find two numbers whose difference is 28 and whose sum is 62.

4. A man bought a harness, a horse, and a carriage for \$400. The horse cost three times as much as the harness, and the carriage twice as much as the horse. Find the cost of each.

5. Divide \$3.15 (315 cents) among three boys, giving to the second 25¢ more than to the first, and to the third twice as much as to the second.

6. Frank and John have 64 marbles. Frank has four more than twice as many as John. How many has each?

7. To three times a number I add 11 and have 65. What is the number?

8. Find four consecutive integral numbers whose sum is 70.*

9. A man bought 3 chairs and 4 tables for \$22. Each table cost twice as much as a chair. Find the cost of each.

10. The difference of two numbers is 6, and three times the smaller number equals twice the larger number. What are the numbers?

11. Mary is 2 years older than John, and three times Mary's age equals four times John's age. Find the age of each.

---

* Let $x$ equal the smallest number; then $x + 1 =$ the next, etc.

## PROBLEMS LEADING TO EQUATIONS CONTAINING FRACTIONS

**1.** If to $\frac{1}{3}$ of Bernard's money you add $\frac{1}{4}$ of his money, the sum is \$63. How much money has he?

Let $x =$ the number of dollars he has

Then $\frac{x}{3}+\frac{x}{4}=63$

Multiplying by 3, $x+\frac{3x}{4}=189$

Multiplying by 4, $4x+3x=756$*

Uniting terms, $7x=756$

Dividing by 7, $x=108$

Thus we find that Bernard has 108 dollars.

**2.** The sum of $\frac{1}{4}$ of a certain number and $\frac{1}{5}$ of the same number, is 45. What is the number?

**3.** A lady upon being asked her age replied: If from $\frac{1}{2}$ of my age you subtract $\frac{1}{5}$ of my age, the remainder will be 9 years. How old was she?

**4.** The sum of two numbers is 72, and the less number equals $\frac{3}{5}$ of the greater number. What are the numbers?

Let $x =$ the greater number.

**5.** The sum of two numbers is 75, and the less number equals $\frac{2}{3}$ of the greater. What are the numbers?

**6.** The difference of two numbers is 30, and the less number equals $\frac{3}{5}$ of the greater number. What are the numbers?

---

* Observe that the equation might have been cleared of fractions by multiplying both its members by 12, the l.c.m. of 3 and 4.

**1.** Henry is 1 fourth as old as his mother and the sum of their ages is 45 years. How old is each?

**2.** George's money is equal to 2 fifths of Harry's money, and Harry has $18 more than George. How much has each?

Let $x =$ the number of dollars Harry has

Then $\frac{2x}{5} =$ the number of dollars George has

and $x = \frac{2x}{5} + 18$

**3.** In a mixture of 729 bushels of grain there are 4 fifths as many bushels of corn as of oats. How many bushels are there of each?

**4.** What number is that which being increased by 5 sixths of itself equals 132?

**5.** Albert had a certain number of marbles and William had 2 thirds as many. Albert gave William 8 marbles and then each had the same number. How many had each at first?

Let $x =$ the number of marbles Albert had

Then $\frac{2x}{3} =$ the number of marbles William had

and $x - 8 = \frac{2x}{3} + 8$

**6.** What number diminished by 1 half of itself equals 1 third of itself plus 10?

**7.** A boy is 12 years old and his father is 36 years old. In how many years will the son be half as old as his father?

**8.** In how many years will the boy whose age is given in problem **12** be 2 thirds as old as his father?

1. Harry has some marbles; Joseph has 8 more than $\frac{1}{2}$ as many as Harry; together they have 68. How many has each?

2. William has 12 more than $\frac{1}{2}$ as many cents as Lucius; together they have 87 cents. How many cents has each?

3. If to the half of a certain number you add a third of the number, the sum will be 5 more than 3 fourths of the number. What is the number?

Let $x =$ the number

Then $\frac{x}{2} + \frac{x}{3} - 5 = \frac{3x}{4}$

4. If to $\frac{1}{2}$ of a number you add $\frac{1}{4}$ of the same number, the sum will be 12 more than $\frac{3}{5}$ of the number. What is the number?

5. If to a certain number you add 5 times the number, the sum will be 24 more than 4 times the number. What is the number?

6. If to $\frac{1}{2}$ of a number you add $\frac{1}{3}$ of the number, the sum will be $a$. What is the number?

Let $x =$ the number

Then $\frac{x}{2} + \frac{x}{3} = a$

Multiplying by 6, $3x + 2x = 6a$

Uniting, $5x = 6a$

Dividing by 5, $x = \frac{6a}{5}$

Observe that you may put in the place of $a$ any number you please; hence any number is $\frac{6}{5}$ of the sum of its half and third.

Reason in a similar manner with reference to the half and fourth of a number. To the third and fourth.

**To find two numbers when their sum and difference are given:**

**1.** The difference of two numbers is 4 and their sum is 20. What are the numbers?

Let $x =$ the smaller number
Then $x + 4 =$ the larger number
and $x + x + 4 = 20$
Transposing, $x + x = 20 - 4$
Uniting, $2x = 16$
Dividing, $x = 8$, the smaller number
$x + 4 = 12$, the larger number

**2.** The difference of two numbers is 9 and their sum is 119. What is the smaller number?

**3.** The difference of two fractions is $\frac{9}{20}$ and their sum is $\frac{3}{4}$. What are the fractions?

**4.** The difference of two numbers is $d$ and the sum is $s$. What is the smaller number?

Let $x =$ the smaller number
Then $x + d =$ the larger number
and $x + x + d = s$
Transposing, $x + x = s - d$
Uniting, $2x = s - d$
Dividing, $x = \frac{s - d}{2}$

Observe that any number you please may be put in the place of $s$, and any number less than $s$ in the place of $d$; so when the sum and the difference of two numbers are given, the *smaller number* may be found by *subtracting the difference from the sum and dividing the remainder by* 2.

**5.** The difference of two numbers is 327, and their sum is 1159. What are the numbers?

**Another method of finding two numbers when their sum and difference are given:**

1. The difference of two numbers is 17, and their sum is 69. What is the larger number?

Let $x =$ the larger number
Then $x - 17 =$ the smaller number
and $x + x - 17 = 69$
Transposing, $x + x = 69 + 17$
Uniting, $2x = 86$
Dividing, $x = 43$, the larger number
$x - 17 = 26$, the smaller number

2. The difference of two numbers is 8.4 and their sum 75.6. What are the numbers?

3. The difference of two numbers is $d$ and their sum is $s$. What are the numbers?

Let $x =$ the larger number
Then $x - d =$ the smaller number
and $x + x - d = s$
Transposing, $x + x = s + d$
Uniting, $2x = s + d$
Dividing, $x = \frac{s + d}{2}$

Observe that any number you please may be put in the place of $s$, and any number less than $s$ in the place of $d$; so when the sum and difference of two numbers are given, the *larger number* may be found by *adding the difference to the sum and dividing the result by* 2.

4. A horse and a harness together are worth $140, and the horse is worth $74. more than the harness. Find the value of each.

**1.** In a school there are 896 pupils. There are three times as many boys as girls. How many girls are there? How many boys?

**2.** A man had 235 sheep. In the second flock there were 15 more sheep than in the first. In the third flock there were 20 fewer than in the first. How many sheep in each flock?

NOTE. Let $x =$ the number in the first flock; then $x + 15 =$ the number in the second, and $x - 20$, the number in the third.

**3.** A man owns three farms. In the second there are half as many acres as in the first. In the third there are twice as many acres as in the first. In all there are 560 acres. How many acres in each farm?

**4.** In an apple and pear orchard containing 296 trees, there were 5 more than twice as many apple trees as pear trees. How many of each kind?

**5.** To a number I add one half of itself and 15 and have 150. What is the number?

**6.** From three times a number I subtract $\frac{2}{3}$ of the number and 5, and have 37 remaining. What is the number?

NOTE. Let $x =$ the number; then $3x - \left(\frac{2x}{3} + 5\right) = 37$. On removing the parenthesis, what sign must be changed? See page 318.

**7.** If to three times a number I add $\frac{2}{3}$ of the number and 18, the sum will be 238. What is the number?

**8.** Two thirds of a number is equal to the number decreased by 56. What is the number?

$$\frac{2x}{3} = x - 56.$$

**To find two numbers when their sum and ratio are given:**

1. The sum of two numbers is 36 and their ratio is 3. What are the numbers?

Let $x =$ the smaller number
Then $3x =$ the larger number
and $x + 3x = 36$
$4x = 36$
$x = 9$, the smaller number
$3x = 27$, the larger number

*Verification.* The ratio of 27 to 9 is 3. $27 + 9 = 36$.

2. The sum of two numbers is 196, and their ratio is 3. What are the numbers?

3. The sum of two numbers is 294, and their ratio is $2\frac{1}{2}$. What are the numbers?

4. The sum of 2 decimals is .42, and their ratio is $2\frac{1}{2}$. What are the numbers?

5. The sum of two numbers is $s$, and the ratio of the larger to the smaller is $r$. What is the smaller number?

Let $x =$ the smaller number
Then $rx =$ the larger number
and $rx + x = s$
or $(r + 1)x = s$
Dividing, $x = \frac{s}{r + 1}$

Observe that any number you please may be put in the place of $s$, and any number greater than 1 in the place of $r$; so that when the sum of two numbers and the ratio of the larger to the smaller are given, the *smaller number may be found by dividing the sum by the ratio plus* 1.

**To find the missing term of a proportion without finding the ratio :**

**1.** Let $a : b = c : d$, stand for any proportion.

Then $$\frac{a}{b} = \frac{c}{d}$$

Clearing of fractions, $$ad = bc$$

But $a$ and $d$ are the extremes, and $b$ and $c$, the means.

Hence, in any proportion in which abstract numbers are employed, *the product of the means equals the product of the extremes.*

Therefore (1) If the extremes and one of the means are given, to find the missing mean, *divide the product of the extremes by the given mean.*

(2) If the means and one of the extremes are given, to find the missing extreme, *divide the product of the means by the given extreme.*

Indicate the operation to find the missing term :

**2.** $a : b = x : d.$*

**3.** $a : x = c : d.$

**4.** $x : b = c : d.$

**5.** $a : b = c : x.$

**6.** Verify problems **2** to **5** by letting $a = 6$, $b = 12$, $c = 4\frac{1}{2}$, and $d = 9$.

**7.** The extremes of a proportion are 15 and 40. One of the means is 20. What is the other mean?

**8.** The means of a proportion are 15 and 12. One of the extremes is 8. What is the other extreme?

**9.** The extremes of a proportion are $\frac{1}{2}$ and $\frac{4}{5}$. One of the means is $\frac{2}{3}$. What is the other mean?

* $x = \frac{ad}{b}$.

**To find the square root of numbers represented by letters and figures.**

### EXPLANATION

Since the square root of a number is one of its two equal factors, the square root of $a^4$, $(a \times a \times a \times a)$, is $a^2$, $(a \times a)$. The square root of $a^2$ is $a$. The square root of $a^6$ is $a^3$. Let $a = 3$, and verify each of the foregoing statements.

Extract the square root as indicated :

**1.** $\sqrt{b^4}$ **2.** $\sqrt{b^6}$ **3.** $\sqrt{b^2}$

**4.** $\sqrt{a^2b^2}$ **5.** $\sqrt{a^2b^4}$ **6.** $\sqrt{a^4b^4}$

**7.** $\sqrt{4\,a^2b^2}$ * **8.** $\sqrt{9\,a^4b^2}$ **9.** $\sqrt{16\,a^2b^4}$

**10.** $\sqrt{25\,x^2y^2}$ **11.** $\sqrt{36\,x^2y^4}$ **12.** $\sqrt{49\,x^4y^4}$

Let $a = 2$, $b = 3$, and $c = 5$, and find the numerical value of each of the following :

**13.** $\sqrt{a^2b^4}$ **14.** $\sqrt{a^4c^2}$ **15.** $\sqrt{b^2c^4}$

**16.** $\sqrt{a^4b^4}$ **17.** $\sqrt{a^2b^2c^2}$ **18.** $\sqrt{a^2b^4c^2}$

Let $a = 2$, $b = 3$, $x = 5$, and $y = 7$, and find the numerical value of each of the following :

**19.** $a\sqrt{x^2y^2}$ **20.** $b\sqrt{x^4y^2}$ **21.** $a\sqrt{x^4y^4}$

**22.** $3\,a\sqrt{x^4}$ **23.** $4\,b\sqrt{y^4}$ **24.** $5\,a\sqrt{x^4y^4}$

Reduce each of the following to its simplest form :

**25.** $7\,a\sqrt{x^2y^2}$ **26.** $5\,b\sqrt{x^4y^4}$ **27.** $6\,c\sqrt{4\,a^2b^2}$

---

* The factors of $4\,a^2b^2$ are 2, 2, $a$, $a$, $b$, $b$.

**To find the cube root of numbers represented by letters and figures:**

### EXPLANATION

Since the cube root of a number is one of its three equal factors, the cube root of $a^6$, $(a \times a \times a \times a \times a \times a)$, is $a^2$, $(a \times a)$. The cube root of $a^3$ is $a$. The cube root of $a^9$ is $a^3$. Let $a = 2$, and verify each of the foregoing statements.

Perform the operations indicated:

**1.** $\sqrt[3]{b^6}$ **2.** $\sqrt[3]{b^3}$ **3.** $\sqrt[3]{b^9}$

**4.** $\sqrt[3]{a^3b^3}$ **5.** $\sqrt[3]{a^6b^3}$ **6.** $\sqrt[3]{a^6b^6}$

**7.** $\sqrt[3]{8a^6}$ **8.** $\sqrt[3]{27b^3}$ **9.** $\sqrt[3]{64a^6}$

Let $a = 2$, $b = 3$, and $c = 5$, and find the numerical value of each of the following:

**10.** $\sqrt[3]{a^3b^6}$ **11.** $\sqrt[3]{8c^3}$ **12.** $\sqrt[3]{27b^3}$

**13.** $2\sqrt[3]{a^6b^3}$ **14.** $3\sqrt[3]{b^3c^3}$ **15.** $4\sqrt[3]{b^6c^3}$

Let $a = 2$, $b = 3$, $x = 5$, and $y = 7$, and find the numerical value of each of the following:

**16.** $ab + \sqrt{x^2y^4}$ **17.** $ab\sqrt{x^2y^4}$ **18.** $2a\sqrt{x^4y^4}$

**19.** $ab + \sqrt[3]{x^3y^6}$ **20.** $ab\sqrt[3]{x^3y^6}$ **21.** $2b\sqrt[3]{x^6y^6}$

**22.** $\frac{1}{4}\sqrt{a^4b^2}$ **23.** $\sqrt{\frac{1}{4}a^4b^2}$ * **24.** $\frac{3}{4}\sqrt{a^4x^2}$

Reduce each of the following to its simplest form:

**25.** $\sqrt{x^2y^2} + \sqrt[3]{x^3y^3}$ **26.** $3\sqrt{x^2y^2} - 2\sqrt[3]{x^3y^3}$

**27.** $\sqrt{a^4b^4} + \sqrt[3]{a^6b^6}$ **28.** $4\sqrt{4a^2b^2} + 3\sqrt[3]{a^3b^3}$

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* The factors of this number are $\frac{1}{2}$, $\frac{1}{2}$, $a$, $a$, $a$, $a$, $b$, $b$.

1. If a piece of land containing 768 square rods is three times as long as it is wide, how wide is it?*

Let $x =$ the width
Then $3x =$ the length
and $x(3x)$ or $3x^2 =$ the area
$3x^2 = 768$
$x^2 = 256$
$x = 16$

2. If a certain room is twice as long as it is wide, and the area of the floor is 968 square feet, what are the length and the breadth of the room?

3. One half of the length of Mr. Smith's farm is equal to the breadth of the farm. The farm contains 80 acres. How many rods of fence will be required to inclose it?

4. Each of four of the faces of a square prism is an oblong whose length is twice its breadth. The area of one of these oblongs is 72 square inches. What is the solid content of the prism? †

5. The width of a certain field is to its length as 2 to 3. Its area is 600 square rods. The perimeter of the field is how many rods?

6. If $\frac{3}{4}$ of the length of an oblong equals the width and its area is 768 square inches, what is the length of the oblong?

7. If to $2\frac{1}{2}$ times the square of a number you add 15, the sum is 375. What is the number?

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[*] To solve this problem arithmetically, one must discern that this piece of land can be divided into three equal squares, the side of each square being equal to the width of the piece.

[†] There are two unequal prisms either of which is in accord with the description here given. Hence there are two answers to this problem.

1. I am thinking of a rectangular surface. Its length is 5 times its breadth. Its area is 45 square decimeters. How long and how wide is the surface?

NOTE. Let $x =$ the number of decimeters in the breadth of the surface.

2. I am thinking of a triangular surface. Its base is three times its altitude. Its area is 8.64 square meters. What is the length of its base?

3. I am thinking of a cube whose entire surface is 150 square centimeters. What is the length of one of its edges?

4. The perimeter of a certain rectangle is 20.4 meters. Its length is twice its breadth. *a.* Find its length and breadth. *b.* Find its area.

5. The difference in the weight of two lead balls is 24 grams. The united weight of the two balls is 1 kilogram. *a.* Find the weight of each ball. *b.* Does the heavier ball weigh more or less than 1 pound?

6. A merchant had three pieces of lace. In the second piece there were twice as many meters as in the first piece. In the third piece there were 6 meters more than in the second piece. In the three pieces there were 106 meters. *a.* How many meters were there in each piece? *b.* Were there more or less than 43 yards in the second piece?

7. John weighs 3.6 kilograms more than Henry. Together they weigh 83.6 kilograms. *a.* Find the weight of each boy. *b.* Does John weigh more or less than 90 pounds?

# DEFINITIONS AND EXPLANATIONS

1. **Arithmetic** treats of numbers.

2. A **unit** is one.

3. A **number** is a unit or it is composed of units.

NOTE. A unit, though regarded as a single thing, is itself made up of minor units; thus, in the expression 1 *dozen* the major unit, *one*, includes twelve minor units, but each of the minor units becomes a major unit when considered with reference to the halves, fourths, tenths, hundredths, thousandths, etc., of which it is composed. So we may have units of hundreds, as 5 hundred; units of thousands, as 5 thousand; units of hundredths, as 5 hundredths. No unit is so small that it is not made up of minor units, and no number is so great that it may not become a minor unit of some greater number.

4. **Notation** is the art of expressing numbers by figures or other characters.

5. In the **Arabic Notation** figures are employed. The figures are : 1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

In the number 7235.468, the figure 5 is said to stand in the first place and represents units of the first order; the figure 3 stands in the second place and represents units of the second order; the figure 2 stands in the third place and represents units of the third order; the figure 7 stands in the fourth place and represents units of the fourth order; the figure 4 stands in the first decimal place and represents units of the first decimal order; the figure 6 stands in the second decimal place and represents units of the second decimal order; the figure 8 stands in the third decimal place and represents units of the third decimal order.

A unit of any order equals ten units of the next lower order.

### 6. Three integral orders form a **period**.

The following shows the names of periods and orders:

| hundreds | tens | units | hundreds | tens | units | hundreds | tens | units | hundreds | tens | units | hundreds | tens | units | tenths | hundredths | thousandths | ten-thousandths | hundred-thousandths | millionths |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 7 | 4, | 3 | 5 | 8, | 6 | 0 | 4, | 3 | 6 | 0, | 2 | 7 | 6. | 1 | 4 | 5 | 6 | 3 | 8 |
| | trillion | | | billion | | | million | | | thous'd | | | units | | | | | | | |

(1) 1 tenth and 4 hundredths = 14 hundredths.

(2) 1 tenth, 4 hundredths, and 5 thousandths = —— thousandths.

(3) 1 tenth, 4 hundredths, 5 thousandths, and 6 ten-thousandths = —— ten-thousandths.

(4) 1 tenth, 4 hundredths, 5 thousandths, 6 ten-thousandths, 3 hundred-thousandths = —— hundred-thousandths.

(5) 1 tenth, 4 hundredths, 5 thousandths, 6 ten-thousandths, 3 hundred-thousandths, and 8 millionths = —— millionths.

(6) In reading decimals, the name of the lowest order is usually given to the entire decimal; thus, .345 is usually read 345 thousandths; not, 3 tenths, 4 hundredths, and 5 thousandths.

(7) A mixed decimal *may* be read by giving the name of the lowest order to the entire number; thus, 6.425 may be read 6425 thousandths; 27.4 may be read 274 tenths.

NOTE. Observe that the name of the 1st period is units; of the 2d, thousand; of the 3d, million; of the 4th, billion, and of the 5th, trillion. The names of the periods above trillion are

as follows: 6th, quadrillion; 7th, quintillion; 8th, sextillion; 9th, septillion; 10th, octillion; 11th, nonillion; 12th, decillion; 13th, undecillion; 14th, duodecillion; 15th, tredecillion; 16th, quatuordecillion; 17th, quindecillion; 18th, sexdecillion; 19th, septendecillion; 20th, octodecillion; 21st, novemdecillion; 22d, vigintillion.

7. In the **Roman Notation**, seven capital letters are employed. The letters are — I, V, X, L, C, D, M. I = 1, V = 5, X = 10, L = 50, C = 100, D = 500, M = 1000.

### PRINCIPLES

(1) When a letter is repeated, its value is repeated. XX = 20. CC = 200. III = 3.

(2) When a letter is placed before one of greater value, the difference of their values must be taken. IX = (10 − 1) = 9. IV = (5 − 1) = 4.

(3) When a letter is placed between two letters, each of greater value, its value is taken from the last letter. XIX = 10 + (10 − 1) = 19.

(4) Placing a short horizontal line over a letter, multiplies its value by 1000. $\overline{\text{X}}$. = 10,000.

Note. Sometimes small letters are employed instead of capitals; thus, ii = 2, iv = 4, etc.

8. **Addition** is the process of uniting two or more numbers into one number.

The **sum** is the number obtained by adding.

The **addends** are the numbers to be added.

The **sign**, +, which is read *plus*, indicates that the numbers between which it is placed are to be added; thus, 6 + 4, means that 4 is to be added to 6.

The **sign**, =, which is read *equal* or *equals*, indicates that what is on the left of the sign is equal to what is on the right of the sign; thus, 3 + 4 = 7. 5 + 4 + 2 = 6 + 5.

Note. There are 45 primary facts of addition which should be perfectly memorized. They are as follows:

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 2 | 3 | 3 | 4 | 3 | 4 | 5 | 4 | 5 | 6 | 4 | 5 | 6 |
| 1 | 1 | 2 | 1 | 2 | 1 | 3 | 2 | 1 | 3 | 2 | 1 | 4 | 3 | 2 |
| 2 | 3 | 4 | 4 | 5 | 5 | 6 | 6 | 6 | 7 | 7 | 7 | 8 | 8 | 8 |
| 7 | 5 | 6 | 7 | 8 | 5 | 6 | 7 | 8 | 9 | 6 | 7 | 8 | 9 | 6 |
| 1 | 4 | 3 | 2 | 1 | 5 | 4 | 3 | 2 | 1 | 5 | 4 | 3 | 2 | 6 |
| 8 | 9 | 9 | 9 | 9 | 10 | 10 | 10 | 10 | 10 | 11 | 11 | 11 | 11 | 12 |
| 7 | 8 | 9 | **7** | **8** | **9** | 7 | **8** | **9** | **8** | **9** | 8 | **9** | **9** | 9 |
| 5 | 4 | 3 | **6** | **5** | **4** | 7 | **6** | **5** | **7** | **6** | 8 | **7** | **8** | 9 |
| 12 | 12 | 12 | **13** | **13** | **13** | 14 | **14** | **14** | **15** | **15** | 16 | **16** | **17** | 18 |

The nine facts given in full-faced type should receive special attention. Pupils seldom fail to memorize the other facts.

9. **Subtraction** is the process of taking one number from (out of) another.

The **minuend** is the number from which another is taken.

The **subtrahend** is the number taken from another number.

The **difference** is the number obtained by subtracting.

The **sign**, –, which is read *minus*, indicates that the number that follows the sign is to be taken from (out of) the number that precedes it; thus, 8 – 3, means that 3 is to be taken from (out of) 8.

**To "prove"** a problem in subtraction: Add the subtrahend and the difference. The sum thus obtained should equal the minuend.

NOTE. There are 81 primary facts of subtraction; that is, 2 for every primary fact of addition except the 1st, 3d, 7th, 13th, 21st, 30th, 37th, 42d, 45th. The facts of subtraction should be learned while learning the facts of addition. If a pupil really knows that 8 and 9 equal 17, he knows also that 17 less 8 = 9, and that 17 less 9 = 8.

10. **Multiplication** is the process of taking one number as many times as there are units in another number.

The **multiplicand** is the number taken or repeated.

The **multiplier** is the number that shows how many times the multiplicand is taken or repeated.

The **product** is the number obtained by multiplying.

The **sign**, ×, which is read *multiplied by*, indicates that the number preceding the sign is to be repeated as many times as there are units in the number following the sign.

NOTE 1. When this sign is followed by a fraction, it indicates that *a certain part* of the number preceding the sign is to be repeated as many times as there are units in the numerator of the fraction following the sign; thus, 12 × $\frac{3}{4}$, means that 1 fourth of 12 is to be repeated 3 times; 50 × .5, means that 1 tenth of 50 is to be repeated 5 times.

NOTE 2. This sign, ×, is sometimes so used that it means *times;* thus, 3 × $6, must be read 3 *times* $6; 3 × 6, may be read 3 *multiplied* by 6 or 3 *times* 6. As employed in this book, *the sign seldom means times.* Instead of 3 × $6, the author prefers $6 × 3. It is believed that the restriction of this sign to one use and to one meaning, at least in the first years of arithmetical study, will promote clearness of thought and accuracy in expression.

NOTE 3. Without danger of ambiguity, the sign, ×, is sometimes used in this book and elsewhere, in place of the word *by;* thus, 1 pc. of 2 × 4, 12 (to be read, 1 *pc. of* 2 *by* 4, 12), means 1 piece of lumber 2 inches thick, 4 inches wide, and 12 feet long.

**To "prove" a problem in multiplication:** Divide the product by the multiplier. The quotient should equal the multiplicand.

NOTE. Besides those problems in which either the multiplicand or the multiplier is 1, and which require no effort on the part of the pupil beyond learning to count, there are 64 primary facts of multiplication that must be perfectly memorized before the pupil can acquire facility in the process. They are as follows:

2 times 2 = 4
2 times 3 = 6
2 times 4 = 8
2 times 5 = 10
2 times 6 = 12
2 times 7 = 14
2 times 8 = 16
2 times 9 = 18

3 times 2 = 6
3 times 3 = 9
3 times 4 = 12
3 times 5 = 15
3 times 6 = 18
3 times 7 = 21
3 times 8 = 24
3 times 9 = 27

4 times 2 = 8
4 times 3 = 12
4 times 4 = 16
4 times 5 = 20
4 times 6 = 24
4 times 7 = 28
4 times 8 = 32
4 times 9 = 36

5 times 2 = 10
5 times 3 = 15
5 times 4 = 20
5 times 5 = 25
5 times 6 = 30
5 times 7 = 35
5 times 8 = 40
5 times 9 = 45

6 times 2 = 12
6 times 3 = 18
6 times 4 = 24
6 times 5 = 30
6 times 6 = 36
6 times 7 = 42
6 times 8 = 48
6 times 9 = 54

7 times 2 = 14
7 times 3 = 21
7 times 4 = 28
7 times 5 = 35
7 times 6 = 42
7 times 7 = 49
7 times 8 = 56
7 times 9 = 63

8 times 2 = 16
8 times 3 = 24
8 times 4 = 32
8 times 5 = 40
8 times 6 = 48
8 times 7 = 56
8 times 8 = 64
8 times 9 = 72

9 times 2 = 18
9 times 3 = 27
9 times 4 = 36
9 times 5 = 45
9 times 6 = 54
9 times 7 = 63
9 times 8 = 72
9 times 9 = 81

Although a knowledge of the "elevens" and "twelves" of the table as it is usually given is convenient and helpful, it will be observed that it is nòt a necessity in the process of multiplication. The pupil who masters the table as given above will have in his mental possession the fundamental facts of simple multiplication.

11. **Division** is one of two processes:

(1) It is finding how many times one number is contained in another number. $18 ÷ $2 = 9.

(2) It is finding one of the equal parts of a number. $18 ÷ 2 = $9.

The **dividend** is the number divided.

The **divisor** is the number by which we divide.

The **quotient** is the number obtained by dividing.

The **sign**, ÷, which is read *divided by*, has two meanings in concrete problems, which correspond to the two cases given above. In Case I, it means, *Find how many times the divisor is contained in the dividend.* In Case II, it means, *Separate the dividend into as many equal parts as there are units in the divisor and note the size of the parts.*

$18 ÷ $2, means, *Find how many times $2 are contained in* $18.

$18 ÷ 2, means, *Find* 1 *half of* $18.

NOTE. The two cases of division are ultimately resolvable into one. But the teacher will find great difficulty, who attempts to make clear to young students the fact that we find one fifth of 15 apples, for instance, by finding how many times 5 apples are contained in 15 apples. "One fifth of 15 apples is as many apples as 5 apples are contained times in 15 apples. 5 apples are contained in 15 apples 3 times; therefore, one fifth of 15 apples is 3 apples." A formal analysis of this kind thrust upon fifth or sixth grade students, with a demand for its reproduction, might be fatal to clear thinking on the part of the student. The author experimented along this line many years ago to his serious discomfiture.

To "prove" a problem in division, multiply the quotient by the divisor. The product thus obtained should equal the dividend.

NOTE 1. There can be no "remainder" in a complete division. Therefore, the product of the quotient and divisor is *always* equal to the dividend. We may have a remainder when the process of dividing is incomplete. In such a case, the quotient, too, is incomplete, and the remainder is the undivided part of the dividend. To the product of the *incomplete quotient* and divisor, add the remainder, and the sum thus obtained will be equal to the dividend.

NOTE 2. Besides the division of a number by a unit, there are 64 primary facts of division to be memorized by the pupil. These are the converse of the 64 facts of multiplication given on pages 259 and 260. Little effort is required for their mastery beyond what must be expended in gaining a working familiarity with the facts of multiplication. The facts of division, however, must receive some attention and must be presented in their double aspect. The pupil must know that because 3 times 5 are 15 (11111 11111 11111), 5 is contained in 15 three times and one third of 15 is 5.

**12.** A **fractional number** is a number of the equal parts of some quantity considered as a unit; as $\frac{3}{4}$, .7.

**13.** An **integral number**, or an **integer**, is a number which is not, either wholly or in part, a fractional number; as, 27, 13, 246.

**14.** A **mixed number** is a number one part of which is integral and the other part fractional; as, $3\frac{2}{5}$, 7.3.

**15.** An **exact divisor** of a number is a number that is contained in the number an integral number of times.

3 is an exact divisor of 12, but not of 1.2.
$12\frac{1}{2}$ is an exact divisor of 100.

**16.** An **even number** is an integral number of which two is an exact divisor; as, 8, 36, 94.

An integral number is exactly divisible by two if its right-hand figure is 0, 2, 4, 6, or 8.

**17.** An **odd number** is an integral number of which two is not an exact divisor; as, 11, 15, 75.

An integral number is not exactly divisible by two if its right-hand figure is 1, 3, 5, 7, or 9.

**18.** A **prime number** is an integral number that has no exact integral divisors except itself and 1; as, 1, 2, 3, 5, 7, 11, 13, 17, 19, etc.

**19.** A **composite number** is an integral number that is not prime. It has one or more exact integral divisors besides itself and 1; as, 4, 6, 8, 9, 10, 12, 75, etc.

**20.** Any divisor of a number may be regarded as a **factor** of the number. An exact integral divisor of a number is an **integral factor** of the number.

3 is an integral factor of 12.
5 is an integral factor of 20.

**21.** An integral factor that is itself a prime number is called a **prime factor.**

3 is a prime factor of 12.
7 is a prime factor of 35.

**22.** An integral factor that is itself a composite number is called a **composite factor.**

6 is a composite factor of 18.
12 is a composite factor of 36.

**23.** *Every composite number is equal to the product of its prime factors.*

$18 = 2 \times 3 \times 3.$ $\qquad 24 = 2 \times 2 \times 2 \times 3.$

NOTE. A factor of a number may be integral, fractional, or mixed. $a$ multiplied by $b = ab$. $a$ and $b$ are factors of $ab$. If $a = 2\frac{1}{2}$, and $b = 2$, then $ab = 5$, and $2\frac{1}{2}$ and 2 are factors of 5. All *prime* factors are integral since all prime numbers are integral.

**24.** A **multiple** of a number is a number that is exactly divisible by that number.

28 is a multiple of 7. It is exactly divisible by 7.
28 is a multiple of 4 and of 2.

**25.** *A multiple of an integral number contains all the prime factors of that number and some other integral factor or factors.*

60 ($2 \times 2 \times 3 \times 5$) is a multiple of 12 ($2 \times 2 \times 3$). It contains all the prime factors of 12, and, besides these, the factor 5. 12 is contained in 60, 5 times.

**26.** A **common multiple** of two or more numbers is a number that is exactly divisible by each of them.

24 and 48 and 72 are common multiples of 8 and 12. Each of these is exactly divisible by 8 and by 12.

A common multiple of two or more integral numbers contains all the prime factors of each of them, and *may contain other prime factors.*

$$420 = 2 \times 2 \times 3 \times 5 \times 7.$$
$$12 = 2 \times 2 \times 3.$$
$$10 = 2 \times 5.$$

420 is a common multiple of 10 and 12. It contains all the prime factors of each of them, and besides these, the prime factor 7.

**27.** The **least common multiple** (l. c. m.) of two or more numbers is the least number that is exactly divisible by each of them.

40 and 80 and 120 are common multiples of 8 and 10. 40 is the least common multiple of 8 and 10.

The least common multiple of two or more integral numbers contains all the prime factors of each of them, *and no other prime factors.*

$60 = 2 \times 2 \times 3 \times 5.$
$20 = 2 \times 2 \times 5.$
$15 = 3 \times 5.$

60 is the l. c. m. of 20 and 15. It contains as prime factors as many 2's, as many 3's, and as many 5's, as either of the numbers. *It contains no other prime factors.*

**28.** A **common divisor** of two or more numbers is any exact divisor of the numbers.

2 is a common divisor of 24 and 32.
4 is a common divisor of 24 and 32.
8 is a common divisor of 24 and 32.

This term as used in arithmetic is usually applied to integral divisors. There is no inaccuracy, however, in applying it to fractional divisors; thus, ½ is a common divisor of 2½ and 7½; 12½ is a common divisor of 25 and 37½.

An integral common divisor of two or more numbers is a prime factor, or the product of prime factors, common to all the numbers.

$24 = 2 \times 2 \times 2 \times 3.$
$36 = 2 \times 2 \times 3 \times 3.$

2 is a common divisor of 24 and 36.
4 (2 × 2) is a common divisor of 24 and 36.
6 (2 × 3) is a common divisor of 24 and 36.
12 (2 × 2 × 3) is a common divisor of 24 and 26.

**29.** The **greatest common divisor** (g. c. d.) of two or more numbers is the highest number that will exactly divide all the numbers.

3 is a common divisor of 12 and 18.
6 is a common divisor of 12 and 18.
6 is the greatest common divisor of 12 and 18.

The greatest common divisor of two or more integral numbers is the product of all the prime factors common to all the numbers.

$$18 = 2 \times 3 \times 3.$$
$$24 = 2 \times 2 \times 2 \times 3.$$

2 and 3 are all the prime factors common to 18 and 24; hence 6 (2 × 3) is the g. c. d. of 18 and 24.

**30.** Two or more integral numbers are said to be **prime to each other** when they have no prime factor in common.

8 (2 × 2 × 2), 9(3 × 3), and 25 (5 × 5) are prime to each other.

When two or more numbers are prime to each other, their least common multiple is their product. The l. c. m. of 8 and 9 is 72. The l. c. m. of 5, 7, and 4 is (5 × 7 × 4) = 140.

**31.** A number that is joined to the name of a standard of measurement is said to be a **concrete number.**

NOTE. Neither *four feet*, *six hours*, nor *eight apples* is a concrete number; the *four*, *six*, and *eight* of these expressions are concrete numbers. If eight apples is a concrete number, then a red apple is a concrete color!

**32.** A number that is not joined to the name of a standard of measurement, as *four*, *five*, *eight*, *ten*, is said to be an **abstract number.**

NOTE 1. An abstract number may *suggest* simply a ratio, or it may suggest both a ratio and a measured magnitude. In the expression, 8 feet ÷ 2 feet = 4, the four suggests pure number, *i.e.*, simply a ratio — the ratio of the magnitude 8 feet to the magnitude 2 feet. The expression 6 + 5 = 11, *may* suggest that 6 inches increased by 5 inches is equal to 11 inches. One may also think the 6 or the 5 or the 11 as the ratio of a certain magnitude to the standard of measurement.

NOTE 2. Strictly speaking, number is ratio and, from its very nature, is always abstract. But the expressions, "*concrete number*," "*denominate number*," etc., have been accorded a place in the language. They are convenient terms to express *certain uses of abstract number* and hence are freely employed on the pages of this book.

**33.** The expression **denominate numbers** is applied to those concrete numbers used in exact measurements.

The six of the expression 6 quarts, the four of the expression 4 inches, are denominate numbers.

**34.** A **simple denominate number** is a denominate number whose units are of one kind. A **compound denominate number** is a denominate number whose units are of two or more kinds.

The twenty-seven of 27 feet is a simple denominate number.

The two and the six of the expression 2 ft. 6 in. together constitute a compound denominate number.

**35.** **Reduction** in arithmetic is the changing of a number of units of one denomination to the equivalent number of units of some other denomination.

Inches may be reduced to feet or feet to inches; pints to quarts or quarts to pints, etc.

**36.** A **fraction** is one or more of the equal parts of a unit.

Seven eighths is a fraction.
Five twelfths is a fraction.

**37.** A fraction may be expressed by two numbers, one of them being written above a short horizontal line and the other below it; thus, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{14}{19}$.

The number above the line is called the **numerator.**

The number below the line is called the **denominator.**

The denominator shows the number of parts into which the unit is divided.

The numerator shows the number of parts taken.

NOTE. The numerator tells the number of things, and the denominator gives their name. In the fraction $\frac{4}{5}$ there are 4 things called fifths.

**38.** A fraction whose numerator is less than its denominator is called a **proper fraction.**

$\frac{3}{4}$, $\frac{5}{11}$, $\frac{1}{8}$, are proper fractions.

**39.** A fraction whose numerator is equal to or greater than its denominator is called an **improper fraction.**

$\frac{8}{8}$, $\frac{9}{8}$, $\frac{11}{8}$, are improper fractions.

**40.** Two or more fractions whose denominators are alike are said to have a **common denominator.**

$\frac{7}{12}$, $\frac{5}{12}$, $\frac{3}{12}$, and $\frac{1}{12}$, have a common denominator.
$\frac{8}{11}$, $\frac{5}{14}$, $\frac{9}{16}$, and $\frac{1}{21}$, do not have a common denominator.

**41.** The **terms of a fraction** are its numerator and its denominator. A fraction is said to be in its **lowest terms** when its numerator and its denominator are integral numbers that are prime to each other.

The terms of $\frac{3}{5}$ are 3 and 5. 3 and 5 are integral numbers that are prime to each other; therefore the fraction is in its lowest terms.

**42.** Such expressions as the following are sometimes called **compound fractions:**

$\frac{2}{3}$ of $\frac{4}{5}$, $\frac{5}{6}$ of $\frac{7}{8}$, $\frac{3}{4}$ of $2\frac{1}{2}$.

**43.** A fraction either of whose terms is itself a fraction or a mixed number is called a **complex fraction.**

$\frac{\frac{2}{3}}{4}$, $\frac{3\frac{1}{2}}{3}$, $\frac{\frac{2}{3}}{\frac{3}{4}}$, are complex fractions.

**44.** Any fraction that is neither compound nor complex is called a **simple fraction.**

**$\frac{5}{7}$, $\frac{3}{8}$, and $\frac{27}{30}$, are simple fractions.**

**45.** A fraction whose denominator is 1 with one or more zeros annexed to it is a **decimal fraction.**

**$\frac{3}{10}$, $\frac{7}{1000}$, $\frac{33}{10000}$, are decimal fractions.**

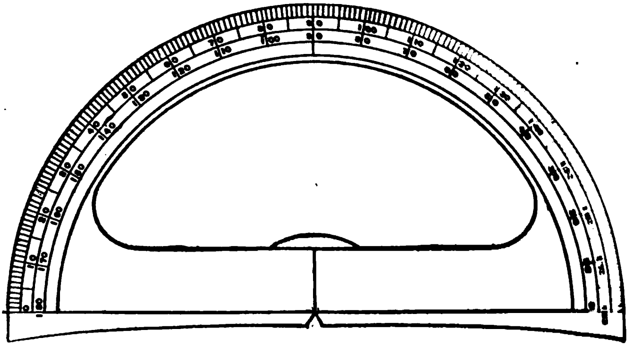
**46.** When a decimal fraction is expressed by figures, the denominator is not usually written. It is indicated by a decimal point; thus, $\frac{7}{10}$ is written, .7; $\frac{8}{100}$ is written, .08, etc. When so written, the expression may be called a **decimal.**

**47.** The **denominator of a decimal** is 1 with as many zeros annexed to it as there are decimal places; thus, the denominator of .8 is 1 with one zero annexed to it, — that is, 10; the denominator of .027 is 1 with three zeros annexed to it, — that is, 1000.

**48.** A **complex decimal** is a decimal and a common fraction combined in one number.

**$.7\frac{1}{3}$, $.25\frac{1}{7}$, $.324\frac{2}{3}$, are complex decimals.**

## PROTRACTOR



Carefully paste this sheet upon cardboard; then cut out the protractor with a sharp knife and preserve it for use in measuring and in constructing angles.